NOVEL CONTROL APPROACHES FOR
WEB TENSION REGULATION

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To my parents and lovely wife…

It is their anticipation that stimulates me to pursue high education!
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ABSTRACT

The web tension control problem is very challenging and important because the system dynamic is a function of many process variables that often vary over a wide range. Traditional linear controllers cannot adequately address this problem even using adaptive methods.

In this research, a new control method is proposed for tension regulation in a web transport system. It is based on the unique active disturbance rejection control (ADRC) strategy, which actively compensates for dynamic changes in the system and unpredictable external disturbances.

The synthetic function for the time optimal control of the second-order discrete time system is obtained by the Isochronic Region method. Based on this function, the novel nonlinear PD controller is proposed and used in conjunction with the ADRC. Two internal compensation methods are used to improve web tension control performance.

A simulation of real industrial application is used to provide realism. The results show the effectiveness of the proposed tension controller in coping with large dynamic variations commonly encountered in web tension applications. The newly developed web tension control approach described in this dissertation is ready for hardware testing and implementations.

Keywords: Web Tension Control, Nonlinear Control, Time Optimal Control.
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NOMENCLATURE

A  System Matrix of the state space control
A₀  Area of the web
B  Input matrix of the state space control
C  Output matrix of the state space control
d  Damping factor
E  Modulus of elasticity
Fₜ  Reference force in the web
fᵢj  Web force between the rollers No. j and k
fᵢj*  Reference web force between the rollers No. j and k
H  Gain vector of the observer
J  Criterion function of state space control
K  Optimal controller gain vector
kₑ  Equivalent gain of the speed control
kₑ  Equivalent gain of the current or torque control
Lₜ  Reference length of the web
lᵢj  Length of the web between the rollers No. j and k
mᵦ  Torque of the motor shaft No. k
nᵦ  Speed of the motor shaft No. k
rᵦ  Radius of the roller No. k
$T_e$ Equivalent time constant of the current control

$T_{en}$ Equivalent time constant of the speed control

$T_{jk}$ Time constant of the web between the rollers No. j and k

$T_N$ Reference time constant of the web ($T_N = L_N / V_N$)

$T_{\theta Nk}$ Time constant of inertia of roller and drive No. k

$u$ Input vector of state space control

$v_k$ Velocity of the web in the section No. k

$v_0$ Average velocity of the web

$V_N$ Reference velocity of the web

$x$ State vector of the state space control

$y$ Output vector of the state space control

$\varepsilon_{jk}$ Strain in the web between the rollers No. j and k

$\varepsilon_N$ Normalized strain

$\rho$ Density of the web

$\Omega_0$ Eigenfrequency of the system

$\Omega_d$ Characteristic frequency of the Symmetrical Optimum
CHAPTER I

INTRODUCTION

The guiding and transport of webs has been studied for many years. Belts (a special case of a web) used for transmission of power were studied by Osborne Reynolds in 1974. D.P. Campbell discussed both the lateral motion (first-order model) and longitudinal tension measurement and feedback control of a moving web [1]. The requirements of web tension control increase because of higher speed demanding in the processing equipment; consequently, better solutions are required.

This chapter introduces the basic concept of web tension regulation and the difficulties of the web tension control problems.
1.1 Web Transport and Tension Control Concept

A web refers to any material in a continuous flexible strip form which is either endless or very long compared to its width, and very wide compared to its thickness. Other terms for a web in industry include film, foil, strip, belt, and fabric. Many types of material are most economically manufactured or processed in web form such as paper, plastic film, cloth fabrics, and even strip steel. Most of the products used by commerce today have as their origins a web form. Examples include all forms of paper, coated plastic wrap for food items, electronic storage media such as floppy disks, photographic film, all forms of fabrics including those used in the cords of automobile tires, and steel strip utilized for automobile bodies and appliances [1].

Existing production plants have continuous moving webs driven by a large number of electrical machines that are controlled by different loops. The web typically has to pass through several processing stations. By its nature, all sections of the continuous process are coupled by the web. There are often “winders” installed at the beginning and at the end of the plant.

Depending on the manufacturing processes, there are different demands on the transport of the web. At the beginning of the process in paper manufacturing, the web consists of about 90% of water; therefore, an auxiliary web of felt is needed for transport. After drying, the paper web is able to be transported under tension. In rolling mills, the web is transported under large forces to deform the material. The demands of plastic foils are quite different: during the transport of the web, no plastic deformation may occur. Similarly in printing machines, the quality of the printed picture requires that no deform
is present in the web. In each case, the transport of the web through the manufacturing processes has to be performed without introducing material defects or losses under a definite tension that is different in separate sections of the processing equipment.

Production plants with a continuous moving web have a complex structure where mechanical and electrical problems are involved. In the system schematically shown in Figure 1, the web will be processed at different stations, known as “nip” sections. There are driven and undriven rollers to transport and to process the web. The web tension must be kept at a desired value to insure product quality. The rollers are driven by electrical motors that controlled with different variables including the current, the speed and sometimes the torque. A superimposed guidance system controls the entire process. The winders at the beginning or at the end of the system are storages of the material. From an analytical point of view, the winders are a special kind of nip section, where the process
begins or ends. That is, they are coupled with the web system only from one side. During the winding process, the tension of the different layers of the material should be controlled in such a way that no plastic deformation of the material occurs [3].

A simple solution to keep constant tension in the web is to use a “dancer” roller [1]. The advantage of a dancer roller is that the tension is decoupled from the speed difference of the nip section. Furthermore, the dancer roller is able to compensate dynamic changes of the web tension caused by a winder running unbalanced. But the usage of a dancer roller is limited to machines running with low-speed [1].

Another simple solution is to control only the speed of the driven rollers of the system. The web forces are controlled in an open loop as a function of the speed difference of the rollers. The disadvantage of this method is that changes in the strain of the web, e.g. during coating or printing, cannot be controlled. Therefore, some nip sections are equipped with load cells to measure the tension so that closed loop control of the web forces is possible. The disadvantage of using load cells is due to the likelihood that random noise will be present at the output signal, which means that the output signal must be smoothed. The filter needed to smooth the signal also introduces a delay. The delay limits the usage of the control system; consequently, acceptable tension control is limited to systems where the physical parameters of the system are changing within a small range.
1.2 Difficulties of Web Tension Control

Tension controls are widely used in web transport and strip processing systems. These systems often either feed material from an existing primary processed roll or coil of material into a process for secondary processing, or wind processed material for temporary storage or final shipment. The main purpose of web tension regulation is to maintain the physical integrity of the material that is being processed.

Factors affecting the longitudinal dynamics of the web include the change of operating conditions (e.g. temperature and moisture), the periodic disturbance from an eccentric roll, the torque variation caused by an unbalance roll, the velocity variation caused by the time-varying radius of roll, and the wound-in tension variation. If severe tension variation occurs, rupture of the material during processing or degradation of product quality may occur, resulting in significant economic losses due to machine damage and interruption of production. The web material may have to pass through several consecutive processing sections in the manufacture of an intermediate or final product. Different web tension levels with varying demands on accuracy may be required in the different processing sections. Therefore, in order to minimize these losses, it is important to monitor and control the tension within required limits within each section.

The tension control problem in web processing applications is a complex one because the system dynamics are a function of many process variables that often vary over a wide range. For example, in rolling mills, these variations include changes in roll diameter, product density, web/strip modulus of elasticity, web cross sectional area, the inner
velocity loop bandwidth and process line-speed. Web tension control is a very stringent and complex problem. Specifically,

- Nonlinear unknowns such as web damping, friction, and slippage make the derivation of the analytical tuning algorithm difficult.
- Parameters such as the inertia of the wind roller change during processing.
- The system dynamic varies between start up and later in processing.

Due to their difficulty and importance in industry, tension problems have drawn the attention of many researchers.

The requirements on tension control become even more demanding because of the higher speed employed in the manufacturing plant; consequently, better solutions are required. Linear control methods are based on the linearized model of the system. Unfortunately, the linearized model does not guarantee an accurate representation of the system, since, in reality there is no such thing as a linear system. For example, the strain in the paper of a coating machine changes during the coating and drying and the friction depends on the temperature. Another example of such disturbances is a winder running unbalance. This fact causes web “flutter” and large variations in the web tension. If the limits of a cascade linear control system are reached, new control methods should be used.

In control theory, nonlinear control is an effective tool to solve complex problems and to improve the dynamic behavior of the control system.

In this research, a new methodology for web tension regulation is described. It is based upon a unique active disturbance rejection control (ADRC) concept. In this approach, the disturbances are estimated using an extended state observer (ESO) and compensated during each sampling period. This method was developed by J. Han [4].
The proposed ADRC control system consists of the ESO and a nonlinear PD controller. It is designed without an explicit mathematical model of the plant. The controller is designed to be inherently robust against process variations. Once it is set up for a class of problems within a predetermined range of variation in system variables, no tuning is needed for start up, or to compensate for changes in the system dynamics and disturbances. The ADRC method, because of its robustness and disturbance rejection capabilities, is particularly suitable for web tension regulation applications.

A new nonlinear PD controller based on time optimal control is discussed in this dissertation. The Isochronic Region method is used to obtain the synthetic function for the time optimal controller of the second-order discrete time system. Based on this function, a novel nonlinear PD controller is proposed and used for tension control in ADRC framework.

In this dissertation, a web tension control method review is given in Chapter II. In Chapter III, mathematical analysis for the web transport system is described. Some research results are discussed. The concept of ADRC and the simulation result of the industrial application are given in Chapter IV. The new nonlinear PD controller based on time optimal control is discussed in Chapter V. Two compensation methods, damping compensation and internal model compensation, are introduced in Chapter VI. Finally, conclusions and future research discuss in Chapter VII.
CHAPTER II
WEB TENSION REGULATION BACKGROUND

The process of a web transport system, web propulsion, and tension measurement are described first in this chapter. Then a general dynamic model of web transport system is developed based on the physical analysis. A simplified second-order web tension model is also introduced. Simulation blocks in Simulink are constructed. A simplified mode for a four sections web transport line used in industry is presented.

Many different control methods have been proposed in web tension regulation. In this chapter, these methods are reviewed, including the speed control with PI controllers, the tension control with PI controller, the state space method, the decentralized control, fuzzy logic control, and the neural networks method. Finally, a comparison of these methods is performed.
2.1 Introduction of Web Transport

**Web Propulsion**

After filaments and webs have been formed, they are propelled. Through some processing operations, they must move at constant velocity, despite external forces or disturbances imported to them. For other processing operations, special time-varying speed patterns may be required. Variable web speed is often desirable so that the throughput of one process can be matched with the throughput of another process in a cascade system.

The winding-up of rollers of finished or partly finished product, the transporting of the rollers to another point in a manufacturing process, and the unwinding of them are a frequently encountered operations. Propulsion is brought about by engaging web of material, drive rollers – generally called pinch rollers. These rollers are connected to motors whose speed can be varied and torque and power level can be adjusted so that the motors can accelerate, decelerate, or continuously propel the web or filament. Several consecutive sets of pinch rollers are sometimes needed in a large process. The different sets of drive rollers are maintained in speed synchronism for propulsion. They may have a speed gradient for combined propulsion and rolling or drawing.

A section of slack web between the rollers may hang loosely in processing baths or soaking pits such as might be found in textile dyeing operations or in steel strip cleaning. Weighted rollers may be attached to webs so that the webs remain taut as they move forward. Except for extreme conditions of acceleration and deceleration, weighted rollers produce tension in webs proportional to a fraction of the dead weight of the roller. Dancer
rollers are also used to hold a web in a taut condition as it propelled through various machines. The object of the dancer roller is to prevent the web from bouncing, vibrating, or snapping into a taut condition from a slack one. Both the weighted roller and the dancer roller tend to be self-regulating tension schemes (whose performance is rather limited) superimposed upon the propulsion system.

As a roller is wound up at the terminal end of a process, the roller starts with a small core diameter. As the diameter increases, the tension in the web of material will tend to increase. Should the web in the machine suddenly break or for some reason become slack, the prime mover which is winding the roller will tend to accelerate.

![Figure 2](image.png)

Figure 2 (a) Pinch Rollers and Guide Rollers in a Web-Propulsion System (b) Unwind and Upwind System [1]
When a roller of material is being unwound in a steady throughput condition, and the process is suddenly stopped, the moment of inertia of the roller of web will cause the roller to continue to unwind. Often a brake is put on an unwind roller to regulate the forward motion of the web. Transients in process throughput can cause the unwind roller to accelerate, in which case the web may become slack in the guide-roller system. Then, if the brake is applied to the unwind roller with improper time phase, the unwind roller can stop suddenly, with the result that the web will snap against the guide roller and possible break under the condition of the severe tension which develops.

