Maple Notes on Smith-Minton Ch. 4 Integration

4.1 Area under a curve

4.1 Homework Problems 5,6,9,13,14,22,29,37,43,49-52, exploratory ex. 1

5.

```
> seq(3*i^2,i=2..6);sum(3*i^2,i=2..6);3*(6*(6+1)*(2*6+1))/6-3;
```

12, 27, 48, 75, 108

270

270

TI-86 calculator syntax: sum(seq(3*i^2,i,2,6))

6.

```
> seq(2*i-1,i=1..5);sum(2*i-1,i=1..5);2*5*6/2-5;
```

1, 3, 5, 7, 9

25

25

TI-86 calculator syntax: sum(seq(2*i-1,i,1,5))

9.

```
> seq(3*i-1,i=1..70);sum(3*i-1,i=1..70);70*71/2*3-70;
```

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 56, 59, 62, 65, 68, 71, 74, 77, 80, 83, 86, 89, 92, 95, 98, 101, 104, 107, 110, 113, 116, 119, 122, 125, 128, 131, 134, 137, 140, 143, 146, 149, 152, 155, 158, 161, 164, 167, 170, 173, 176, 179, 182, 185, 188, 191, 194, 197, 200, 203, 206, 209

7385

7385

TI-86 calculator syntax: summing sequence term by term: sum(seq(3*i-1,i,1,70)) or using summation rules: 70*71/2*3-70

13.

```
> seq(i^2-3*i+2,i=1..100);sum(i^2-3*i+2,i=1..100);100*(100+1)*(2*100
  +1)/6-3*100*(100+1)/2+2*100;
```

0, 0, 2, 6, 12, 20, 30, 42, 56, 72, 90, 110, 132, 156, 182, 210, 240, 272, 306, 342, 380, 420, 462, 506, 552, 600, 650, 702, 756, 812, 870, 930, 992, 1056, 1122, 1190, 1260, 1332, 1406, 1482, 1560, 1640, 1722, 1806, 1892, 1980, 2070, 2162, 2256, 2352, 2450, 2550, 2652, 2756, 2862, 2970, 3080, 3192, 3306, 3422, 3540, 3660, 3782, 3906, 4032, 4160, 4290, 4422, 4556, 4692, 4830, 4970, 5112, 5256, 5402, 5550, 5700, 5852, 6006, 6162, 6320, 6480, 6642, 6806, 6972, 7140, 7310, 7482, 7656, 7832, 8010, 8190, 8372, 8556, 8742, 8930, 9120, 9312, 9506, 9702

323400

323400

TI-86 calculator syntax: umming sequence term by term: sum(seq(i^2-3*i+2,i,1,100)) or using summation rules 100*(100+1)*(2*100+1)/6-3*100*(100+1)/2+2*100

14

```
> seq(i^2+2*i-4,i=1..140);sum(i^2+2*i-4,i=1..140);140*(140+1)*(2*140
  +1)/6+2*140*(140+1)/2-4*140;
```

-1, 4, 11, 20, 31, 44, 59, 76, 95, 116, 139, 164, 191, 220, 251, 284, 319, 356, 395, 436, 479, 524, 571, 620, 671, 724, 779, 836, 895, 956, 1019, 1084, 1151, 1220, 1291, 1364, 1439, 1516, 1595, 1676, 1759, 1844, 1931, 2020, 2111, 2204, 2299, 2396, 2495, 2596, 2699, 2804, 2911, 3020, 3131, 3244, 3359, 3476, 3595, 3716, 3839, 3964, 4091, 4220, 4351, 4484, 4619, 4756, 4895, 5036, 5179, 5324, 5471, 5620, 5771, 5924, 6079, 6236, 6395, 6556, 6719, 6884, 7051, 7220, 7391, 7564, 7739, 7916, 8095, 8276, 8459, 8644, 8831, 9020, 9211, 9404, 9599, 9796, 9995, 10196, 10399, 10604, 10811, 11020, 11231, 11444, 11659, 11876, 12095, 12316, 12539, 12764, 12991, 13220, 13451, 13684, 13919, 14156, 14395, 14636, 14879, 15124, 15371, 15620, 15871, 16124, 16379, 16636, 16895, 17156, 17419, 17684, 17951, 18220, 18491, 18764, 19039, 19316, 19595, 19876

943670

943670

TI-86 calculator syntax: summing sequence term by term: sum(seq(i^2+2*i-4,i,1,140)) or using summation rules 140*(140+1)*(2*140+1)/6+2*140*(140+1)/2-4*140

22

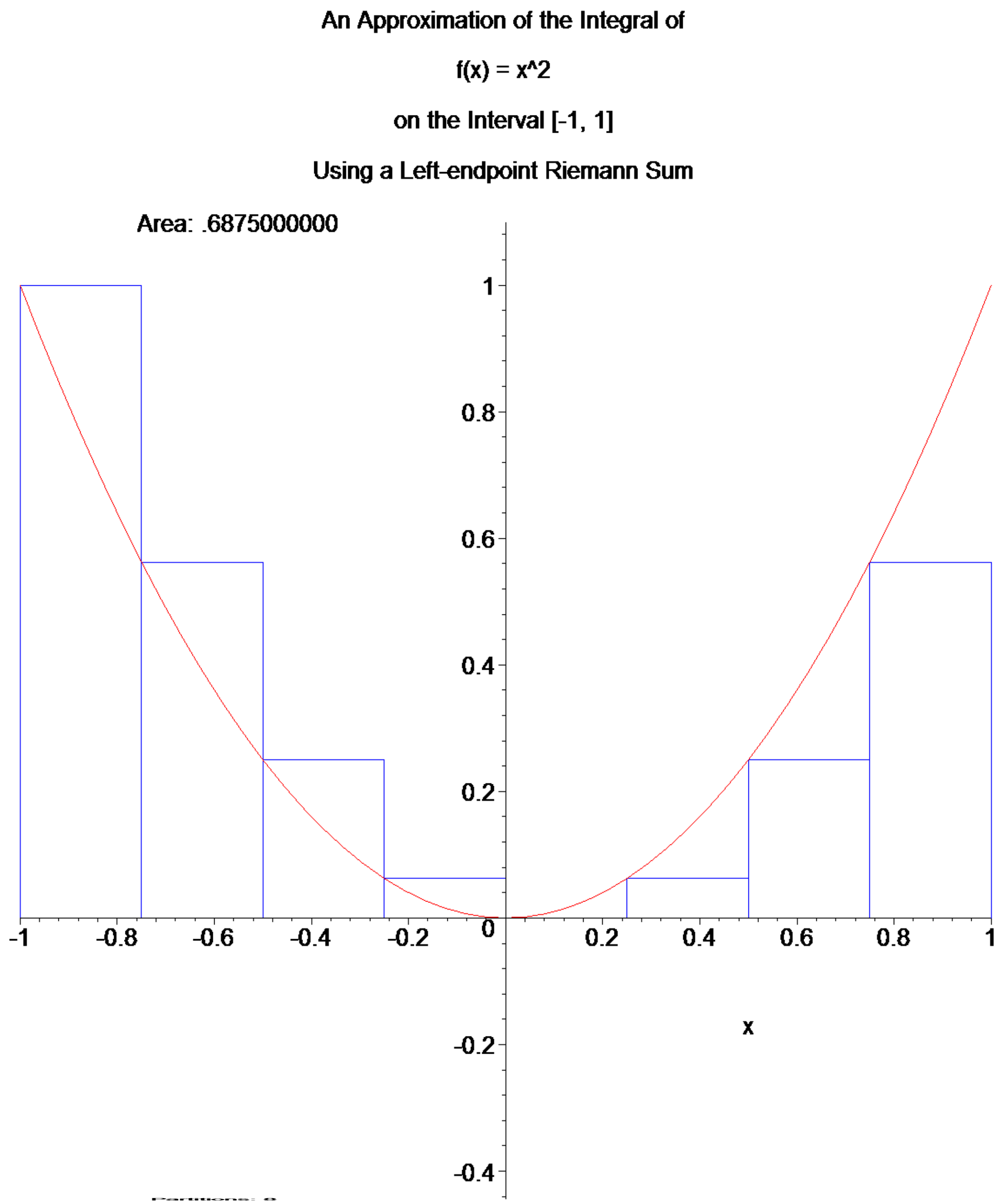
```
> 12*20/60+14*30/60+18*10/60+15*40/60;
```

24

The runner will have travelled 24 miles.

