Practice Proficiency Worked Solutions

1. $F(x)=\sin(x^2)$

C $\frac{dF}{dx} = \cos(x^2)\frac{d}{dx}[x^2] = 2x\cos(x^2)$

2. $F(x) = e^{4x-5}$

D $\frac{dF}{dx} = e^{4x-5}\frac{d}{dx}[4x-5] = 4e^{4x-5}$

3. $F(x)=\ln(x^2-6x+8)$

E $\frac{dF}{dx} = \frac{1}{x^2-6x+8}\frac{d}{dx}[x^2-6x+8] = \frac{2x-6}{x^2-6x+8}$

4. $F(x) = x^3\tan x$

E $\frac{dF}{dx} = x^3\frac{d}{dx}\tan x + \frac{d}{dx}[x^3]\tan x = x^3\sec^2 x + 3x^2\tan x$

5. $F(x) = (4x^2+9)^{3/2}$

C $\frac{dF}{dx} = \frac{3}{2}(4x^2+9)^{1/2}\frac{d}{dx}(4x^2+9) = 8x\left[\frac{3}{2}(4x^2+9)^{1/2}\right] = 12x\sqrt{4x^2+9}$

6. $F(x) = \sin^{-1}(4x)$

B $\frac{dF}{dx} = \frac{1}{\sqrt{1-(4x)^2}}\frac{d}{dx}(4x) = \frac{4}{\sqrt{1-16x^2}}$

7. $F(x) = \frac{3x-2}{x^2-4x+5}$

B $\frac{dF}{dx} = \frac{\left[\frac{d}{dx}(3x-2)\right](x^2-4x+5) - (3x-2)\frac{d}{dx}[x^2-4x+5]}{(x^2-4x+5)^2}$

$= \frac{3(x^2-4x+5) - (3x-2)(2x-4)}{(x^2-4x+5)^2} = \frac{3x^2-12x+15-[6x^2-16x+8]}{(x^2-4x+5)^2}$

$= \frac{-3x^2+4x+7}{(x^2-4x+5)^2}$

key

1	C	8	B
2	D	9	B
3	E	10	D
4	E	11	D
5	C	12	E
6	B	13	B
7	B	14	F
		15	D

8. $\lim_{x\to 6} \frac{x^2-8x+12}{x^2-3x-18} \approx \frac{0}{0}$

L'Hôpital's rule

B $\lim_{x\to 6} \frac{x^2-8x+12}{x^2-3x-18} = \lim_{x\to 6} \frac{2x-8}{2x-3} = \frac{2x-8}{2x-3}\Big|_{x=6} = \frac{4}{9}$

or $\lim_{x\to 6} \frac{x^2-8x+12}{x^2-3x-18} = \lim_{x\to 6} \frac{(x-6)(x-2)}{(x-6)(x+3)} = \frac{(x-2)}{(x+3)}\Big|_{x=6} = \frac{4}{9}$

9. $\lim_{x\to\infty} \frac{3x^2-7x-2}{9x^2+4x+2} \approx \frac{\infty}{\infty}$

L'Hôpital's Rule

$\lim_{x\to\infty} \frac{x^2-8x+12}{x^2-3x-18} \frac{3x^2-7x-2}{9x^2+4x+2} = \lim_{x\to\infty} \frac{6x-7}{18x+4} \approx \frac{\infty}{\infty}$

B L'Hôpital's Rule

$\lim_{x\to\infty} \frac{6x-7}{18x+4} = \frac{6}{18} = \frac{1}{3}$

10. $y = e^x \quad x = 2.$

D $\frac{dy}{dx} = e^x$

$m = \frac{dy}{dx}\Big|_{x=2} = e^2$

$y = e^2x + b$

$b = e^2 - 2e^2 = -e^2$

$y = e^2x - e^2$

11. $y = x^3 \quad x = 2$

D $\frac{dy}{dx} = 3x^2$

$m = \frac{dy}{dx}\Big|_{x=2} = 12$

$y = 12x + b$

$b = 8 - 24 = -16$

$y = 12x - 16$

12. E

13. B

14. F (CBA)

15. 1) $G'(\frac{3}{2}) = 1$ F

2) $\lim_{x\to 1^-} G(x) = 1$

$\lim_{x\to 1^+} G(x) = 0$

D $1 \neq 0$ limit Does not exist

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3) ~~Let~~ If $G(3)$ were zero instead of 1 function would be continuous there. T