Figure 2 shows the rudimentary schematic drawings which define propulsion of webs, winding and unwinding [1].

**Web Tension Measurement**

A conventional method to measure web tension is by sensing the reaction force from a roller which is wrapped by the web. Another method is by installing a dancer subsystem which consists of a dancer roller, a damper, and a spring, in series with other system components. The web tension is measured based on the displacement of the dancer roller. Both methods have been widely used in web handling machinery. However, little is known about proper application to specific problem.

In Shelton’s report (Shelton, 1991) [8], the frequency response approach has been adopted to develop a quantitatively useful tool to determine optimum arrangements of sensors for various configurations of web handling lines. The same approach also can be used for the study of the web longitudinal behavior and the attenuation or amplification of system disturbances by the sensing transducers.
In addition to the above mentioned methods for the measurement of web tension, methods such as simplification of web handling machinery by sensing with driven rollers and dancers with multiple rollers, flywheels on dancer rollers, and other variations could be possible candidates to improve measurement accuracy (Shelton, 1991) [8]. On the other hand, a noncontact method using a point-source pulse is an advanced technique for the measurement of web tension. The use of such a method can result in a more compact and less complicated system than now exists (Lowery et al., 1992) [3].

2.2 Web Tension Model

Keeping tension in a web constant requires that the web shall neither elongate nor contract with respect to time. If two consecutive sets of pinch rollers rotate in identical positional synchronism, and if they remain positively engaged with a web at all times, the web tension will not change unless the properties of the section of web between the pair of pinch rollers changes as the web is exposed to a changing environmental conditions such as drying, chemical treatment, or mechanical deformation.

When processing actions are performed upon webs, they become subject to a variety of manipulating forces that tend directly or indirectly to deform the web so that their tension changes. The manipulating forces sometimes affect the speed of the drive mechanisms. This variance in speed in turn brings about changes in the web tension.

The guiding and transport of webs have been studied for many years. Belts (a special case of a web) used for the transmission of power were studied by Osborne Reynolds
Reynolds discussed the behavior of the creeping of a belt around a pulley and the effects of the elastic modulus of the belt on the efficiency of the transmission of power. He showed that with an elastic belt, there must be a loss of speed of the driven pulley due to the extension and contraction of the belt, and that this loss of speed must increase as the tension difference increases. A related concept was advanced by H.W.Swift (Swift, 1928) in a significant and substantial paper. He recognized that a flat belt used for power transmission has an inactive arc on the exiting side over which no creep occurred and an active arc on the exiting side over which creep occurred as the belt made the transition from the entering to the exiting value of strain. As a result, if friction is sufficient to avoid complete slippage, the inactive arc will isolate upstream tension from downstream tension disturbances in a velocity-controlled web transport system. However, ideal friction in any amount is still incapable of preventing disturbances of downstream tension by upstream tension, as the upstream strain is transported to the downstream span (Shelton, 1991) [8]. This phenomenon, known as transport of strain, appears to be poorly understood by many researchers and is a principal reason for the development of incorrect mathematical models. Swift published another substantial article (Swift, 1932) which addressed imperfections in the geometric arrangement of belt drives due to the misalignment of pulleys. He also discussed lateral tracking of belts in conventional and “coned” pulleys. This work is considered fundamental to steering of belts and is a precursor to open-loop lateral control. The basic approaches are developed for belts and the principles cannot be generalized to webs of large width.

D. P. Campbell [1] discusses both the lateral motion (first-order model) and longitudinal tension measurement and feedback control of a moving web. He
concentrated on electromechanical drivers for web transport, but unfortunately overlooked the phenomenon of transport of strain and its implications on tension control. In addition, Campbell discusses the forming of webs by extrusion and the speed control associated with transport.

After D. P. Campbell, many researchers published papers about web handling and web tension control, including Brandenberg (1976), King (1973), Schroder (1987) and W. Wolfermann (1995) [3], etc.

This body of research is motivated by the complex control problem encountered in a web line testing facility, referred to as the Lab-line and is shown in Figure 3 and Figure 4. It was used to evaluate web handling control strategies. A simplified one-line diagram is shown in Figure 5. Below the mechanical system, electrical drives and the web tension subsystem are studied. Finally, a general web transport system model is given.
Mechanical System

The mechanical system is composed of the transported web and the rollers in the nip sections.

Behavior of the web. The behavior of the web under tension can be elastic, viscoelastic, or plastic non-linear. In many cases, it can be assumed that the web behavior is elastic and thin, therefore an un-dimensional web system exists. For these conditions, Hooke’s law can be written as:

\[ F_{jk} = e_{jk} \cdot E \cdot A_0 \]  \hspace{1cm} (2.1)
where \( F_{jk} \) Web force between the rollers No. j and k;

\( \varepsilon_{jk} \) Strain in the web between the rollers No. j and k;

\( E \) Modulus of elasticity;

\( A_0 \) Area of the web.

Throughout this research, normalized quantities are used. The normalized strain \( \varepsilon_N \) denotes the strain in the web if the normal values are acting. From equation (2.1), the normalized strain of the web in the machine direction is:

\[
\varepsilon_N = \frac{F_N}{EA_0}
\]  

(2.2)

where \( \varepsilon_N \) Normalized strain;

\( F_N \) Reference force in the web.

There are various types of rollers to produce, transport, and form the web. But all rollers are nip sections with slip and non-slip zones. Because of the non-slip zone, it can be assumed that the speed of the web is equal to the peripheral speed of the roller. During the transport of the web through the machine under dynamic change of the stress and strain, the mass has to be constant. To describe this behavior, the law of conservation of mass in a control volume can be used, which is well known in the theory of fluid dynamics.

\[
\frac{d}{dt} \int_{Vol} \rho \cdot dVol = -\int_{A_r} \rho \cdot V \cdot dA
\]  

(2.3)

where \( \rho \) Density of the web;

\( V \) Reference velocity of the web.
Equation (2.3) describes, on the left side, the temporal change of mass in the control volume, whereas on the right side the difference of the input and output of the mass of the control volume is shown. The solution of equation (2.3) yields the following non-linear differential equation:

$$\frac{d}{dt} \left( \frac{L_{jk}(t)}{1 + \varepsilon_{jk}(t)} \right) = \frac{V_j(t)}{1 + \varepsilon_{jk}(t)} - \frac{V_k(t)}{1 + \varepsilon_{jk}(t)}$$

(2.4)

where $L_{jk}$ is the length of the web between the rollers No. $j$ and $k$. If it is in a steady state, equation (2.4) gives the result:

$$\frac{V_k}{V_j} = \frac{1 + c}{1 + \varepsilon_{ij}}$$

(2.5)

Equation (2.5) shows that the strain $\varepsilon_{jk}$ in a web is created by the relation of the output to input velocity $\frac{V_k}{V_j}$ of a web section and the incoming web strain $\varepsilon_{ij}$. As the strain $\varepsilon$ is normally very small, the relation of the velocities is nearly 1, the difference is only some thousandth part. This fact is important if the tension of the web is controlled in an open loop via the difference of the velocities. In this case, a very precise control of the speed of the drives is needed.

Equation (2.4) must be considered if changes in the system are large, e.g. during start up of the machine. If the steady state or dynamic behavior is studied and the changes in variables are acceptably small, equation (2.4) can be linearized. All variables then treated as small changes from initial steady-state values. Linearization yields the following result

$$T_{jk} \frac{d}{dt} \left( \Delta \varepsilon_{jk} - \frac{\Delta l_{jk}}{l_{jk}} \right) = \frac{\Delta V_k}{V_0} - \frac{\Delta V_j}{V_0} - \Delta \varepsilon_{jk} + \Delta \varepsilon_{ij}$$

(2.6)
The time constant $T_{jk}$ is the time needed to transport the web from nip section $j$ to section $k$. It depends on the average transport velocity $v_0$ and the length $l_{jk}$ of the web between section $j$ and $k$.

$$T_{jk} = \frac{L_{jk}}{V_0} = \frac{l_{jk}}{v_0} \cdot T_N$$  \hspace{1cm} (2.7)

It should be mentioned that the time constant is not actually constant, it is a function of the variable average velocity $v_0$ of the machine.

To achieve the linear signal-flow graph, equation (2.6) has to be transformed into the frequency domain. The result is:

$$sT_{jk} \left( \frac{\Delta \varepsilon_{jk}}{l_{jk}} \right) = \frac{\Delta v_k}{v_0} - \frac{\Delta v_j}{v_0} - \Delta \varepsilon_{jk} + \Delta \varepsilon_{ij}$$  \hspace{1cm} (2.8)

The second term in the brackets on the left side in the equation (2.6) and (2.8) describe a change of the length of the web between section $j$ and $k$. This change in length occurs when a dancer roller is used. Two cases must be considered.

Case 1: System with dancer roller.

The transfer function will be:

$$\frac{\Delta l_{jk}}{l_{jk}} = \left( \frac{\Delta v_j}{v_0} - \frac{\Delta v_k}{v_0} - \Delta \varepsilon_{ij} \right) \frac{1}{sT_{jk}} + \Delta \varepsilon_{jk} - \frac{1}{sT_{jk}} + \frac{1}{sT_{jk}}$$  \hspace{1cm} (2.9)

The change of the length $\Delta l_{jk}$ depends on the difference of the velocities $\Delta v_j$ and $\Delta v_k$, and its time behavior is integrated. The strain $\Delta \varepsilon_{jk}$ is not a function of the speed difference; conversely, it depends on the force which acts on the dancer roller.

Case 2: System without dancer roller.

Here, $\Delta l_{jk} = 0$ and the transfer function will be:
\[
\Delta \varepsilon_{jk} = \left( \frac{\Delta v_k}{v_0} - \frac{\Delta v_j}{v_0} + \Delta \varepsilon_{ij} \right) \frac{1}{1 + s T_{jk}}
\]

(2.10)

The change of the strain \( \Delta \varepsilon_{jk} \) depends on the difference of the velocities \( \Delta v_j \) and \( \Delta v_k \).

The time behavior is a first order lag element, \( v_0 \) is the variable average velocity of the machine.

**Behavior of the roller.** If a stiff mechanical coupling of the drives and rollers is assumed, the moment of inertia of the drive and the roller can be added to a resultant moment of inertia \( \Theta_{NK} \). After normalization, the well-known equation is:

\[
T_{\Theta_{nk}} \frac{dn_k}{dt} = \Delta m_k - \Delta m_{ak}
\]

(2.11)

where

- \( m_k \) Torque of the motor shaft No. k;
- \( n_k \) Speed of the motor shaft No. k.

The load torque \( \Delta m_{ak} \) is determined by the forces acting on both sides of the motor shaft.

\[
\Delta m_{ak} = (\Delta f_{kl} - \Delta f_{jk}) r_k
\]

(2.12)

where

- \( f_{jk} \) Web force between the rollers No. j and k;
- \( r_k \) Radius of the roller No. k.

**Electrical Drives**

The rollers are driven by electrical motors, which can be provided by DC or AC. The dynamic behavior of DC motor is simple. If the flux is constant, the speed is proportional to the voltage of the armature. The variable voltage is generated by ac-dc converter. The
current $i_k$ of the armature is proportional to the torque $m_k$, and it is controlled in a closed loop.

The dynamic behavior of an AC motor is more complex. To change the speed, it needs a variable voltage and frequency which are produced by an inverter. If a field-oriented control of voltage, current, and flux is employed then it is assumed that the dynamic behavior is nearly the same as that of a current controlled DC motor. Therefore, a simple transfer function of the first order is often used to describe the electrical drives, independent of the kind of the machine. In doing so, a lag element is obtained as an equivalent function of the controlled current or torque:

$$\frac{m_k}{m'_k} = \frac{k_m}{1 + sT_e}$$  \hspace{1cm} (2.13)

$k_m$ is the gain and $T_e$ the equivalent time constant. Depending on the quality of the control, the value of $T_e$ typically is between 1 to 10 ms [3].

In the research described here, DC motors are used to drive the web transport system.

**The Web Tension Subsystem**

The driving motors and the rollers are coupled to each other by the web as shown in Figure 6. This causes the propagation of disturbances within the system in the direction of the transport of the web, and contrary to that direction. This fact forms a multi-dimensional system. To understand the control direction of the web forces, the subsystem of shown in Figure 6 should be considered.