29

```
> ApproximateInt(x^2, -1..1, output=animation, partition=8,
  refinement=halve, subpartition=width,
  method=left,showpoints=false);
```



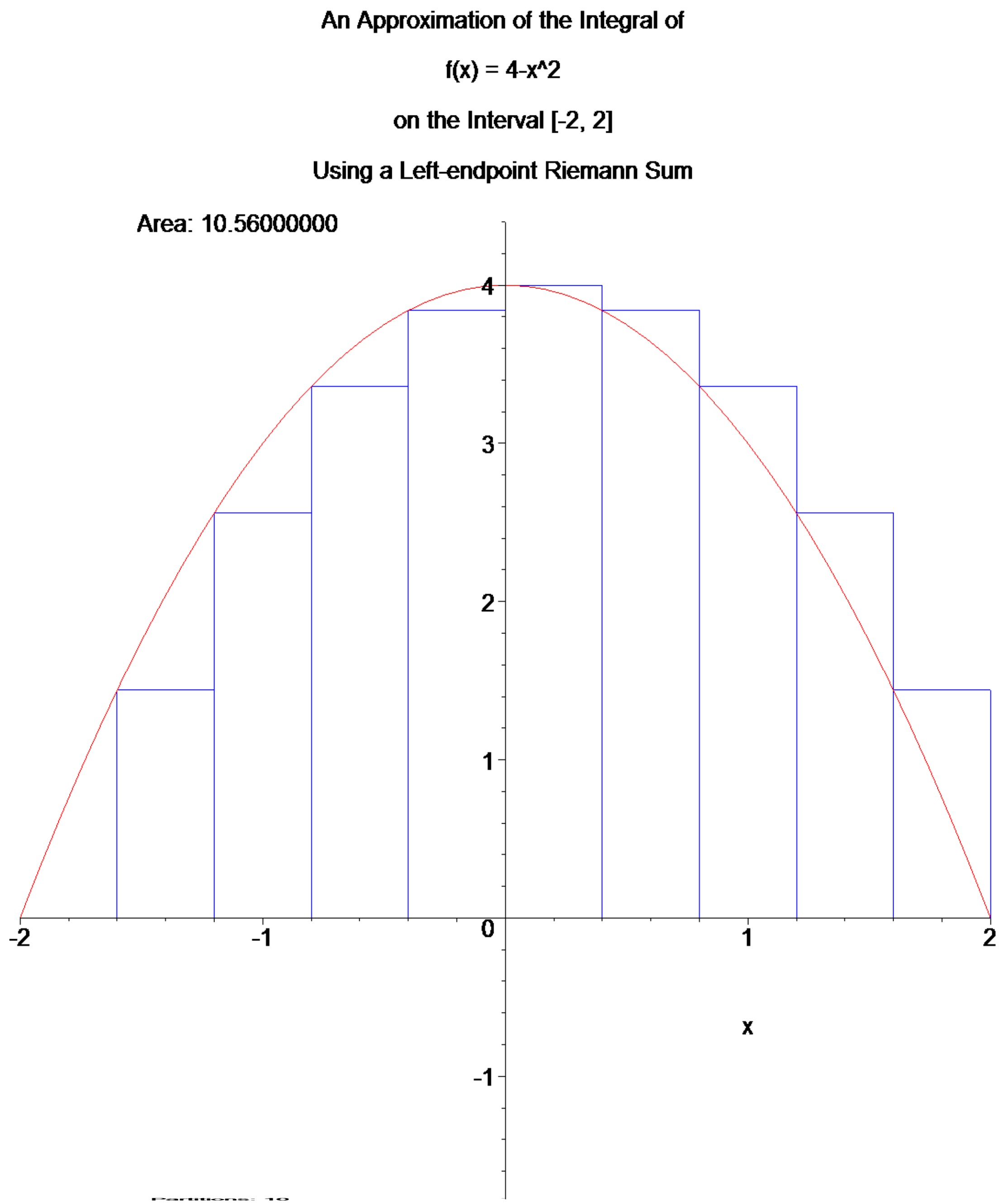

```
> sum((-1.+2*i/8)^2,i=0..7)*2/8;
```

0.6875000000

TI-86 calculator syntax: summing sequence term by term: sum(seq((-1.+2*i/8)^2,i,0,7))*2/8

37

```
> ApproximateInt(4-x^2, -2..2, output=animation, partition=10,
  refinement=halve, subpartition=width,
  method=left,showpoints=false);
```



Left-hand endpoint method

```
> mylist:=[10,50,100,500,5000]:lmeth:=seq(subs(n=mylist[j],sum(4-(-2.+4*i/n)^2,i=1..n)*4/n),j=1..5);
```

lmeth := 10.56000000, 10.66240000, 10.66560000, 10.66662400, 10.66666625

Right-hand endpoint method

```
> rmeth:=seq(subs(n=mylist[j],sum(4-(-2.+4*i/n)^2,i=0..n-1)*4/n),j=1..5);
```

rmeth := 10.56000000, 10.66240001, 10.66560000, 10.66662401, 10.66666625

Midpoint method

```
> mmeth:=seq(subs(n=mylist[j],sum(4-(-2.+4*i/n-2/n)^2,i=1..n)*4/n),j=1..5);
```

$$mmeth := 10.72000000, 10.66880000, 10.66720000, 10.66668800, 10.66666689$$

```
> for m from 1 to 5 do
  mytable[m]:=[mylist[m],lmeth[m],rmeth[m],mmeth[m]];od;
```

$$mytable_1 := [10, 10.56000000, 10.56000000, 10.72000000]$$

$$mytable_2 := [50, 10.66240000, 10.66240001, 10.66880000]$$

$$mytable_3 := [100, 10.66560000, 10.66560000, 10.66720000]$$

$$mytable_4 := [500, 10.66662400, 10.66662401, 10.66668800]$$

$$mytable_5 := [5000, 10.66666625, 10.66666625, 10.66666689]$$

TI-86 calculator syntax: summing sequence term by term: sum(seq(4-(-2.+4*i/10)^2,i,1,10))*4/10 for left hand endpoint method n=10; x[i]=-2.+4*i/10. Syntax is similar for other table entries.