The motor and the gearbox have been omitted, and the assumptions detailed as follow:

- Unstretched web is introduced into the tension zone.
• Bridle No.1 is an ideal speed regulator.

Figure 6 Physical Plant of the Web

The tension dynamics associated with the conveyance of a web through a single tension zone is described in (2.14). It is based upon the principle of conservation of mass in a mass-flow system [9].

\[
\frac{L_i}{dt} \frac{dT_i}{dt} = E_i \cdot A_i \cdot (V_i - V_{i-1}) + V_{i-1} \cdot T_{i-1} - V_i \cdot T_i \tag{2.14}
\]

where

- \( L_i \) \( i^{th} \) length [engineering units of choice];
- \( T_i \) \( i^{th} \) tension [units of force];
- \( E_i \) \( i^{th} \) modulus of elasticity [units of force per units of A];
- \( A_i \) \( i^{th} \) web cross sectional area [engineering units of choice];

and

\[
V_i = \frac{R_i}{GR_i} \cdot \frac{2\pi}{60} \cdot (\omega_i) \tag{2.15}
\]

where

- \( R_i \) \( i^{th} \) roll radius [engineering units of choice];
- \( GR_i \) \( i^{th} \) gear ratio;
- \( \omega_i \) \( i^{th} \) roll rotational speed [rad/sec].
The motor/load torque equation is:

\[ J_i \frac{d\omega_i}{dt} = \tau_i + \frac{R_i}{GR_i} \cdot (T_{i+1} - T_i) \] (2.16)

where \( \tau \) is the motor torque [engineering units of choice].

The above equations can be presented in block diagram format as shown in Figure 7. This modular representation includes the hooks that allow coupling multiple sections together. However, it does not include any damping terms. It should be noted that a rigorous representation of equation (2.14-2.16) requires the integrators in Figure 7 to be preset to their respective initial conditions [9].

![Figure 7 Block Diagram of a Web Tension Zone](image)

For a given operating line velocity \( L_{S_i} \), an approximate linear representation of equations (2.14-2.16) can be obtained [9]. A block diagram of the linearized model is shown in Figure 8, with the assumptions that 1) the tension in the web entering the tension zone is zero; and 2) the system speed reference \( V_{i-1} \) is constant.
Figure 8 Linearized Block Diagram of a Web Tension Zone

Of particular interest is the behavior of the tension loop at stall (that is when the web stops moving). At this point, with some simplifications, a tension transfer function can be approximated as a typical second order system

$$\frac{T}{\tau} = \frac{k}{s^2 + \frac{2\zeta}{\omega} s + 1} \tag{2.17}$$

with $\omega \in [\omega_{\text{min}}, \omega_{\text{max}}]$ , $\zeta \in [\zeta_{\text{min}}, \zeta_{\text{max}}]$ and $k \in [k_{\text{min}}, k_{\text{max}}]$ . With a corresponding differential equation of:

$$\ddot{\tau} = -2\zeta \omega \dot{\tau} - \omega^2 \tau + k \omega^2 \tau \tag{2.18}$$

Considering the tension coupling, dead zone and other nonlinearities in the system as well as external disturbances, a more generic and realistic differential equation for the tension dynamics is

$$\ddot{y} = f(t, y, \dot{y}, w) + bu \tag{2.19}$$

where $y$ is tension, $u$ is motor torque, and $w$ is the disturbance. Here, $f(t, y, \dot{y}, w)$ is generally a time-varying nonlinear function that represents the true system dynamics. The difficulty in tension control is mainly due to the fact that the function $f(t, y, \dot{y}, w)$
changes significantly during operation. Thus, better control strategies are needed to solve this practical problem.

The simulation block diagram developed and used by practicing engineers is shown in Figure 9. It should be noted that there are three tension zones but only one tension feedback loop, which is located on the winder side of the process line. The most critical area for tension regulation is at the winder. With the tension loop in place, it is optional to use an inner speed loop in the winder tension regulation scheme. Using the inner speed loop helps to dampen natural frequencies that are lower than the speed loop bandwidth [10]. The optional inner speed loop is not employed here for the sake of simplifying the design, and also to make the control problem more challenging.
The Matlab Simulink diagram is shown in Figure 10.

The parameter’s m-file of this model is shown in Appendix A. As the web is moving through the process, if the system dynamic changes over a very wide range, the web tension control problem becomes very challenging. In the following chapter, a novel nonlinear control structure will be used to solve this problem.

The Simulink model shown in Figure 10 is used in all web tension simulation in this research.
Figure 10 Simulink Block Diagram for Web Transport System
2.3 Web Tension Control Problem and Literature Review

The web material may have to pass through several consecutive processing sections in the manufacture of an intermediate or final product. Different web tension levels and accuracies may be required in the different processing sections.

In this section, a few control methods proposed in the literature are reviewed.

**Tension Control with PI or PID Controllers**

The proportional-integral (PI) or proportional-integral-derivative (PID) control approaches are the primary feedback control law used in industry. They are also widely used in web tension regulation. Some control approaches commonly used in web processing industries for tension control are closed-loop progressive set-point coordination control, open-loop draw control, and tension feedback control. In progressive set-point coordination control, once the set point is provided to an upstream driven roller, an input of the same magnitude is automatically provided to each of the driven rollers downstream. This approach is effective for the start-up or shut-down of a web processing plant; however, it is not an effective scheme for continuous operation since it forces tension in the downstream web span to be automatically changed when only the tension in the upstream span needs to be changed. Essentially, it is impossible to control the tension in each web span independently in a multispans web transport system using progressive set-point coordination [3].

In many processing machines used in the plastic-, textile- and paper industries only the speed of the rollers is controlled. In this case, the web tension is indirectly regulated
as a function of the speed difference. In the “draw control” scheme, tension in a web span is controlled in an open-loop fashion by controlling the velocities of the rollers at either end of the web span. W. Wolfermann and D. Schroder [3][12] used an optimal output feedback method to control the speed of the driven rollers. As shown in Figure 11, a decentralized observer was designed to decouple the drives from the web tension acting on the driven rollers, and this information is used to improve the speed control of the driven rollers. This method leads to considerable improvement in the speed responses of the driven rollers. An inherent drawback of indirectly controlling tension through speed control is its dependency on the open loop relationship between the speed and tension. This control method cannot reject disturbances due to “tension transfer” from adjacent web spans or interaction between adjacent web spans linked through an intermediate driven roller. It is important to note that tension is also affected by the change in temperature, material, thickness, as well as other operating variables. It is also very sensitive to noise in the speed feedback devices.

Figure 11 The Decoupling Observer
The disadvantage of only controlling the speed is that changes in the strain of the web during coating or printing cannot be controlled. An alternative method is to measure the tension directly so that closed loop control of the web tension can be applied. Such tension control system can be configured in a cascade structure (Figure 12). The inner circuit is the current control, the next the speed control and the outer circuit is the tension control.

![Figure 12 Tension Control with PI Controllers](image)

Due to the significant variations in system dynamics, PI or PID controller alone has been shown to be inadequate. K. Reid, K. Shin, and K. Lin [5][6][7] proposed the fixed-gain and variable-gain PID control of web tension in the winding section. For variable gain PID, the control parameters are continuously updated based on the diameter of the roller, which is a major contributor to the system dynamics. This method uses pole placement techniques, and is illustrated in Figure 13. The variable-gain controller continuously updates the PID gains, based on the feedback of the signal R₂, to place the system’s closed-loop poles at the required locations.
The advantage of a variable-gain PID control compared to some adaptive control methods is its simplicity in design. Nonetheless, it requires the measurement of the radius of the winding roll.

**State Space Method**

**Introduction**

The requirements on tension control have become greater because of higher speed employed in the plant and new solutions are required. The state space method is proven to be an effective tool to solve complex problems and to improve the dynamic behavior of the control. As a state space control of the complete system is quite complex, and often impractical solution for the web tension problem in industrial plants, this control is transformed into a so-called cascade state space control. It not only provides the advantages of state space control, but also retains the advantageous of cascade structure.
In the state space control, the current, speed and the strain or tension are defined as the state variables. The system to be controlled is described with linear state equations:

\[ \dot{x} = Ax + Bu \]
\[ y = Cx \] (2.20)

The feedback law is defined as,

\[ u = -K \cdot x \] (2.21)

where \( K \) is the feedback control gain that can be designed using various methods. Figure 14a shows the configuration of the state space control.

The objective of optimal control is to minimize the quadratic criterion function,

\[ J = \int_{0}^{\infty} \left[ x^TQx + u^TRu \right] dt \] (2.22)
The matrix $Q$ is the valuation of the state values in the control while the matrix $R$ is the valuation of the inputs of the control. In practice, matrices should be in diagonal form. With the choice of $Q$ and $R$, the engineer is able to design the control system.

The feedback gain $K$ is calculated with the Matrix Riccati Equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0$$  \hspace{1cm} (2.23)

The matrix $P$ is calculated from equation (2.23) and the optimal gain $K$ is derived from:

$$K = R^{-1}B^TP$$  \hspace{1cm} (2.24)

Fortunately, there are efficient software tools to transform the system from the signal-flow graph or transfer description to the state space description corresponding to equation (2.21) and (2.23):

$$\dot{x} = (A - BK) \cdot x + B\omega$$
$$y = C \cdot x$$  \hspace{1cm} (2.25)

The vector $\omega$ describes the reference inputs of the control. All states must be available.

The advantage of this control is that the system behavior can be changed by the gain $K$. There are no limitations, an oscillating system or even an unstable system can be controlled with satisfaction.

**Cascade State Space Control**

Each state space control can be transformed into an equivalent cascade structure. This provides the advantages of both the state space control and the cascade structure of the control. Because the feedback of the state space control is a gain, the common feedback of the states can be transformed into a cascade feedback of all separated states. This approach leads to cascaded circuits with P controllers. To achieve no state-state errors,
the main control value, e.g. the tension, is fed back with an integrator. This leads in the cascade state space control to a PI controller with a delay in the reference input. The structure of a cascade state space control is shown in Figure 15. The parameters of the P and PI controllers are calculated with the equations (2.21), (2.24), and (2.25).

![Figure 15 Cascade State Space Control](image)

Observers

As mentioned above in a state space control, all state values have to be available. Unfortunately, in a real plant situation not all state values can be measured, or the quality of the measurement is poor. In these cases, observers can be used.

An observer is a feedback model of the system which estimates the state values \( \hat{x} \) from well measurable inputs and outputs of the system. The feedback is fed by the error \( (y - \hat{y}) \) of the system and model. The basic equations of an observer in state space are as follow:

\[
\begin{align*}
\dot{x} &= A\hat{x} + Bu + H(y - \hat{y}) \\
\hat{y} &= C\hat{x}
\end{align*}
\]

(2.26)
The feedback gain $H$ is calculated similarly to the gain $K$ of the state space control and gives the dynamic of the observer. Figure 14b shows the structure of an observer with an integrator. The integrator compensates the errors caused by disturbances. From the point of view of control, such an observer is a highly dynamic measuring device with no steady-state error.

**Decentralized Control**

Introduction

As a state space control of the total system is complex and often impractical in industrial plant, decentralized control methods should be used, where the state space control is designed with subsystems of low order. To design a decentralized control, the total system has to be separated into subsystems. As shown in Figure 16, the subsystems are the roller, the electrical drive, and the web section on the left side of the roller. Separation in this manner approximates the actual system which consists of drives, rollers, and web sections.

As mentioned before, each subsystem can be controlled with a low order state space control. But if the controller is designed with the isolated subsystem, it experiences significant deterioration of the dynamic behavior in the total system. Because of the influence of the coupling quantities, oscillations occur, and the forces of the neighboring subsystems also have large dynamic changes. This is a consequence of neglecting the quantities of coupling during the design of the control.
To get a proper dynamic behavior, the quantities of coupling must be taken into account. There are three possibilities to accomplish this as detailed before:

1) The design of decoupling networks;

2) The use of so called equivalent terminating models;

3) The decentralized decoupling.

The first possibility requires the design of a special decoupling network and presupposes the measurement of the quantities of the coupling.

The second possibility requires the design of a low order equivalent model of the controlled neighboring subsystems. If there are only a few subsystems (less than three or four), this method is successful and gives results equivalent to method three. But if the
number of subsystems increases, this method is not recommended because of many iterations needed during the design of the control. The third possibility avoids the disadvantage described. The goal of the method three is to design a controller which minimizes the influence of the remaining system. The state space controller has two functions:

- to guarantee the desired dynamic and stability of the total system, and
- to minimize the influence of the remaining system.

The solution to design such a controller is to consider the sensitivity of the eigenvalues. The advantage of this approach is that no measurements of the quantities of coupling are required. It is only necessary to know where the influences of coupling are active in the subsystem. The designed control is robust against changes of the parameters in a wide range.

**Fuzzy Logic Control**

The linear optimal control methods will provide us a controller which is guaranteed to be the best possible to control the linearized model of the system. Unfortunately, the linearized model is not guaranteed to represent the system accurately, since in reality there is no such thing as a linear system. For example, the strain in the paper of a coating machine changes during the coating and dying, and the friction depends on the temperature. In some cases, fuzzy logic can outperform a linear controller, and sometimes by a wide margin.

A fuzzy logic controller is in principle a non-linear P controller. To find the setting of the fuzzy logic controller it is not necessary to have a mathematical description of the
process. But it requires a good physical knowledge of the process. The rules of a fuzzy logic controller are made with if...than conditions. This way of thinking is close to that of people. In the conventional control, the process is modeled, but in fuzzy logic control the expert is modeled. This fact may explain why some difficult problems are solved better, and in a shorter time, with fuzzy logic control that with a conventional non-linear control.

There are three steps to design a fuzzy logic control.

**Step 1** is the so-called fuzzyfication. This is done with membership functions.

**Step 2** is to create the rules with if...than conditions and

**Step 3** is the defuzzification to get a definite output of the controller.

![Figure 17 Fuzzy Logic Control with Decoupling](image)

Unfortunately, there are no rules like criterion functions to find an optimal fuzzy logic controller. Usually, the optimum is found with the trial and error method. Nevertheless, in many cases, the fuzzy logic control is a method that provides better results.
The block diagram of the fuzzy logic control of the three subsystems is shown in Figure 17 [3]. It should be mentioned that the design of the fuzzy logic controllers was made only of a single subsystem and without knowledge of the influence of coupling.

In large systems, it is advantageous to add decoupling networks to achieve the best results. Here the decoupling was realized with fuzzy logic. The design of the fuzzy logic controller was made with trapezoidal membership functions for the inputs, whereas the membership functions of the output are singletons. In the reference path, an integrator was added to avoid steady-state errors [3].

**Neural Networks**

Today, more and more neural networks are being used to control non-linear systems. In the field of web tension control, there are limitations of the current or torque of the electrical drives. The limitations exist in the steady-state value and in the derivation of the current because of the inverter and motor. With a neural network, it is possible to superpose on a conventional control a self-learning circuit to get a time-optimal control [3]. Another example is the use of neural network to learn the unknown time dependent friction of the mechanical system for compensation [3]. A third example is the compensation of disturbances if a winder runs unbalance. The neural network is able to learn such disturbances. [3]
Summary of The Literature

From the literature review above, it is indicated that web tension control is a very stringent and complex problem. Many researches have been done and several methods have been attempted, but there are some problems with these control methods.

• The PI controller is tuned to provide a responsive system for the entire range of product process as through the enter system. This results in a system that is detuned for a large range of products and optimally tuned for a small range of the products.

• The PI controller is tuned based on observed system performance. Typically this results in stable tension regulation until such time as a product with an extreme physical parameter is processed through the system. Very often this results in unstable tension regulation, which in turn requires re-tuning the loop.

• The advanced control methods, such as state space method and decoupling network, are too complex to implement in industry.

• Nonlinear control methods, such as fuzzy logic control and neural network, are also difficult to design and implement actual practice.

In this research discuss here, a novel nonlinear control structure is introduced and used in web tension control.
CHAPTER III

ACTIVE DISTURBANCE REJECTION CONTROL FOR WEB TENSION REGULATION

The active disturbance rejection control (ADRC) is based on the idea that in order to formulate a robust control strategy, one should start with the original problem in equation (2.19), not its linear approximation in equation (2.18). Although the linear model makes it feasible for us to use powerful classical control techniques such as frequency response based analysis and design methods, it also limits our options to linear algorithms and makes us overly dependent on the mathematical model of the plant.

In this chapter, the concept of ADRC is first introduced. Then, simulation results for a simple model web tension control are given.
3.1 Active Disturbance Rejection Control Concept

Instead of following the traditional design path of modeling and linearizing $f(t, y, \dot{y}, w)$ and then designing a linear controller, the ADRC approach seeks to actively compensate for the unknown dynamics and disturbances in the time domain. This is achieved by using an extended state observer (ESO) to estimate $y$, $dy/dt$, and $f(t, y, \dot{y}, w)$ iteratively. Once $f(t, y, \dot{y}, w)$ is estimated, the control signal is then used to actively compensate for its effect and reduce (2.19) to a double integrator, which in turn becomes a relatively simple control problem. This new method is based on Han’s pioneer work. More details of this novel control concept and associated algorithms can be found in [4] [11].

The Extended State Observer (ESO)

For a general $n$th order plant with unknown dynamics and external disturbances,

$$y^{(n)} = f(y, \dot{y}, \cdots, y^{(n-1)}, w(t)) + u$$  \hspace{1cm} (3.1)

Han proposed a nonlinear observer of the form

$$\begin{cases} 
\dot{z}_1 = z_2 - g(z_1 - y(t)) \\
\vdots \\
\dot{z}_n = z_{n+1} - g_n(z_1 - y(t)) + u \\
\dot{z}_{n+1} = -g_{n+1}(z_1 - y(t)) 
\end{cases} \hspace{1cm} (3.2)$$

where $g_i(\bullet)$, $i = 1, 2, \ldots, n+1$, are appropriate nonlinear functions, $z_i$ is the estimate of $y^{(i-1)}$ ($i \in n$), $z_{n+1}$ is the estimate of the extended state a(t).
This observer is denoted as the Extended State Observer (ESO). Interestingly, if we choose \( g_i(e) = \beta_{0i} e(i = n + 1) \), the ESO takes the form of the classical Luenberger Observer. On the other hand, if \( g_i(e) = \beta_{0i} (e + k_i \cdot \text{sign}(e)) \), then it is consistent with the variable structure method \([11]\).

In order to estimate \( f(t, y, \dot{y}, w) \) without knowing its analytical form, the plant in (2.19) is augmented as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + bu, \quad x_3(t) \Delta f(t, x_1, x_2, w) \\
\dot{x}_3 &= h(t) \\
y &= x_1
\end{align*}
\]  

(3.4)

where \( h(t) \) is the derivative of \( f(t, y, \dot{y}, w) \) and is unknown. The reason for increasing the order of the plant is to make \( f(t, y, \dot{y}, w) \) a state set such that a state observer can be used to estimate it. One such observer is given as \([11]\)

\[
\begin{align*}
\dot{z}_1 &= z_2 - \beta_{01} \cdot \text{fal}(z_1 - y(t), \alpha_i, \delta_i) \\
\dot{z}_2 &= z_3 - \beta_{02} \cdot \text{fal}(z_1 - y(t), \alpha_2, \delta_2) + b_0 u \\
\dot{z}_3 &= -\beta_{03} \cdot \text{fal}(z_1 - y(t), \alpha_3, \delta_3)
\end{align*}
\]  

(3.5)

where \( \beta_{01}, \beta_{02} \) and \( \beta_{03} \) are observer gains, \( b_0 \) is the normal value of \( b \) and \( \text{fal}(\cdot) \) is defined as

\[
\text{fal}(\varepsilon, \alpha, \delta) = \begin{cases} 
\varepsilon^{\alpha} \cdot \text{sign}(\varepsilon), & |\varepsilon| > \delta \\
\varepsilon, & |\varepsilon| \leq \delta
\end{cases}
\]  

(3.6)
This observer is denoted as the extended state observer (ESO) and is the cornerstone of the ADRC method.

Remarks:

1. The nonlinear function in (3.6) is used to make the observer more efficient. It was selected heuristically based on experimental results. Intuitively, it is a nonlinear gain function where small errors correspond to higher gains. This technique is used widely in industrial applications. The graphical interpretation seen in Figure 18 shows that the new function $f_{\text{al}}(x, \alpha, \delta)$ introduces a small linear region in the gain function. The purpose of it is to prevent excessive gain when the error is small, which was known to cause high frequency chattering in some simulation studies.

![Figure 18 Comparison of Linear and Nonlinear Gains](image)

2. If $\alpha_i = 1, 2, 3$, are chosen as unity, then (3.5) is equivalent to the well known Luenburger observer found in linear system theory;
3. Similar to linear observers, the observer equation (3.5) reflects our best knowledge of the plant. The information on the range of $b$ in (2.19) is needed for the selection of $b_0$ in the ESO. If we know more about the plant, such as the roll diameter, which gives us part of $f(t, y, \dot{y}, w)$, then this information should be incorporated into the observer to make it more efficient.

4. It should be noted that there is a linear segment in (3.6) in the neighborhood of the origin. It was discovered experimentally that the nonlinear function in (3.6) makes the observer converge faster than its linear counterpart and the linear section in (3.6) and makes the output of the observer smoother.

5. The proper selection of the gains and functions in (3.5) are critical to the success of the observer. One approach is to design the linear observer first and then gradually increase the nonlinearity to improve the performance. This is particularly helpful for investigators experimenting with this method for the first time.

**The Control Law**

![Figure 19 Structure of the ADRC](image)

- **Figure 19 Structure of the ADRC**
The architecture of the ADRC is shown in Figure 19. It consists of three components: the Profile Generator which provides the desired transient trajectory for tension to follow from the initial value to the set point; the ESO which is described above, and the control law which is defined as

$$u(t) = u_0(t) - z_3(t)/b_0$$  \hspace{3cm} (3.7)

This control law reduces the plant to a double integration and is controlled by the nonlinear PD controller:

$$u_0(t) = K_p f a l(e_p, \alpha_p, \delta_p) + K_D f a l(e_D, \alpha_D, \delta_D)$$  \hspace{3cm} (3.8)

For example, in positioning applications, $e_p = v_1 - z_1$, $e_D = v_2 - z_2$, are “position” and “velocity” error, respectively, $k_p$ and $k_D$ are the gains of the PD controller, and $f a l(\cdot)$ is the nonlinear function defined in (3.6).

It should be noted that the profile generator generates the desired output trajectory, $v_1(t)$, and its derivative, $v_2(t)$. They are then compared to the filtered output, $z_1(t)$, and its derivative, $z_2(t)$. Clearly, the differentiation of the error is obtained without taking the direct differentiation of the set point or the output. This makes the algorithm much less sensitive to noise in the output and discontinuities in the set point $r(t)$.

The critical component here is obviously the ESO. Its parameters need to be tuned properly for the ADRC to work. We find it useful to get a rough linear model from testing data of the real system, based on which the ESO parameters and feedback gains are tuned. It was discovered that once the ESO is properly set up, the performance is quite insensitive to the plant variations and disturbances.
3.2 Web Tension Control Using ADRC

The Matlab/Simulink package from Mathworks was used for the simulation in this research. The simulation is carried out in two stages. The validity of the ADRC is examined in the first stage using the simple linear model in (2.18). Once the parameters are well tuned and successful results are obtained, the ADRC is then tested on a simulated industrial web-line in the second stage.

![Simulink Block Diagram for ESO Tuning](image)

**Figure 20 Simulink Block Diagram for ESO Tuning**

**Proof of the concept**

1. **ESO open loop tuning**

The ESO is first tuned and simulated using the linearized plant in (2.18) with parameters in the ranges of \( \omega \in [3.2,13] \), \( \zeta \in [0.2,0.9] \) and \( k \in [0.8,1.2] \). A discrete version of (3.5-3.8) is used with a sampling rate of 1 kHz. For simplicity, Euler’s formula is used for the discrete approximation. It also shows that this controller should not be
sensitive to numeric inaccuracies. The Simulink diagram is shown in Figure 20. The plant is a linearized second-order model as shown in Figure 21. Its outputs are $y$, $\dot{y}$ and $f(\cdot)$ as discussed above. These make it very easy to compare the ESO outputs with the plant real variables.