43,

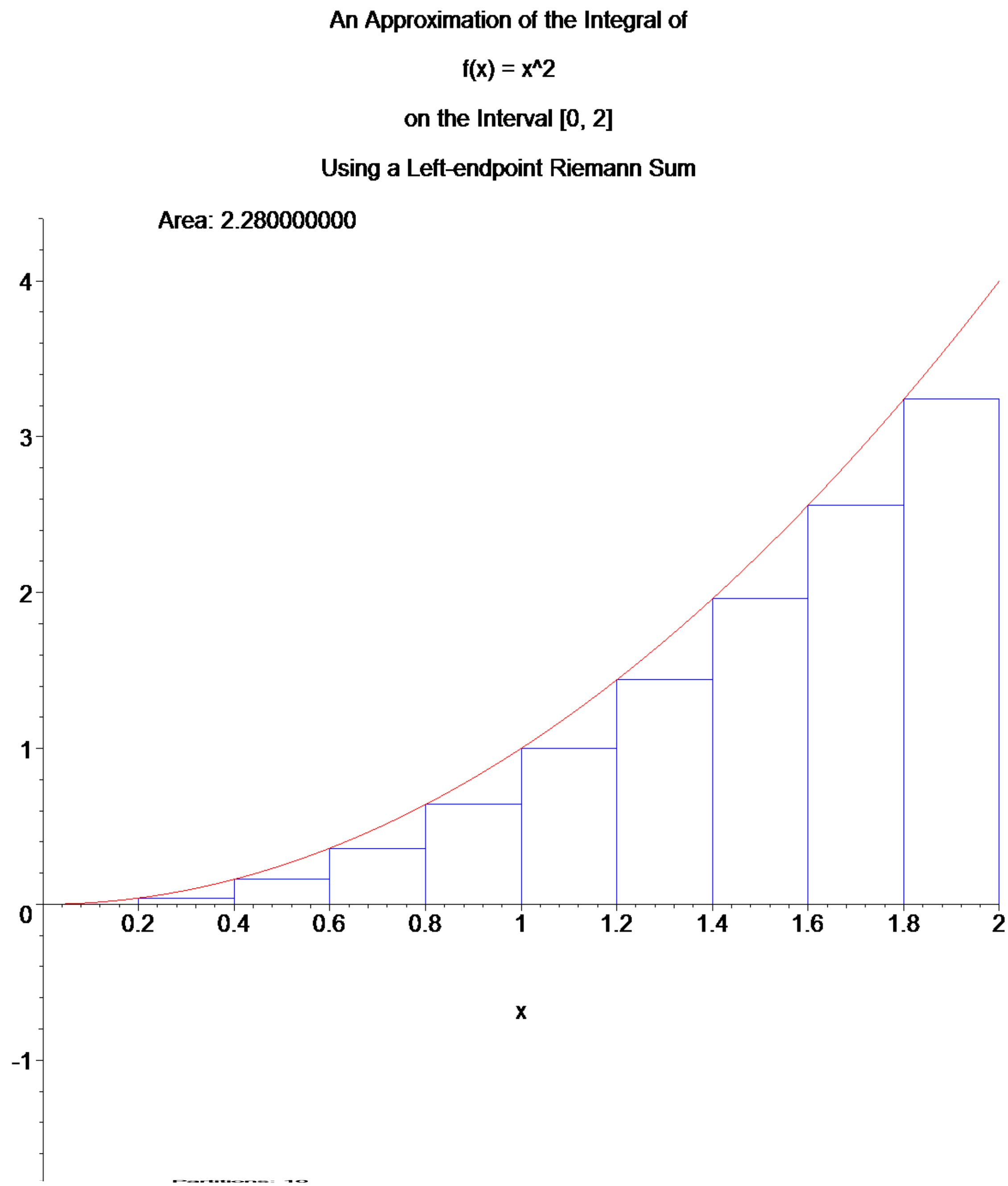
```
> (2+2.4+2.6+2.7+2.6+2.4+2+1.4)/10;(2.4+2.6+2.7+2.6+2.4+2+1.4+.6)/10
  ;
```