![Simulink Diagram of the Second-Order System](image)

Figure 21 Simulink Diagram of the Second-Order System

![Performance of ESO for Simplified Model](image)

Figure 22 The Performance of ESO for Simplified Model ($\omega \in 13 \quad \zeta \in 0.9 \quad k \in 1.2$)
The ESO output and the plant output are compared in Figure 22 and Figure 23, respectively. It should be noted that all three outputs of the ESO track their targets quite well, employing one set of ESO parameters. In the three states of the plant, $z_3$ is more difficult to estimate because it always has a bigger change than the others.
2. ADRC close loop tuning

Based on the ESO parameters from open loop tuning, the whole ADRC can be turned in a close loop with the second order system model. Figure 24 exhibits the Simulink diagram.

Figure 24 Simulink Diagram for ADRC with Second Order System

The ADRC subsystem in Simulink is shown in Figure 25. The resulting PD design becomes quite straightforward. The most encouraging part of the simulation is that similar performance is observed for various parameter settings within the range, demonstrating the robust properties of the controller. Figure 26 and Figure 27 are the simulation result for the second order system. From Figure 26 and 27, it is observed that although ESO cannot follow the y’ and f(.) perfectly, the tension output (y) is very good for both full roll and low roll.
Figure 25 Simulink Diagram of ADRC Subsystem

Figure 26 Simulation Result of ESO in Close Loop for Second Order System

\( \omega = 13, \ \zeta = 0.9, \ k = 1.2 \)
Testing in an industrial setting

Following the successful simulation with the simple model, the ADRC was tested in a full-scale simulation of the Lab-line shown in Figure 3 and Figure 10, which is a four-section process line. The ADRC subsystem in Simulink is shown in Figure 25. There are three tension zones but only one tension feedback loop, which is located on the winder side of the process line. The most critical area for tension regulation is at the winder. A speed control inner loop was not employed for the sake of simplifying the design, and also to make the control problem more challenging.

As the web moves through the line, the diameter of the winder roll varies from 3 inches to 24 inches, which roughly corresponds to variations in the linear model of: \( \omega \)
from 3.2 to 13, $\zeta$: from .2 to .9, and $k$: from .8 to 1.2. Understandably, such changes pose a significant challenge for the controller to be designed for stability and performance, over the entire operating range.

Figure 28 shows the response of web tension loop using the ADRC. The sample period is $t_s=10\text{ms}$, which is consistent with many existing industrial tension application hardware capabilities, such as the Rockwell AutoMax DPS UDC Controller. The parameters are fixed for the ADRC while the tests are conducted at different operating conditions, particularly for different winder diameters. The design specifications call for a 1.5 sec settling time and overshoot of less than 10%. The ADRC meets both of these transient requirements for all diameter changes. Two extreme cases are shown in Figure 28. A constant 20% rated torque pulse disturbance is also added at $t=5\text{ sec}$ to show disturbance rejection properties. Overall, the results are very encouraging.

![Figure 28 Tension Responses At Empty and Full Roll with 20% Torque Disturbance at $t=5\text{ sec}$. ($t_s=10\text{ms}$)](image-url)
Fixed gain controllers cannot come close to this performance. Even the variable gain PID controller used in the industrial application did not perform as well in simulation.

Figure 29 Tension Responses at Empty and Full roll 20% Torque Disturbance at t=5sec. (t_s=1ms)

It was discovered that if a faster sample rate can be used, the ESO can produce a much better estimate, which leads to significantly better results. Figure 29 shows the results of the ADRC, using a t_s=1 ms sample period, for the same two extreme operational cases: the empty roll and full roll. The results are astonishing. There is barely any difference between the two responses. With the analog to digital converter running in the range of a microsecond, a 1 millisecond sampling rate will not be an application problem in the near future.
CHAPTER IV

TIME OPTIMAL CONTROL

In this chapter some new results concerning analysis and synthesis of time optimal control systems are given. The synthetic function for the time optimal control of the second-order discrete time system is obtained by the Isochronic Region method. Based on this new synthetic function, a new nonlinear PD is introduced and used in the new nonlinear control construct. Simulation results for web tension control are given.
4.1 Time Optimal Control Concept

The class of optimization problems for which the sole measure of performance is the minimization of the transition time from an initial state to a target set is called the class of minimum-time problems. One well-known special case of minimal-time controls is the predictor control or “bang-bang” servo.

The guiding principle of time optimal control is the so-called maximum principle. A very general and precise formulation of the maximum principle has been given fairly by a well-known mathematician, L. S. Pontryagin, and his students V. G. Boltyanskii and R. V. Gamkrelidze. References [14]–[17] provide detail.

Time Optimal Intercept Problem

Given the dynamical system (with state $x(t)$, output $y(t)$, and control $u(t)$) defined by the equations

\[
\begin{align*}
\dot{x}(t) &= f(x(t), t) + B(x(t), t)u(t) \\
y(t) &= h(x(t))
\end{align*}
\]

(4.1)

where

- $x(t)$ is an n-dimensional vector
- $y(t)$ is an m-dimensional vector
- $u(t)$ is an r-dimensional vector

$n \geq r \geq m \geq 0$

Thus, $f$ is an n-vector-valued function, $B(x(t); t)$ is a $(n \times r)$-matrix valued function, and $h$ is an m-vector-valued function. Moreover, assume that the components
\( u_1(t), u_2(t), \ldots, u_r(t) \) of the control vector \( u(t) \) are restricted in magnitude by the inequalities
\[
\left| u_j(t) \right| \leq m_j \quad j = 1, 2, \ldots, r
\] (4.2)

Let \( z(t) \) be a vector with \( m \) components. The desired output is termed as \( z(t) \).

Let
\[
e(t) = y(t) - z(t)
\] (4.3)
be the error vector.

Let \( t_0 \) be the initial time and let \( x(t_0) \) be the initial state of the dynamical system.

The goal of time optimal control is to find the control(s) which:

1. Satisfy the constraint (4.2)

2. Drive the system in such a way that at the terminal time \( T \)
\[
e(T) \in E
\] (4.4)

where \( E \) is some specified subset of \( \mathbb{R}_m \)

3. Minimize the response time \( T - t_0 \)
4.2 The Synthetic Function for A Discrete Time Double Integrator

Many research has been done to investigate the time optimal control problem. The simplest second order system, a double integrator, was studied. Professor Han did a lot of work and developed the synthetic function for this simple second-order discrete time system. Here is some result of his research [18].

Given a continuous double integrator system

\[ \dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad |u| \leq r \quad (4.5) \]

The discrete time equation of this system is

\[
X(k + 1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ h \end{bmatrix} u(k) \quad |u| \leq r
\]

where \( h \) is the sampling interval, \( X(k) = [x_1(k) \quad x_2(k)]^T \) is the state at \( k^{th} \) step. The solution of this equation with \( X(0) = [x_1(0), x_2(0)]^T \) as the initial condition is

\[
X(k + 1) = \begin{bmatrix} 1 & (k + 1)h \\ 0 & 1 \end{bmatrix} X(0) + \begin{bmatrix} 1 & kh \\ 0 & 0 \end{bmatrix} u(0) + \cdots + \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} u(k - 1) + \begin{bmatrix} 0 \\ h \end{bmatrix} u(k) \quad (4.7)
\]

where \( u(i) \) is the control single at \( i^{th} \) step. Thus, the initial states, from which the state can be brought back to the origin in \( k+1 \) steps, can be calculated as

\[
X(0) = \begin{bmatrix} (k + 1)h^2 \\ -h \end{bmatrix} u(k) + \begin{bmatrix} kh^2 \\ -h \end{bmatrix} u(k - 1) + \cdots + \begin{bmatrix} 2h^2 \\ -h \end{bmatrix} u(1) + \begin{bmatrix} h^2 \\ -h \end{bmatrix} u(0) \quad (4.8)
\]

With the condition of \( |u| \leq r \), let \( G(k) \) denote the set of all states (initial points) from which the state can be brought back to the origin in \( k \) steps. The set \( G(k) \) is called the K-Isochronic Region [18].
Plotting based on equation (4.8), the isochronic region diagram in the phase plane with \( k \leq 5 \) is shown in Figure 30.

In Figure 30, the switch curve \( \Gamma_1 \) is the connection of \( \ldots, a_{-k}, a_{-(k-1)}, \ldots, a_{-2}, b_{-2}, b_{-3}, \ldots, b_{-(k-1)}, b_{-k}, \ldots \). Below \( \Gamma_1 \) is the region where the control signal is \( u = +r \). The switch curve \( \Gamma_2 \) is the connection of \( \ldots, a_{-k}, a_{-(k-1)}, \ldots, a_{-2}, b_{+2}, b_{+3}, \ldots, b_{+(k-1)}, b_{+k}, \ldots \). Above \( \Gamma_2 \) is the region where the control signal is \( u = -r \). Inside the region between \( \Gamma_1 \) and \( \Gamma_2 \), when \( k \geq 2 \), if the initial state is \( \alpha a_{-k} + (1-\alpha)b_{+k} \), \( 0 < \alpha < 1 \), the control signal chooses \( u = \alpha r - (1-\alpha)r = (2\alpha - 1)r \).

This range is called linear range.

Outside isochronic region \( G(2) \), the points \( a_{zi}, b_{zi}, c_{zi}, i \geq 2 \), are located on six trajectories described algebraically as:
\[ x_1 = \frac{x_2^2 - hr x_2}{2r}, \quad x_2 < 0 \]
\[ x_1 = \frac{x_2^2 + hr x_2}{2r}, \quad x_2 > 0 \]
\[ x_1 = \frac{x_2^2 - 5hr x_2 + 2h^2r^2}{2r}, \quad x_2 < 0 \]  \hspace{1cm} (4.9)
\[ x_1 = \frac{x_2^2 + 5hr x_2 + 2h^2r^2}{2r}, \quad x_2 > 0 \]
\[ x_1 = \frac{x_2^2 - 3hr x_2}{2r}, \quad x_2 < 0 \]
\[ x_1 = \frac{x_2^2 + 3hr x_2}{2r}, \quad x_2 > 0 \]

Let  
\[ z_1 = x_1 + hx_2, \quad z_2 = x_2, \]

Such that the six trajectory functions change to
\[ z_2 - \text{sign}(z_1) \frac{r}{2} (h - \sqrt{\frac{8|z_1|}{r} + h^2}) = -\text{sign}(z_1) h r \]
\[ z_2 - \text{sign}(z_1) \frac{r}{2} (h - \sqrt{\frac{8|z_1|}{r} + h^2}) = \text{sign}(z_1) h r \]  \hspace{1cm} (4.10)
\[ z_2 - \text{sign}(z_1) \frac{r}{2} (h - \sqrt{\frac{8|z_1|}{r} + h^2}) = 0 \]

Let \( Z = (z_1, z_2)^T \), then
\[ g(Z) = z_2 - \text{sign}(z_1) \frac{r}{2} (h - \sqrt{\frac{8|z_1|}{\alpha r} + h^2}) \]  \hspace{1cm} (4.11)

When \( i \geq 2 \), (outside \( G(2) \)),
Then, the time optimal synthetic function is

\[ u(z_1, z_2) = -r \cdot \text{sat}(g(Z), hr) \]  

(4.12)

where

\[ \text{sat}(x, \delta) = \begin{cases} 
\text{sign}(x), & |x| > \delta \\
\frac{x}{\delta}, & |x| \leq \delta 
\end{cases} \]  

(4.13)

Inside the isochronic region \( G(2) \), the states can be forced to reach \((0,0)\) in two steps, the optimal control is

\[
\begin{align*}
0 &= x_1(2) = x_1(1) + hx_2(1) = x_1(0) + hx_2(0) + h(x_2(0) + hu(0)) \\
0 &= x_2(2) = x_2(1) + hu(1) = x_2(0) + hu(0) + hu(1)
\end{align*}
\]

(4.14)

The solution is

\[ u(0) = \frac{x_1(0) + 2hx_2(0)}{h^2} = -\frac{z_1(0) + hz_2(0)}{h^2} \]  

(4.15)

where

\[ |z_1(0)| \leq h^2 r, \quad |z_2(0)| \leq h^2 r \]

Let \( g(Z) = \frac{z_1}{h} + z_2 \), then the control law inside \( G(2) \) is

\[ u(z_1, z_2) = -r \cdot \text{sat}(g(Z), hr) \]  

(4.16)

So the final synthetic functions for time optimal control are:

\[ \delta = hr, \quad \delta_1 = h\delta \]

\[ z_1 = x_1 + hx_2 \quad z_2 = x_2 \]
\[
g(Z) = \begin{cases} 
    z_2 - \text{sign}(z_1) \frac{r}{2}(h - \sqrt{\frac{8|z_1|}{r} + h^2}), & |z_1| \geq \delta_i \\
    z_2 + \frac{z_1}{h}, & |z_2| \leq \delta_i 
\end{cases}
\] (4.17)

\[
sat(x, \delta) = \begin{cases} 
    \text{sign}(x), & |x| > \delta \\
    \frac{x}{\delta}, & |x| \leq \delta 
\end{cases}
\]

\[
u(x_1, x_2) = -r \text{sat}(g(z), \delta)
\]

The phase plane diagram of this time optimal control is shown in Figure 31.