$$1.810000000$$

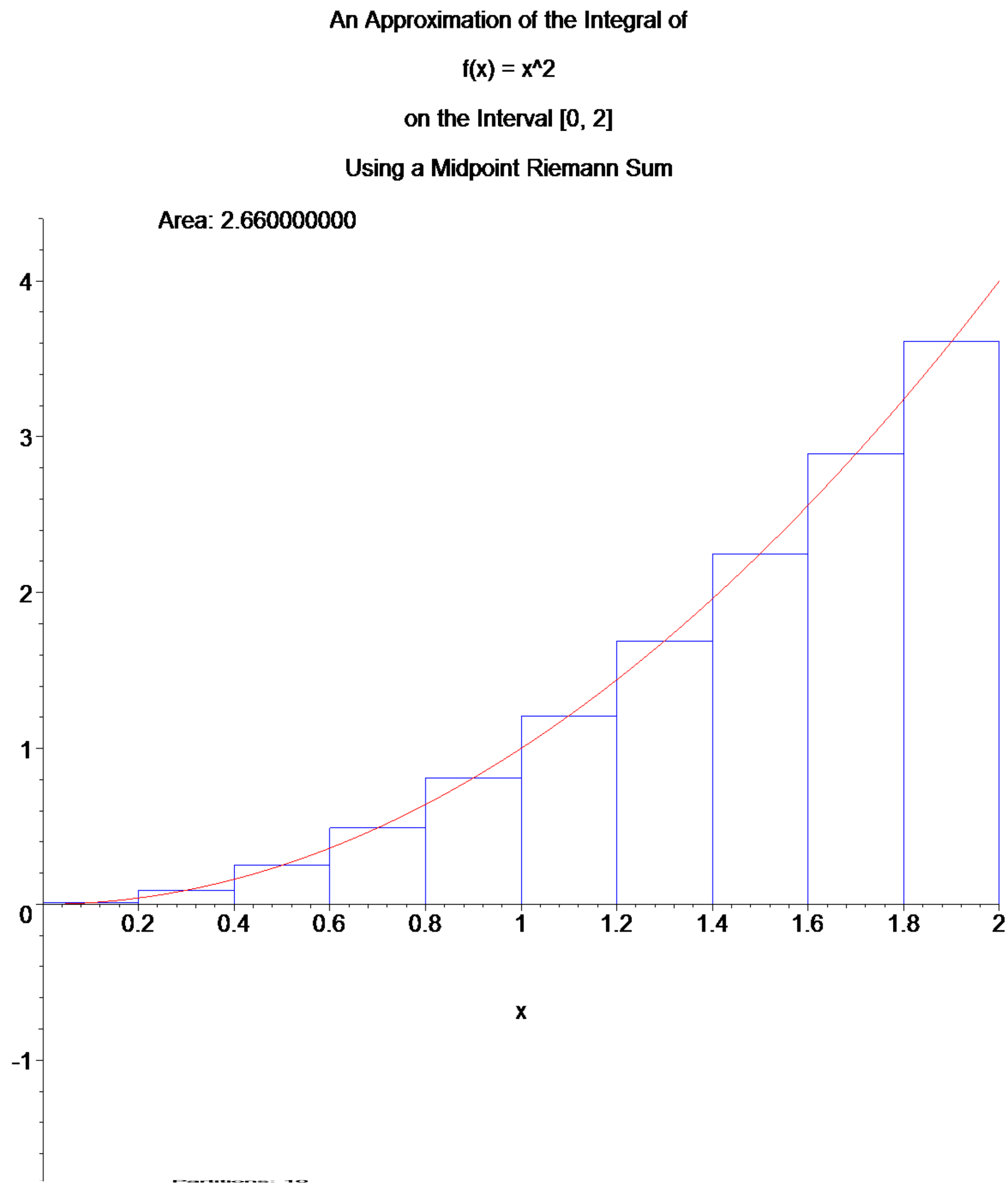
$$1.670000000$$

49-52

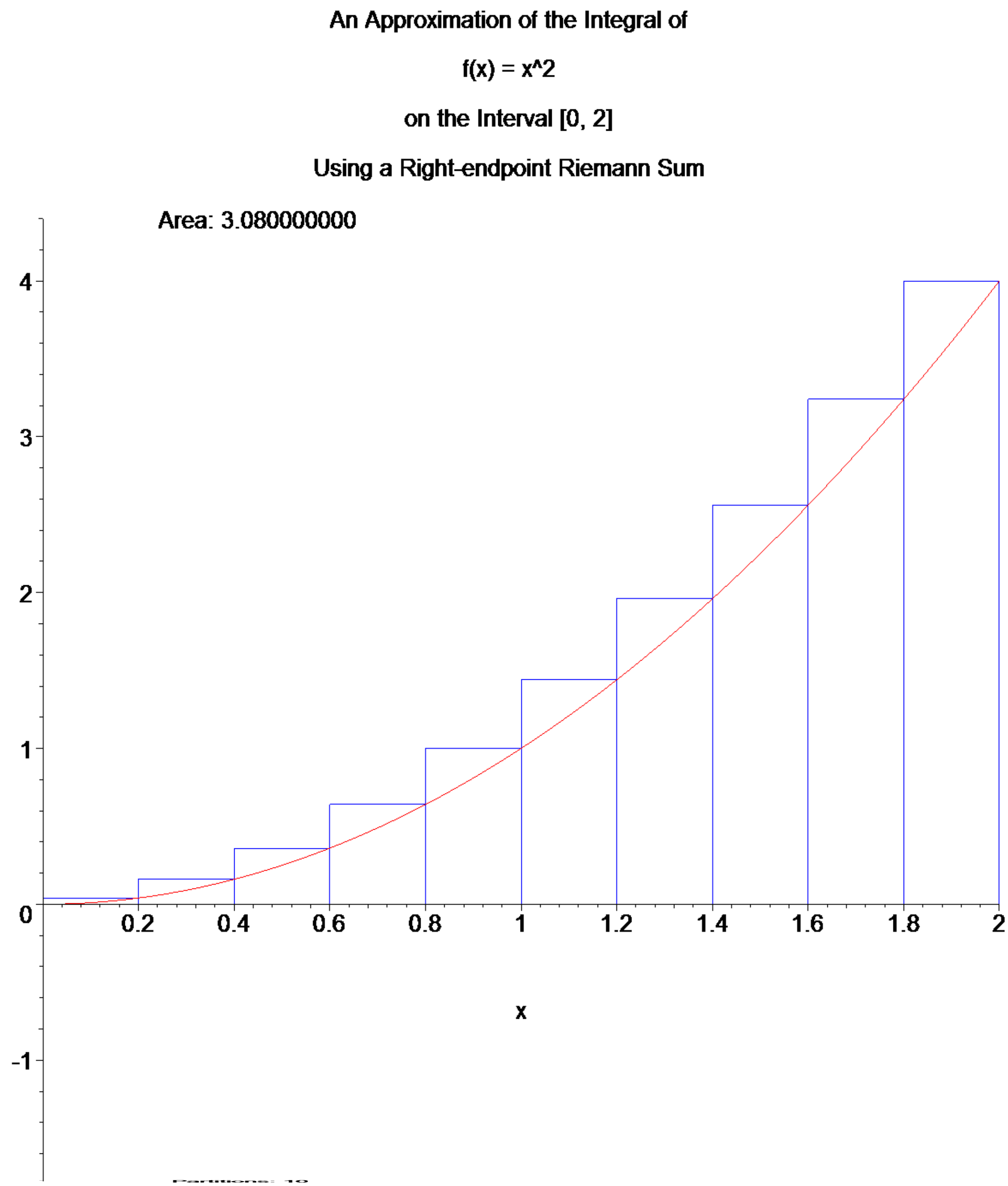
```
> ApproximateInt(x^2, 0..2, output=animation, partition=10,
  refinement=halve, subpartition=width,
  method=left,showpoints=false);
  lower;ApproximateInt(x^2, 0..2, output=animation, partition=10,
  refinement=halve, subpartition=width,
  method=midpoint,showpoints=false);
  lower;ApproximateInt(x^2, 0..2, output=animation, partition=10,
  refinement=halve, subpartition=width,
  method=right,showpoints=false);greater;
```


An Approximation of the Integral of
f(x) = x^2
on the Interval [0, 2]
Using a Left-endpoint Riemann Sum
Area: 2.280000000
4
3
2
1
0
-1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x

lower


An Approximation of the Integral of
f(x) = x^2
on the Interval [0, 2]
Using a Midpoint Riemann Sum
Area: 2.660000000
4
3
2
1
0
-1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x

lower



greater

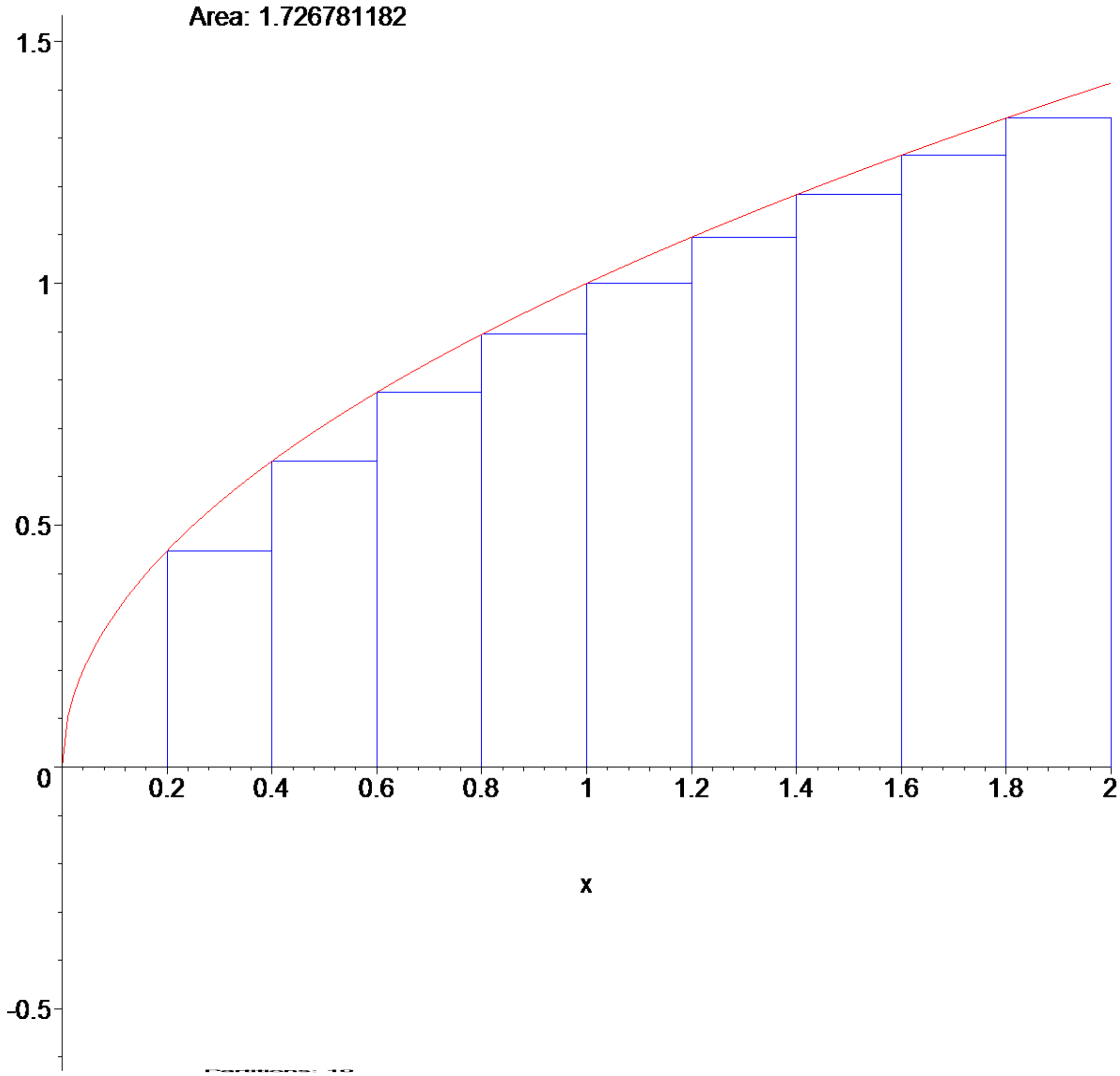
```
> ApproximateInt(x^(1/2), 0..2, output=animation, partition=10,
  refinement=halve, subpartition=width,
  method=left,showpoints=false);
  lower;ApproximateInt(x^(1/2), 0..2, output=animation,
  partition=10, refinement=halve, subpartition=width,
  method=midpoint,showpoints=false);
  greater;ApproximateInt(x^(1/2), 0..2, output=animation,
  partition=10, refinement=halve, subpartition=width,
  method=right,showpoints=false);greater;
```

An Approximation of the Integral of

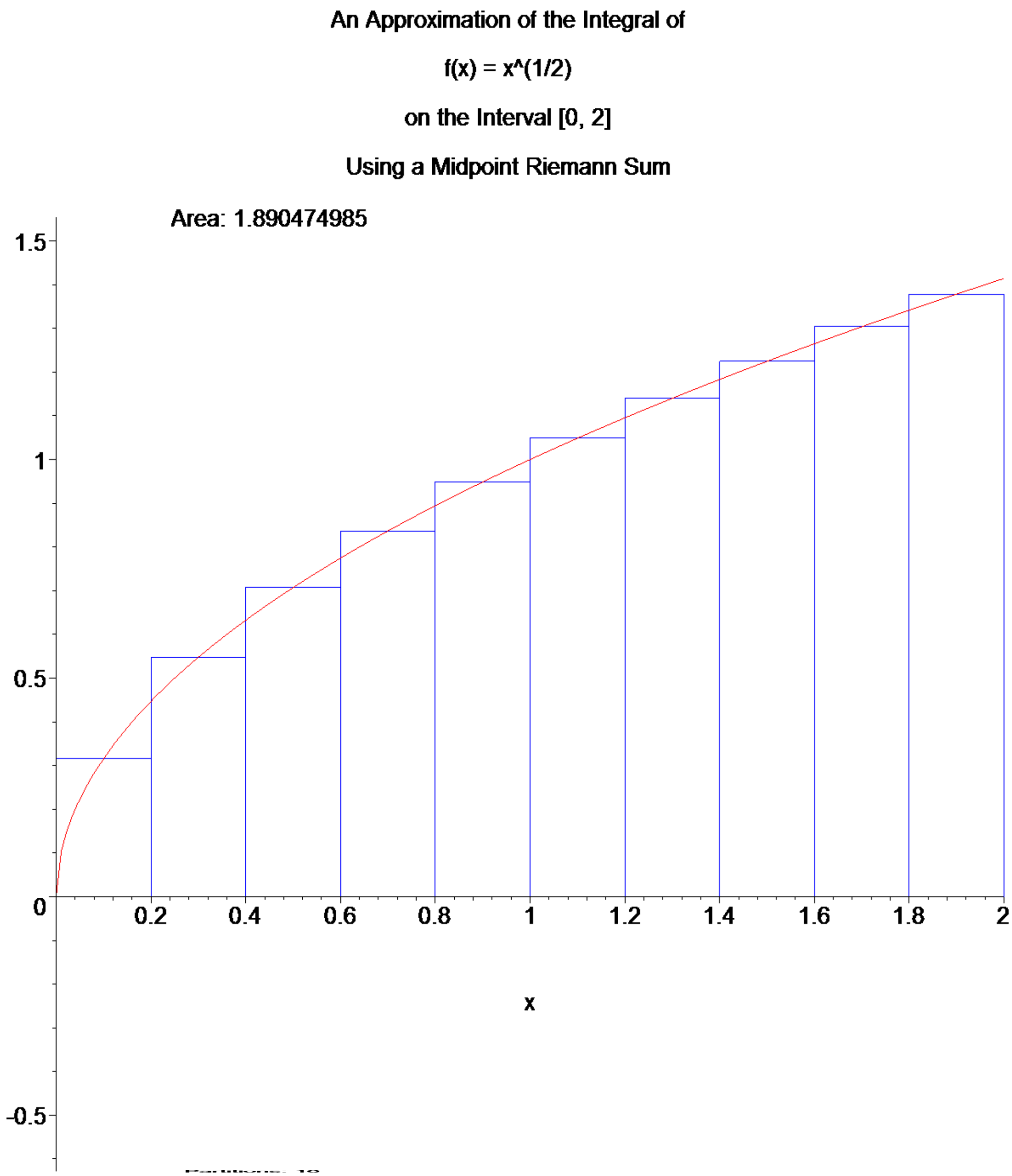
f(x) = x^(1/2)

on the Interval [0, 2]