In Figure 31, when initial state is located outside the linear region which is bounded by curves (4), (7) and (5), (6), the control signal is +r or –r. This control signal can force the state to reach to the linear region as shown by trajectories a and b. When it is inside the linear region, the control signal is chosen within [-r, +r] which forces the system to
reach the curves (4) and (5). Then the state moves on curves (4) or (5) and reaches G(2). Then go to (0,0) in one or two steps.

### 4.3 The Synthetic Function for Discrete Time Second-order System

Based on the research of Professor Jingqing Han on discrete time optimal control for double integrator plant [11], the synthetic function for second-order system is further studied here so that it can be extended to a wider class of plants. The synthetic functions are derived, first for a double integrator with an arbitrary gain, and then for a typical second-order system with a nonzero pole.

**The synthetic function for a double integrator with gain**

The transfer function for a double integrator with a gain is:

\[
Y = \frac{\alpha}{s^2} U
\]  

(4.18)

The state space equations are:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \alpha u \\
|u| &\leq r
\end{align*}
\]  

(4.19)

The discrete-time equation is:

\[
X(k+1) = \begin{bmatrix} x_1(k) + hx_2(k) \\ x_2(k) + \alpha hu \end{bmatrix} = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ \alpha h \end{bmatrix} u(k) \\
|u| &\leq r
\]  

(4.20)

where \( h \) is the sampling interval, \( X(k) = [x_1(k) \quad x_2(k)]^T \) is the state at \( k^{th} \) step.
\[ X(k+1) = \begin{bmatrix} 1 & (k+1)h \\ 0 & 1 \end{bmatrix} X(0) + \begin{bmatrix} 1 & kh \\ 0 & 0 \end{bmatrix} u(0) + \cdots + \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} u(k-1) + \begin{bmatrix} 0 \\ \alpha h \end{bmatrix} u(k) \]

(4.21)

where \( u(i) \) is the control single at \( i^{th} \) step. So the initial states, from which the state can be brought back to the origin in \( k+1 \) steps, can be calculated as

\[ X(0) = \begin{bmatrix} \alpha(k+1)h^2 \\ -\alpha h \end{bmatrix} u(k) + \begin{bmatrix} \alpha kh^2 \\ -\alpha h \end{bmatrix} u(k-1) + \cdots + \begin{bmatrix} 2\alpha h^2 \\ -\alpha h \end{bmatrix} u(1) + \begin{bmatrix} \alpha h^2 \\ -\alpha h \end{bmatrix} u(0) \]

(4.22)

Let: \( z_i = x_i + hx_i, \ z_2 = x_2, \ Z = [z_1 \ z_2]^T \), and

\[ g(Z) = z_2 - \text{sign}(z_i) \frac{\alpha r}{2} (h - \sqrt{\frac{|z_i|}{\alpha r}} + h^2) \]

(4.23)

when \( i \geq 2 \), that is \( |z_i| \geq h^2 r \),

\[ g(c_+) = g(c_-) = 0; \quad g(a_+) = g(b_-) = -h\alpha r; \quad g(a_-) = g(b_+) = h\alpha r \]

Consider \( i \leq 2 \), the final time optimal control synthetic functions are:

\[ \delta = h\alpha r, \ \delta_i = h\delta \]

\[ z_1 = x_1 + hx_2, \ z_2 = x_2 \]

\[ g(Z) = \begin{cases} z_2 - \text{sign}(z_i) \frac{\alpha r}{2} (h - \sqrt{\frac{|z_i|}{\alpha r}} + h^2), & \text{if } |z_i| \geq \delta_i \\ z_2 + \frac{z_1}{h}, & \text{if } |z_i| \leq \delta_i \end{cases} \]

(4.24)

\[ \text{sat}(x, \delta) = \begin{cases} \text{sign}(x), & |x| > \delta \\ x, & |x| \leq \delta \end{cases} \]

\[ u(x_1, x_2) = -r\text{sat}(g(Z), \delta) \]
The only different between this function (4.24) and the function described above (4.17) is that \( r \) is replaced by \( \alpha r \) in \( g(Z) \).

The simulation examples using this time optimal control method to control a double integrator with gain are now given. Figure 32 is the block diagram in Simulink. The inputs of the synthetic function are the error and its derivative. Figure 33 shows the simulation results for different gain \( k \), which is the same as the above \( \alpha \) in (4.24). The result shows the efficiency of this time optimal controller.
Figure 33 Simulation Result of Time Optimal Controller for Double Integrator with a Gain
**Synthetic function for second-order system**

The second order system is shown as:

$$Y(s) = \frac{B}{s(s + A)}U(s) \quad (4.25)$$

Most motion control problems can be reduced to this function.

The state space equations are:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -Ax_2 + Bu \\
\end{align*}
\]

$$\left| u \right| \leq r \quad (4.26)$$

The discrete-time equation is:

\[
X(k+1) = \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^{k+1} X(0) + \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^k \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(0) + \cdots + \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix} \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k-1) + \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k)
\]

$$\quad (4.27)$$

where $h$ is the sampling interval, $X(k) = [x_1(k) \ x_2(k)]^T$ is the state at $k^{th}$ step. Let $\alpha \equiv 1 - Ah$.

\[
X(k+1) = \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^{(k+1)} X(0) + \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^k \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(0) + \cdots + \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix} \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k-1) + \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k)
\]

$$\quad (4.28)$$

where $u(i)$ is $i^{th}$ step control single. So the initial states, from which the state can be brought back to the origin in $k+1$ steps, can be calculated as:

\[
\begin{bmatrix}
0 \\
0
\end{bmatrix} = \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^{(k+1)} X(0) + \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^k \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(0) + \cdots + \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix} \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k-1) + \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k)
\]

\[
\begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^{(k+1)} X(0) = -\begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix}^k \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(0) - \cdots - \begin{bmatrix}
1 & h \\
0 & \alpha
\end{bmatrix} \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k-1) - \begin{bmatrix}
0 & \alpha
\end{bmatrix} u(k)
\]
\[ X(0) = \begin{bmatrix} 1 & h^{-(k+1)} & 0 & 0 \\ 0 & \alpha & B h & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & B h \end{bmatrix} u(k) - \begin{bmatrix} 1 & h^{-k} & 0 \\ 0 & \alpha & B h \\ \vdots & \vdots & \vdots \\ 0 & 0 & B h \end{bmatrix} u(k-1) - \cdots - \begin{bmatrix} 1 & h^{-1} & 0 \\ 0 & \alpha & B h \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} u(0) \]

then

\[ X(0) = \left( \frac{B}{\alpha} + \frac{B}{\alpha^2} + \cdots + \frac{B}{\alpha^{k+1}} \right) h^2 \left( -\frac{B}{\alpha^{(k+1)}} h \right) u(k) + \cdots + \left( \frac{B}{\alpha} + \frac{B}{\alpha^2} \right) h^2 \left( -\frac{B}{\alpha^2} h \right) u(1) + \frac{B}{\alpha} h^2 \left( -\frac{B}{\alpha} h \right) u(0) \]

\[ X(0) = \begin{bmatrix} \alpha^{(k+1)} - 1 & -h^2 \\ -h \end{bmatrix} Bu(k) + \cdots + \begin{bmatrix} \frac{1}{\alpha} & \alpha^2 h^2 \\ -h \end{bmatrix} Bu(1) + \begin{bmatrix} \frac{1}{\alpha^2} & \frac{1}{\alpha} h^2 \\ -h \end{bmatrix} Bu(0) \] (4.29)

Usually, \( 0 < Ah \ll 1 \) so \( \alpha = 1 - Ah \approx 1 \),

\[ \lim_{\alpha \to 1} \frac{\alpha^n - 1}{\alpha^n (\alpha - 1)} = \lim_{\alpha \to 1} (\alpha^{n-1} + \alpha^{n-2} + \cdots + \alpha + 1) = n \] (4.30)

So finally

\[ x(0) = \begin{bmatrix} (k+1)h^2 \\ -h \end{bmatrix} Bu(k) + \cdots + \begin{bmatrix} 2h^2 \\ -h \end{bmatrix} Bu(1) + \begin{bmatrix} h^2 \\ -h \end{bmatrix} Bu(0) \] (4.31)

This result is the same as (4.24) above for \( y = \frac{\alpha}{s^2} u \) when \( B = \alpha \).

For an example, if \( \alpha = 1 - Ah = 0.9 \),

\[ x(0) = \begin{bmatrix} 1.1h^2 \\ -1.1h \end{bmatrix} Bu(0) + \begin{bmatrix} 2.35h^2 \\ -1.23h \end{bmatrix} Bu(1) + \begin{bmatrix} 3.35h^2 \\ -1.37h \end{bmatrix} Bu(2) + \cdots \]

So the synthetic functions for this second-order system (when \( 0 < Ah \ll 1 \)) are the same as those for double integrator with a gain (4.24).
The simulation examples using these functions to control the second-order system are shown below.

Figure 34 Block Diagram of Time Optimal Controller for Second-Order System

Figure 34 is the block diagram in Simulink. Here $A$ is the same as in (4.25) and $k$ as $B$ in (4.25). Figure 35 shows the result for different $A$ and $k$ using these synthetic functions (4.24).
Figure 35 Simulation Result of Time Optimal Controller for Second-Order System
Based on the bang-bang control concept, a nonlinear filter-differentiator is proposed by J. Han as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -M \text{sign}(x_1 - r(t)) + \frac{x_1 |x_2|}{2M}
\end{align*}
\] (4.32)

where the output \( y = x_1 \) tracks the reference input \( v(t) \) in shortest time possible without overshoot. Here \( M \) represents the maximum acceleration the system can obtain and is a function of the maximum actuation available in the system.

It was shown [11][19] that \( \forall \varepsilon > 0 \text{ and } T > 0 \) \( \exists M_0 > 0 \) such that if \( M > M_0 \) \( \int_0^T |x_1(t) - v(t)| \, dt < \varepsilon \). Here, the only design parameter of the filter is the gain \( M \), which corresponds to the upper bound of acceleration, if \( x_1 \) can be viewed as a position signal. Parameter \( x_1 \) can track \( v(t) \) arbitrarily fast as long as \( M \) can be chosen arbitrarily large. The parameter \( x_2 \) is the derivative of \( x_1 \) and therefore it approximates the derivative of \( v(t) \), even though it may not exist at certain points. This nonlinear filter is denoted as the Tracking Differentiator (TD).

To improve the numerical properties and avoid high frequency oscillations, a discrete time realization of TD was derived from equation (4.17). [11][19]

\[
\begin{align*}
v_1(t + h) &= v_1(t) + h \, v_2(t) \\
v_2(t + h) &= v_2(t) + h \, \text{fst}_2(v_1(t), v_2(t), v(t), r, h)
\end{align*}
\] (4.33)

where \( v_1 \) and \( v_2 \) are the state variables, \( v(t) \) is the input signal, \( h \) is the step size and the function \( \text{fst}_2 \) is defined as:
The impact of TD is profound. First, as a noise filter, it blocks any part of the signal with acceleration rates exceeding M. In practice, we often know the physical boundary of a signal in terms of its acceleration rate. This knowledge can be conveniently incorporated into TD to reject noise based on the understanding of the physics of the plant. On the other hand, the traditional linear filter can only attenuate noises based on its frequency contents.

Secondly, it is shown [11][19] that as a filter TD has a very desirable frequency response characteristics. In particular, it has a much smaller phase shift compared to linear filters, while maintaining an extremely flat gain within the pass band.

Finally, perhaps the most important role of TD is its ability to obtain the derivative of a noisy signal with a good signal to noise ratio. It is well known that a pure differentiator is not physically implementable. The error is often not differentiable in practice due to the noise in the feedback and the discontinuities in the reference signal. This explains why the PID controller is used primarily as a PI controller in most applications. The use of the “D” part has been quite limited due to the extreme amplification of noise by differentiation, or its approximations. This noise problem is resolved in TD because $x_2$ is obtained via
integration. This idea of using integration to obtain differentiation goes back to 1920s when N. Wiener proposed the definition of “fractional differentiation” based on integration. It leads to the concepts of generalized function and generalized derivative, which were used widely in the theory of partial differential equations.