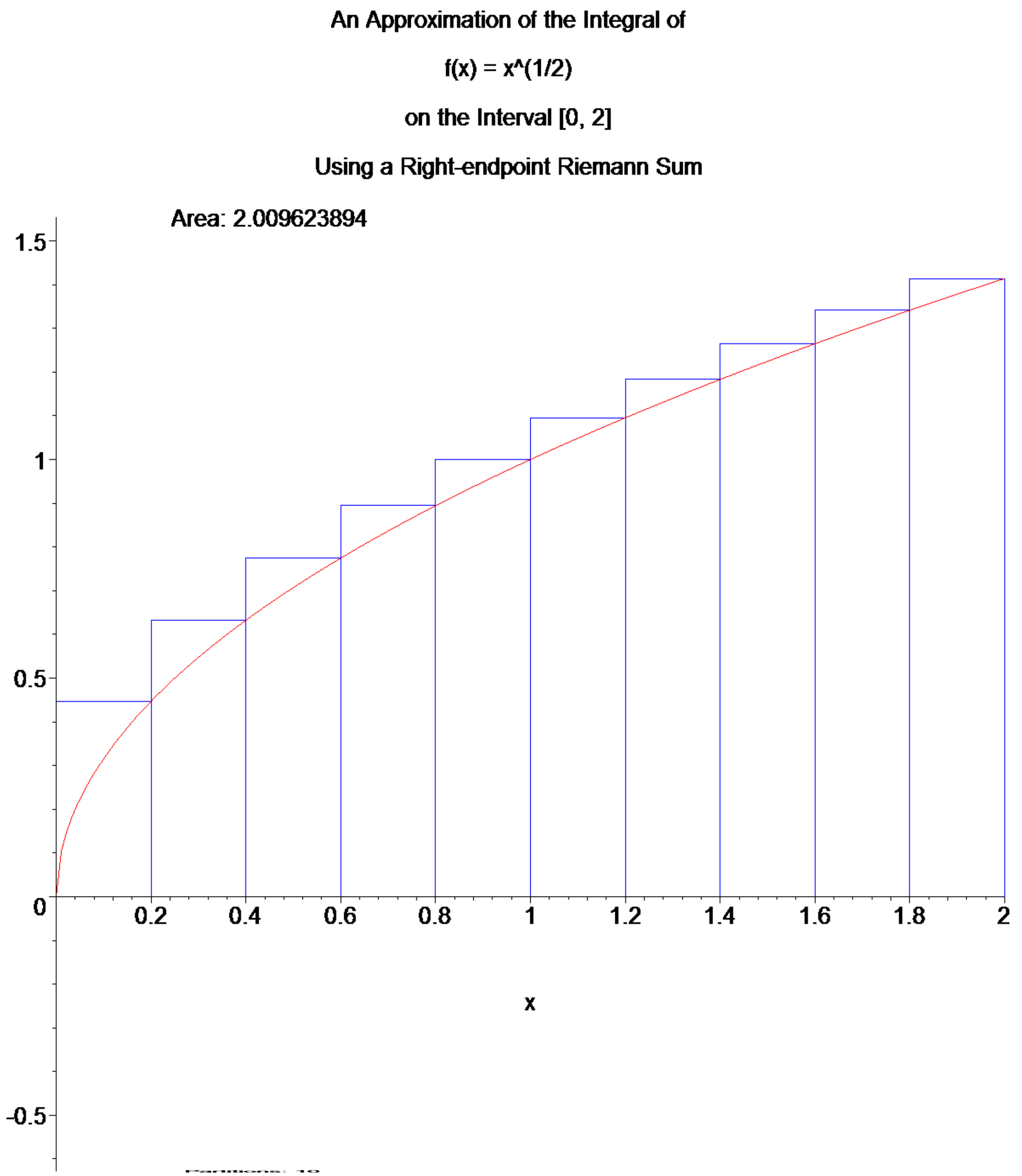
Using a Left-endpoint Riemann Sum



lower

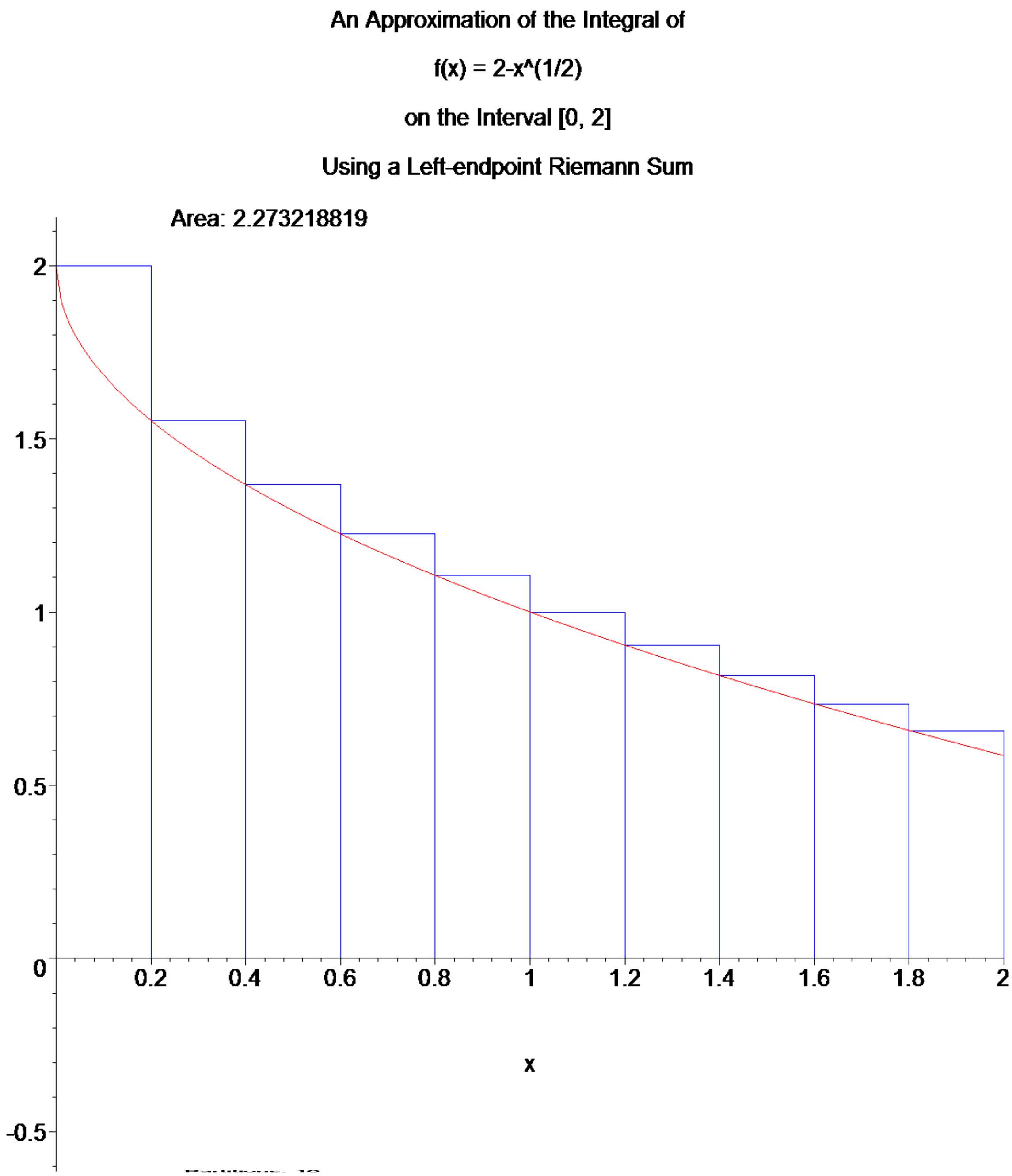

An Approximation of the Integral of
f(x) = x^(1/2)
on the Interval [0, 2]
Using a Midpoint Riemann Sum
Area: 1.890474985
1.5
1
0.5
0
-0.5
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x

greater



greater

```
> ApproximateInt(2-x^(1/2), 0..2, output=animation, partition=10,
  refinement=halve, subpartition=width,
  method=left,showpoints=false);
  greater;ApproximateInt(2-x^(1/2), 0..2, output=animation,
  partition=10, refinement=halve, subpartition=width,
  method=midpoint,showpoints=false);
  lower;ApproximateInt(2-x^(1/2), 0..2, output=animation,
  partition=10, refinement=halve, subpartition=width,
  method=right,showpoints=false);lower;
```


An Approximation of the Integral of
f(x) = 2-x^(1/2)
on the Interval [0, 2]
Using a Left-endpoint Riemann Sum
Area: 2.273218819
2
1.5
1
0.5
0
-0.5
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x

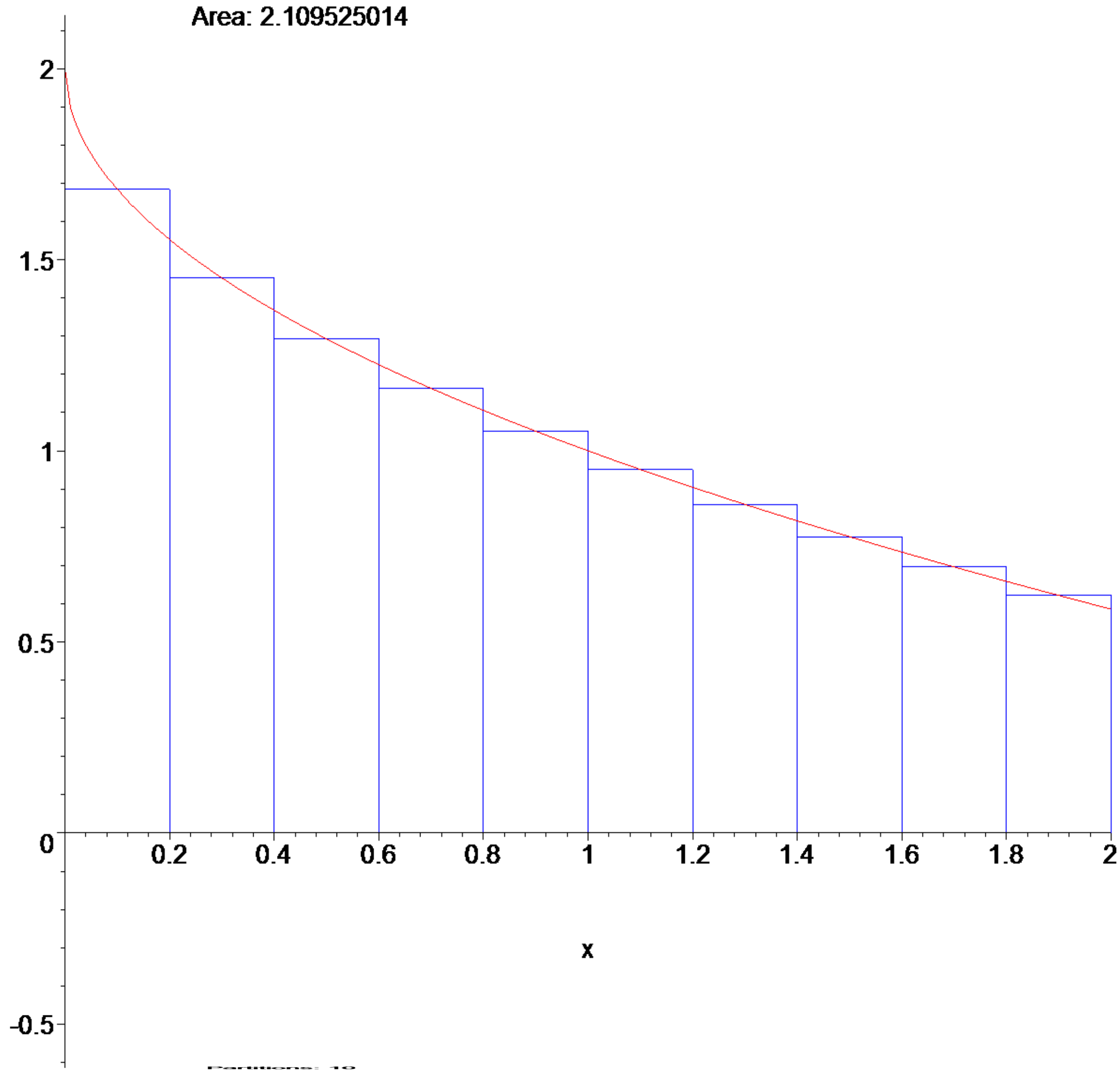
greater

An Approximation of the Integral of

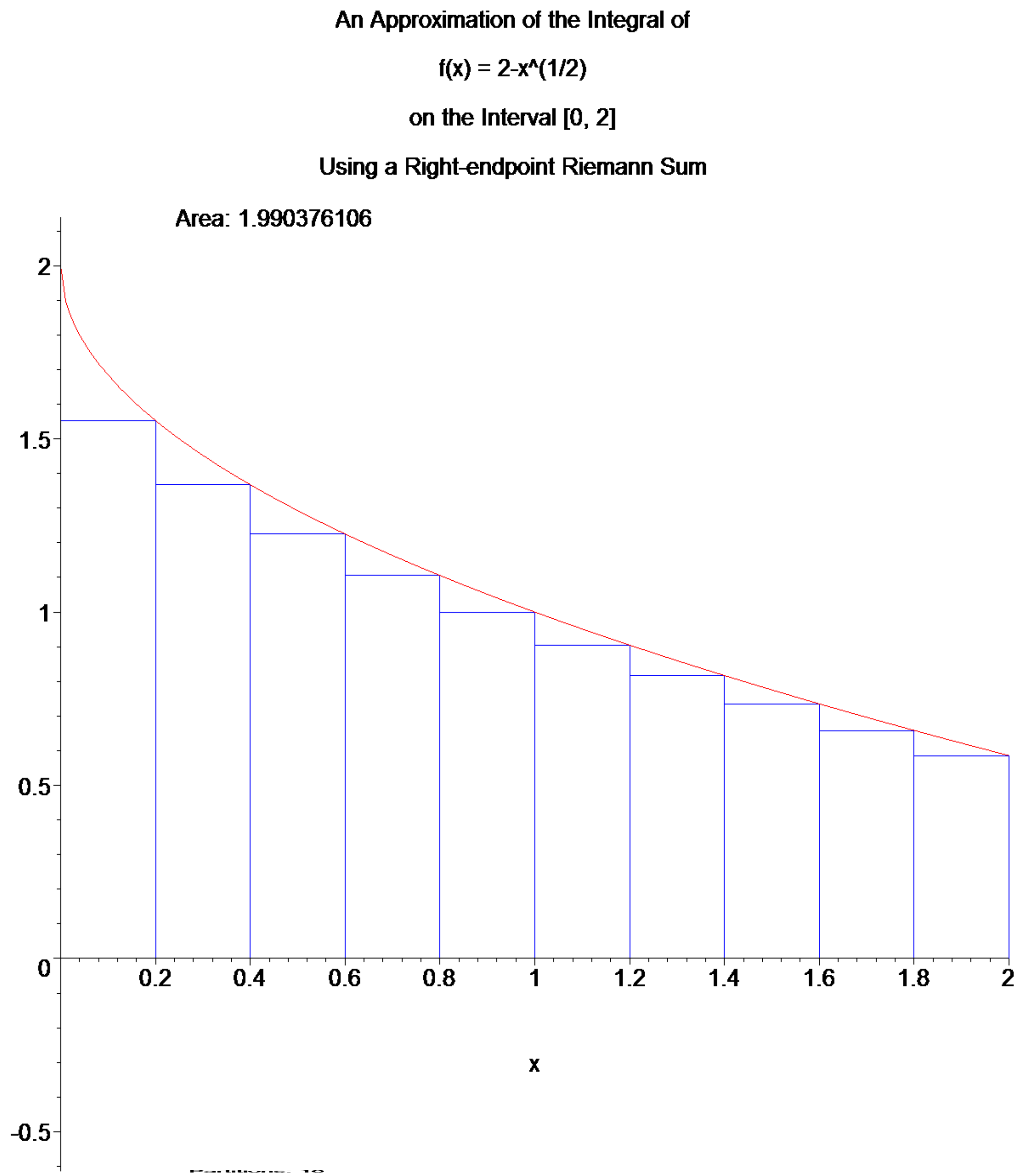
f(x) = 2-x^(1/2)

on the Interval [0, 2]