4.5 A New Control Scheme Based on Time Optimal Control

The equation (4.17) can be thought of as a new nonlinear PD based on time optimal control method. First the error signal of the set point and the feedback signal are passed through a second order TD. The TD generates the error signal and its derivative, and inputs these signals into a nonlinear PD, NPD. Function g(t) is based on equation (4.24). Figure 36 is the diagram of new nonlinear controller. The new controller also introduces the motion profile method into control algorithm. By setting the motion profile, the system is made robust to oscillations and it gets the desired speed signal.

![Figure 36 Diagram of New PD Controller Based on Time Optimal Control](image-url)
Using this method in the web tension control, if the controller step size is too big (say 10 ms), the controlled result will have a large steady state error. The error becomes large because there is no integrator in the controller. A linear integrator can be used in the controller, but this would result in a slow response. Therefore, an ESO is added to the controller to deal with the steady state error. Because ESO estimates the plant’s output and its derivative, TD is deleted from the system. The structure of the new controller is shown in Figure 37.

ESO changes the plant into a double integrator, and the nonlinear PD, which is driven from double integrator time optimal controller, will control it easily. But it makes the controller more complex, and the ESO tuning becomes the key to the whole controller design.

![Figure 37 Diagram of New Nonlinear PD with ESO](image)

Another problem is the $r$ in the $g(t)$ function (4.24). Usually $r$ is chosen large enough to let the system run fast which rejecting disturbance. But in the real world, $r$ is the maximum output of the controller, and it cannot be extremely large. This problem is solved by letting $r$ large enough in the $g(t)$ function and multiply a small gain outside the
g(t) function. This makes the control output less than or equal to the maximum physical controller output, and it makes the controller have a fast response and adequate disturbance rejection. This has been proved in section 4.2.

In order to get the ESO parameters and test the performance of the new nonlinear PD, the new controller is tuned first with the simpler second order model. The plant is shown as in Figure 21 and is configured to get the f(.) signal and to tune the ESO parameters. The simulation block in Simulink is shown in Figure 38.

The controller is tuned in two sampling times: 10 ms and 20 ms. The second model has parameters in the ranges of $\omega \in [3.2, 13]$, $\zeta \in [0.2, 0.9]$ and $k \in [0.8, 1.2]$. A discrete version of (4.24) is used with a sampling rate of 100 Hz first, then changes to 50Hz. This low sample rate (20ms) does not work using ADRC as discussed in chapter III. The step responses are shown in Figure 39 to 42. During controller tuning, it is found that after
determining the ESO parameters, it is very simple to design the time optimal based nonlinear PD. Each parameter in the new nonlinear PD has its physical meaning. Based on these, the parameter setup is much easier.

Figure 39 Tension Response for Second-Order System Using Nonlinear PD with ESO
\( (\omega \in 13 \; \zeta \in 0.9 \; k \in 1.2 \; Ts=20ms) \)
Figure 40 Tension Response for Second-Order System Using Nonlinear PD with ESO
\((\omega \in 3.2 \quad \zeta \in 0.2 \quad k \in 0.8 \quad Ts=10ms)\)

Figure 41 Tension Response for Second-Order System Using Nonlinear PD with ESO
\((\omega \in 13 \quad \zeta \in 0.9 \quad k \in 1.2 \quad Ts=20ms)\)
Figure 42 Tension Response for Second-Order System Using Nonlinear PD with ESO

\[ (\omega \in 3.2 \quad \zeta \in 0.2 \quad k \in 0.8 \quad T_s=20\text{ms}) \]

4.6 Web Tension Model Simulation

The industrial lab-line model shown in Figure 3 and Figure 4 is used to tune the new nonlinear controller. As in ADRC tuning, no speed inner loop is used for the tension regulation. Two sample rates are used to tune the controller, 10 ms and 20 ms. This shows the power of the new nonlinear controller. Note that ADRC cannot adequately support with the tension control using the 20 ms sample rate. The new controller subsystem diagram implemented in Simulink is shown in Figure 43.
Figure 43 Simulink Subsystem Diagram of Nonlinear PD with ESO

Figure 44 Tension Response for Web Tension Control Using Nonlinear PD with ESO
(Full Roll, Ts=10ms)
Figure 45 Tension Response for Web Tension Control Using Nonlinear PD with ESO (Low Roll, Ts=10ms)

Figure 46 Tension Response for Web Tension Control Using Nonlinear PD with ESO (Full Roll, Ts=20ms)
It was discovered that even if a slower sample rate is used, the new nonlinear controller still functions. If a faster sample rate can be used, the ESO can produce a much better estimate, which leads to significantly improved results.

These results show the new nonlinear PD, that is based on time optimal control theory, has better performance then the $f(t)$ function which is used in ADRC. The meaning of each parameter in the nonlinear PD is very clear.

In this chapter and the previous chapter, the speed inner loop is not used when tuning the new nonlinear controller. By doing so, the design is simplified and the control problem is made more challenging. But if the speed loop was used, the nonlinear controller would become easier to tune and the performance would be better. Some results of tension control using speed inner loop are shown in Figure 48 and 49.
Figure 48  Tension Response Using Nonlinear PD with ESO and Speed Inner Loop (Full Roll, 0.01s)

Figure 49  Tension Response Using Nonlinear PD with ESO and Speed Inner Loop (Low Roll, 0.01s)
Using a speed inner loop, the plant is made readily controllable. The benefit of ESO is also to change a plant to a double integrator, thereby simplifying control system design. Based on this idea, internal compensation methods will be used to increase the controller power as described in the next chapter.
CHAPTER V
ADRC WITH INTERNAL COMPENSATIONS

The primary problem of web tension control is due to the plant changing greatly while processing. In this chapter, two types of internal compensations will be introduced. Changing the plant’s damping is a simple and efficient method for web tension control. The idea is that some information on the plant conditions can be obtained from the ESO. If this information is used to make the plant change in a small range, the controller design can become simpler.
5.1 ADRC with Speed Compensator – The Damping Factor

Consider a standard motion equation \[11\]

\[
J\ddot{y} = -\alpha \dot{y} + k_i u
\]  

(5.1)

where \(y\) is the position of the motor shaft, \(u\) is the current in the motor armature, \(J\) is the inertia, \(\alpha\) is the friction coefficient and \(k_i\) is the motor torque constant. If \(\alpha\) is very small, the plant will be highly under-damped, making it harder to control. In web tension regulation, when the roller is very small or even empty, the system is in highly under-damped situation. This can be changed, however, with a derivative control. For example, the control law \[11\]

\[
u = -\beta \dot{y} + u_0
\]  

(5.2)

will result in a dynamically compensated plant

\[
J\ddot{y} = -(\alpha + \beta) \dot{y} + k_i u_0
\]  

(5.3)

which is less under-damped and easier to control. Here \(u_0\) is the new control input which can be designed, for example using (3.8).

For the above reason, it can be readily understood why derivative control adds “damping” to the system. This notion can also be generalized to other linear and nonlinear systems. For example, with the standard second order plant of

\[
\ddot{y} = -2\zeta \omega_n \dot{y} + \omega_n^2 y + \omega_n^2 u
\]  

(5.4)

the control law of

\[
u = \frac{-2\zeta \omega_n}{\omega_n} \dot{y} + u_0
\]  

(5.5)

we get
\[ y = -2(\zeta + \zeta_1)\omega_n y + \omega_n^2 y + \omega_n^2 u_0 \]  \hspace{1cm} (5.6)

and this changes the damping ratio of the plant from \( \zeta \) to \( (\zeta + \zeta_1) \).

Based on this consideration, the ADRC can be changed to the new control structure shown in Figure 50.

![Diagram](image)

*Figure 50 Structure of ADRC with a Damping Compensator*

The second order system is FIRST used to test the concept. The Simulink subsystem block diagram is shown in Figure 51. Figure 52 and 53 are the ESO output for this method. A better result is achieved than where using ADRC only.
Figure 51 Simulink Subsystem Diagram for ADRC with Damping Compensator

Figure 52 ESO Output for Second Order System Using ADRC with Damping Compensator ($\omega = 13$, $\zeta = 0.9$, $k = 1.2$)
The new controller is then tuned with the industrial tension model. The parameters are fixed for the ADRC while the tests are conducted at different operating conditions, particularly for different winder diameters. The design specifications call for a 1.0~1.2 sec settling time and overshoot of less than 3%. The ADRC meets both of these transient requirements for all diameter changes. Two extreme cases are shown in Figure 54. A constant 20% rated torque pulse disturbance is also added at t=5 sec to show disturbance rejection properties. Compared to only using the ADRC (Figure 28), the results are very encouraging.
5.2 ADRC with Internal Model Compensator

From the analysis of the ADRC configuration in Chapter III, it is shown that the ADRC is a new control method dealing with the problem, which the model of the plant is unknown or the change of the plant is unknown. The key to solve this problem is the nonlinear observer ESO as shown in equation (3.5). The nonlinear gains in ESO are intended to make the observer converge faster. The state $z_3$ in ESO is used to estimate the plant unknown and the change of the plant. If the plant has a large change, $z_3$ will also have a large changing range. Adequate closed loop control with ADRC will require an accurate estimation of $z_3$. Without the knowledge of the plant, say $f(t, y, \dot{y}, w)$, the
observer will be hard pressed to provide reasonably accurate estimations in real time, or
the observer tuning will be very difficult.

One way to get a good estimation of \( f(t, y, \dot{y}, w) \) is to reduce the amplitude of the
plant change. If all or part of the model of the plant, say \( f_i(t, y, \dot{y}, w) \), is known, then it
should be incorporated into the observer as

\[
\begin{align*}
\dot{z}_1 &= z_2 - \beta_{01} f_1(z_1 - y(t), \alpha_1, \delta_1) \\
\dot{z}_2 &= f_i(t, y, \dot{y}, w) + z_3 - \beta_{02} f_2(z_1 - y(t), \alpha_2, \delta_2) + b_0 u \\
\dot{z}_3 &= -\beta_{03} f_3(z_1 - y(t), \alpha_3, \delta_3)
\end{align*}
\] (5.7)

In this equation, \( z_1 \) and \( z_2 \) are still used to estimate \( y \) and \( \dot{y} \). But \( z_3 \) will now track
\( a(t) = f(t, y, \dot{y}, w) - f_i(t, y, \dot{y}, w) \) and make the observer more efficient. The control law
(3.7) will be changed to

\[
u(t) = u_0(t) - \frac{z_3(t) + f_i(t, y, \dot{y}, w)}{b_0}
\] (5.8)

Even if \( f_i(t, y, \dot{y}, w) \) is only part of \( f(t, y, \dot{y}, w) \), it will still reduce the difficulty of
the observer to estimate the plant. Figure 55 is the configuration of this new controller
method -- ADRC with internal model compensator. The \( f_i(t, y, \dot{y}, w) \) is used to simplify
the design of the observer and the controller. The remainder of the controller will be not
changed. The nonlinear PD can use equation (3.8) or equation (4.24) which is based on
time optimal control. The Simulink subsystem diagram is shown in Figure 56.

The simulation results for the second order system and for the web tension regulation
are show in Figure 57, 58 and 59. A fixed model is used because the exact changes of the
plant is not known in the processing cycle. The results are better then using ADRC only.
Figure 55 ADRC with Internal Model Compensator Configuration

Figure 56 Simulink Subsystem Diagram for ADRC with Internal Model Compensator
Figure 57 ESO Output for Second Order System Using ADRC with Internal Model Compensator ($\omega = 13$, $\zeta = 0.9$, $k = 1.2$)

Figure 58 ESO Output for Second Order System Using ADRC with Internal Model Compensator ($\omega = 3.2$, $\zeta = 0.2$, $k = 0.8$)
Figure 59 Tension Responses At Empty and Full Roll with 20% Torque Disturbance at $t=5\text{sec}$ Using ADRC with Internal Model Compensator. ($t_s=10\text{ms}$)
CHAPTER VI

SUMMARY AND CONCLUSION

Research performed in this dissertation is summarized in this chapter including web tension control literature review, web plant modeling, proposed control structure and design approach, and controller design and simulation. Implementation issues and future research recommendation are also discussed.
6.1 Summary

Most of the products we use today have as their origins a web form. The guiding and transport of webs have been studied for many years. Tension controls are widely used in web transport and strip processing systems. The main purpose of web tension regulation is to maintain the physical integrity of the material that is being processed.