Using a Midpoint Riemann Sum

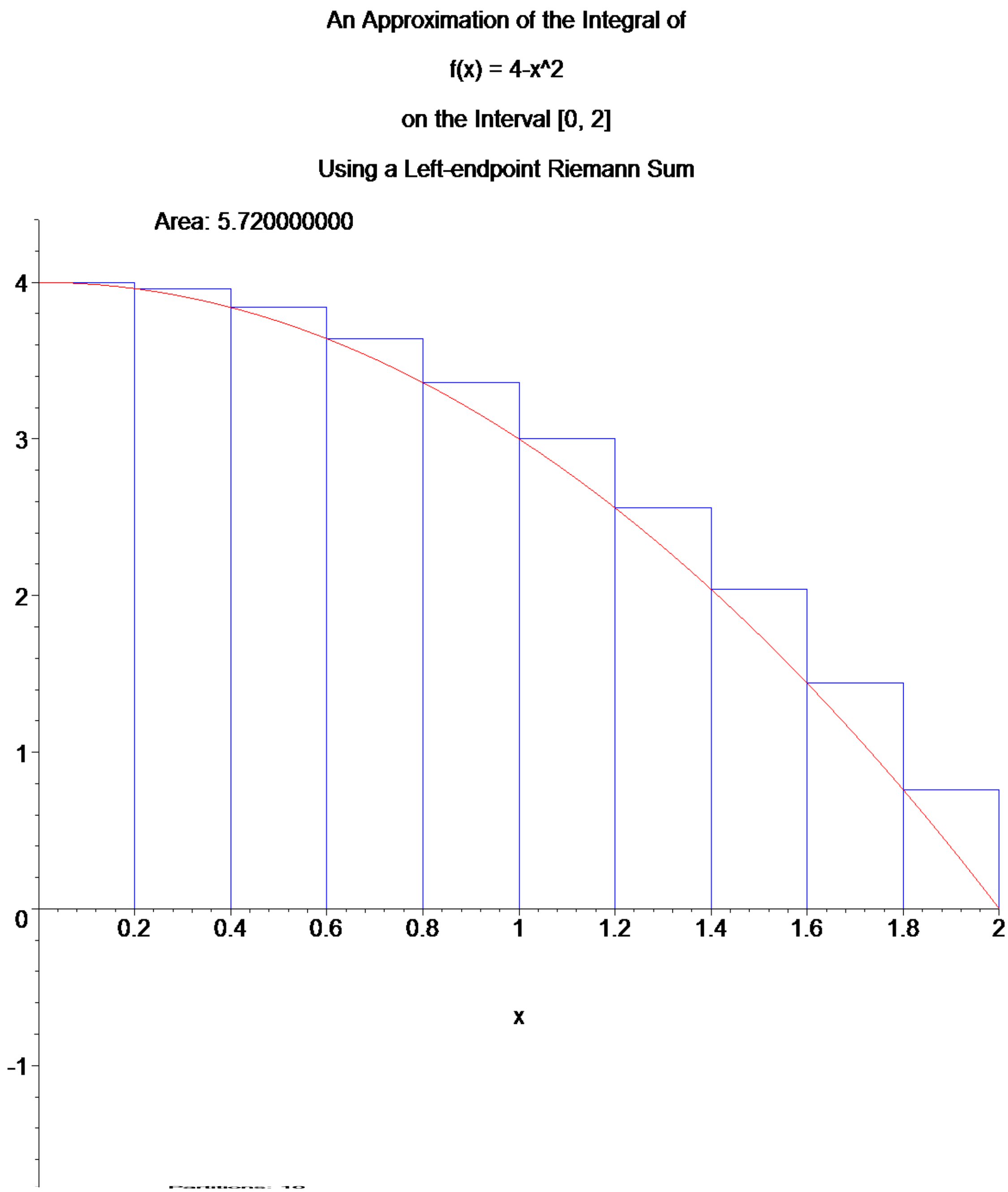


lower

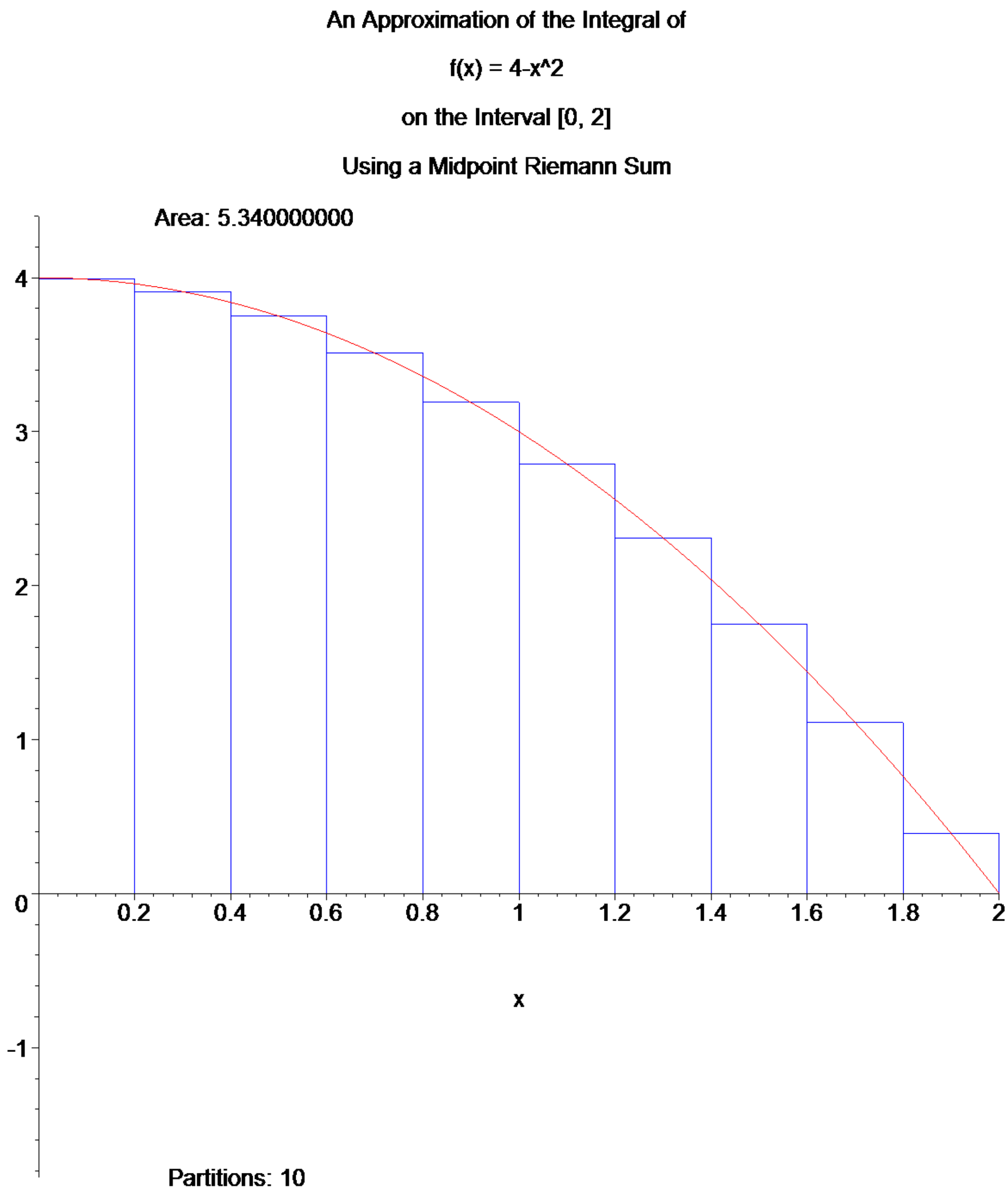


lower

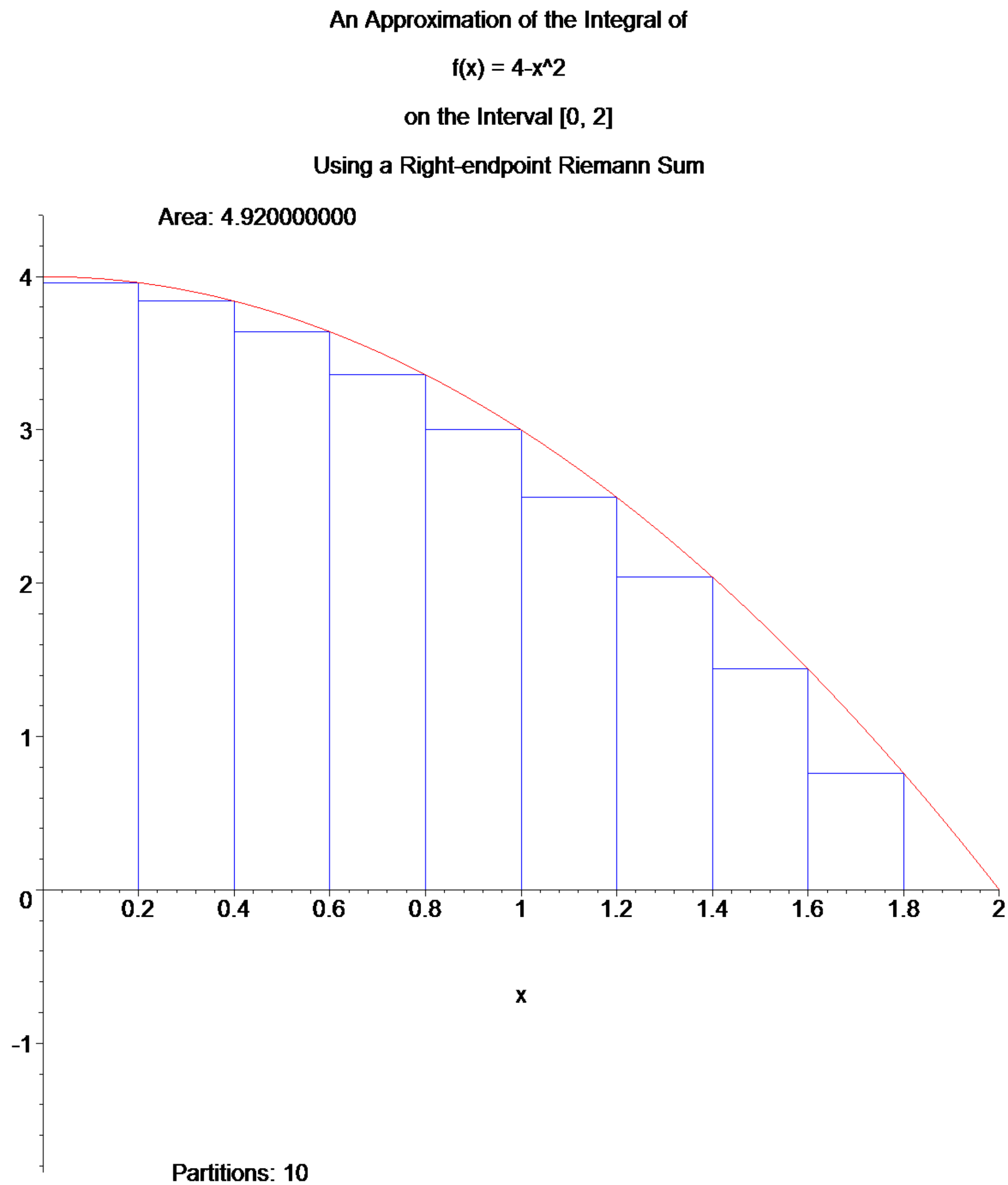
```
> ApproximateInt(4-x^2, 0..2, output=animation, partition=10,
  refinement=halve, subpartition=width,
  method=left,showpoints=false);
  greater;ApproximateInt(4-x^2, 0..2, output=animation,
  partition=10, refinement=halve, subpartition=width,
  method=midpoint,showpoints=false);
  greater;ApproximateInt(4-x^2, 0..2, output=animation,
  partition=10, refinement=halve, subpartition=width,
  method=right,showpoints=false);lower;
```


An Approximation of the Integral of
f(x) = 4-x^2
on the Interval [0, 2]
Using a Left-endpoint Riemann Sum
Area: 5.720000000
4
3
2
1
0
-1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x

greater


An Approximation of the Integral of
f(x) = 4-x^2
on the Interval [0, 2]
Using a Midpoint Riemann Sum
Area: 5.340000000
4
3
2
1
0
-1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x
Partitions: 10

greater



lower

exploratory ex. 1

>

Digits := 10

>