The requirements on a tension control are becoming more demanding because of higher speeds in the plant, therefore better control solutions are required. The tension control problem in strip/web processing applications is a complex one because the system dynamics are a function of many process variables that often change over a wide range. Due to their importance in industry and relative difficulty, tension problems have drawn the attention of many researchers. One problem is the establishment of a proper mathematical model. Campbell and Brandenburg studied the longitudinal dynamics of a moving web.

There are two common approaches used in web processing industries for tension control: open-draw control and closed-loop control. In the “draw control” scheme, tension in a web span is controlled in an open-loop fashion by controlling the velocities of the rollers at either end of the web span. W. Wolfermann and D. Schroder used an optimal output feedback method to control the speed of the driven rollers. A decentralized observer was designed to decouple the drives from the web tension acting on the driven rollers and this information is used to improve the speed control of the driven rollers. This method leads to considerable improvement in the speed responses of the driven rollers. An inherent drawback of indirectly controlling tension through speed control is its dependency on the open loop relationship between the speed and tension.
This control method cannot reject disturbances due to “tension transfer” from adjacent web spans and interaction between adjacent web spans through an intermediate driven roller. It should be noted that tension is also affected by the change in temperature, material, thickness, as well as other operating variables. This method is also very sensitive to noise originating in the speed feedback devices.

The proportional-integral-derivative (PID) control approach is the primary feedback control law used in industry. For tension feedback control, however, because of the significant variations in system dynamics, PID alone has been shown to be inadequate. K. Reid, K. Shin and K. Lin proposed the fixed-gain and variable-gain PID control of web tension in winding section. For variable gain PID, the control parameters are continuously updated based on the diameter of the roller, which is a major contributor to the system dynamics. This method uses pole placement techniques.

A system model is almost always needed for controller design. This research is motivated by the complexity of the control problem encountered in a web line testing facility, referred to as the Lab-line, and is shown in Figure 4 ~ Figure 6. It was used to evaluate web handling control strategies. After the mathematical analysis for web transport system is described, an industrial web transport system model and a simplified model are derived. Finally a generic nonlinear function (2.19) is summarized.

The Active disturbance rejection control (ADRC) is based on the idea that in order to formulate a robust control strategy, one should start with the original problem in (2.19), not its linear approximation in (2.18). Instead of following the traditional design path of modeling and linearizing $f(t, y, \dot{y}, w)$ and then designing a linear controller, the ADRC approach seeks to actively compensate for the unknown dynamics and disturbances in the
time domain. This is achieved by using an extended state observer (ESO) to estimate $y$, $dy/dt$, and $f(t, y, \dot{y}, w)$ iteratively. Once $f(t, y, \dot{y}, w)$ is estimated, the control signal is then used to actively compensate for its effect and reduce (2.19) to a double integration, which in turn becomes a relatively simple control problem. Simulation shows the efficacy of this new control structure.

The class of optimization problems for which the sole measure of performance is the minimization of the transition time from an initial state to a target set is called the class of minimum-time problems. The synthetic function for second order system is driven and proved based on the Isochronic Region method. The synthetic function can be used as the new nonlinear PD. The first example of the usage of this synthetic function is to build a nonlinear filter, which is denoted as the Tracking Differentiator (TD). The most important role of TD is its ability to obtain the derivative of a noisy signal with a good signal to noise ratio. Then the new nonlinear PD is configured into the ADRC structure. The simulation results show the effectiveness of the new controller in coping with large dynamic variations commonly seen in web tension applications.

Finally, two compensation methods are introduced with the ADRC configuration. Changing the plant’s damping is a simple and efficient method for under-damped system control. The most difficult part of web tension control is the large change of the plant while the web is processing. If we can use a mathematical method to reduce this change, the controller design will be simplified. The nonlinear observer ESO can estimate the states of the plant. If all or part of the model of the plant, say $f_i(t, y, \dot{y}, w)$, is known, then it should be incorporated into the observer in an inner loop and reduce the plant’s variations. The effectiveness of these two methods is proved by simulation results.
Through this research, the Active Disturbance Rejection Control (ADRC), including time optimal nonlinear PD and some compensator, is applied to deal with significant dynamic change in the web transport processes. The new control algorithm is simulated in digital form on a simulation of an industrial process with very encouraging results. This is a promising new solution for web applications because: 1) it’s intuitive; 2) it does not require an explicit mathematical model of the plant under control; 3) it is inherently robust. It was shown that once the ADRC controller is set up properly, it could handle a large range of dynamic changes.

### 6.2 Future Research

Based on the study and tuning experience of the new nonlinear control structure, we find the critical component here is obviously the ESO. Its parameters need to be tuned properly for the ADRC to function properly. We find it useful to get a rough linear model from test data of the real system, based on which the ESO parameters and feedback gains are tuned. Once the ESO is properly set up, the performance is quite insensitive to the plant variations and disturbances. The ideal solution is to find how to tune the observer automatically.

Instead of using the nonlinear function fal(x, α, δ) in ESO, if linear gains are used, the observer becomes a linear observer. We can first determine the linear observer, then design the ESO based on this information.
Since the web tension regulation problem is studied based on an industrial application model, and detailed aspects like sampling time and plant parameters are all considered, the whole approach proposed in this thesis is realistic and desirable for hardware-in-the-loop validation and further hardware implementation.
REFERENCES


A. Matlab m-file for Lab-line Model Parameters

% LABLINEU.M  Simplified model - Lab Line
% Lab Line Model parameters
% Yi Hou, Cleveland State University
% December, 2000
% Low_Side: Di0=24, Di3=3
%

WR2ec0=6.5; % Inertia (empty-core, Drive 0) [lb ft^2]
WR2pr0=5.9; % Inertia (product, Drive 0) [lb ft^2]
WR21=1.5; % Inertia (Drive 1) [lb ft^2]
WR22=0.8; % Inertia (Drive 2) [lb ft^2]
WR2ec3=6.5; % Inertia (empty-core, Drive 3) [lb ft^2]
WR2pr3=5.9; % Inertia (product, Drive 3) [lb ft^2]
Dec0=3; % Diameter (empty-core, Drive 0) [in.]
Dfr0=24; % Diameter (full-roll, Drive 0) [in.]
Dbar0=Dfr0/Dec0; % DBAR (Drive 0)
Di0=24; % Diameter Unwind (instantaneous, Drive 0) [in]
%Di0 recalc below (2/18/97 change)
D0=Di0; % label used for diameter in K01 and K02 (below)
Dbar0=Di0/Dec0; % DBAR_i (instantaneous, Drive 0)
N0=WR2pr0/WR2ec0/(Dbar0^4-1); % per-normal ratio for WR2 calc
WR2i0=1+N0*(Dbari0^4-1); % WR2BAR_i (instantaneous, Drive 0)
D1=12; % Diameter (Drive 1)
D2=6; % Diameter (Drive 2)
Dec3=3;
Dfr3=24;
Dbar3=Dfr3/Dec3;
Di3=3; % Diameter Wind (instantaneous, Drive 3) [in.]
Di0 = sqrt(24^2+3^3-Di3^2); % Calculated based on: Di3 gains what Di0 loses
D0=Di0;
D3=Di3;
Dbar3=Di3/Dec3;
N3=WR2pr3/WR2ec3/(Dbar3^4-1);
WR2i3=1+N3*(Dbari3^4-1);
GR0=3.795; % Gear Ratio
GR1=13.95;
GR2=7.01;
GR3=3.795;

HP0=3; % Horsepower [hp]
HP1=3;
HP2=1.5;
HP3=3;

V=350/60; % Line Speed [ft/sec]

% Motor speeds, Drive 0 [rpm]
Smin0=400; % minimum
Sb0=Smin0; % base
Swf0=1200; % fully-weakened field
Si0=60*V*GR0/(pi*Di0/12); % instantaneous
Fbar0=Swf0/Sb0;
Fbari0=Si0/Sb0;
if Fbari0 < 1
    Fbari0 = 1;
end
if Fbari0 > Fbar0
    Fbari0 = Fbar0;
end

Sb1=1750; % Motor speed, Drive 1 (base) [rpm]
Sb2=1750; % Motor speed, Drive 2 (base) [rpm]
Smin3=400; % Motor speeds, Drive 3 [rpm]

Sb3=Smin3;
Swf3=1200;
Si3=60*V*GR3/(pi*Di3/12);
Fbar3=Swf3/Sb3;
Fbari3=Si3/Sb3;
if Fbari3 < 1
    Fbari3 = 1;
end
if Fbari3 > Fbar3
    Fbari3 = Fbar3;
end

% Conversion factors: per-normal ® engineering units
% rotational speed (motor-side) ® linear speed (load-side)
K01=Sb0*D0*pi/60/12/GR0;
K11=Sb1*D1*pi/60/12/GR1;
K21=Sb2*D2*pi/60/12/GR2;
K31=Sb3*D3*pi/60/12/GR3;
% Conversion factors: engineering units ® per-normal
% tension (load-side) ® torque (motor-side)
K02=D0*Sb0/24/GR0/5250/HP0;
K12=D1*Sb1/24/GR1/5250/HP1;
K22=D2*Sb2/24/GR2/5250/HP2;
K32=D3*Sb3/24/GR3/5250/HP3;

E=2e5;       % Modulus of elasticity [psi]
W=14;        % Web width [in.]
H=.0045;     % Web thickness [in.]
A=W*H;       % Cross sectional area [in.^2]
L1=20;       % Web length [ft]
L2=12;
L3=15;
KT1=E*A/L1;   % Elastic (spring) constant [lb/ft]
KT2=E*A/L2;
KT3=E*A/L3;
J0=WR2i0*WR2ec0*Sb0^2/(308*5250*HP0); % JBARi
J1=WR21*Sb1^2/(308*5250*HP1);
J2=WR22*Sb2^2/(308*5250*HP2);
J3=WR2i3*WR2ec3*Sb3^2/(308*5250*HP3);

% Crossover frequencies, desired [rad/sec]
Wco0_s=15;    % Speed-loop, Drive 0
Wco1_s=15;    % "        Drive 1
Wco2_s=15;    % "        Drive 2
Wco3_s=15;    % "        Drive 3
Wco0_p=4;    % Position-loop, Drive 0
Wco2_c=Wco2_s*4/3; % Current-loop, Drive 2
Wco3_t=7;    % Tension-loop, Drive 3

LIM0=1.5;     % LIM_BAR
LIM1=1.5;
LIM2=1.5;
LIM3=1.5;

PC0=.1;       % Percent-control factor [fraction]
PC1=.1;
PC2=.1;
PC3=.1;

Tmax=240;     % Maximum tension, Drive 3 [lb]
Zspd0=1.1;    % Damping ratio, speed-loop, Drive 0
Zpos0=3;      % Damping ratio, position-loop, Drive 0
Zspd1=1.1;    % "        Drive 1
Zspd2=1.5;    % "        Drive 2
Zspd3=1.1;    % "        Drive 3

% Drive 0
% Speed-control parameters
Kps0=Wco0_s*J0*Fbar0*Fbari0/LIM0;
if Kps0 > 60 % Kps_max = 60
Kps0 = 60;
Wco0_s = Kps0*LIM0/(J0*Fbar0*Fbari0);
end
Wlds0 = Wco0_s/(4*Zspd0^2);

% Position-control parameters
Wfldp = Wco0_s/Zpos0^0.5; % Feedback lead
Wflgp = Wfldp*20; % Feedback lag
if Wfldp > 60
Wfldp = 60;
end
if Wflgp > 120
Wflgp = 120;
end
Wldp0 = Wco0_p/(5*Zpos0);
L = 44/12; % 4 strands x 11"/strand
PBAR = V; % Max line speed
Kpp0 = Wco0_p*L/PBAR/PC0;
M = 10/32.2; % Mass of dancer [slugs]

% Drive 1
% Speed-control parameters
Kps1=Wco1_s*J1/LIM1;
if Kps1 > 60
Kps1 = 60;
Wco1_s = Kps1*LIM1/J1;
end
Wlds1 = Wco1_s/(4*Zspd1^2);

% Drive 2
% Speed-control parameters
Kps2=Wco2_s*J2/LIM2;
if Kps2 > 60
Kps2 = 60;
Wco2_s = Kps2*LIM2/J2;
end
Wlds2 = Wco2_s/(4*Zspd2^2);

% Current-control parameters
Wldc2=2.0*Wco2_c;
Kpc2=1/(PC2*Wldc2*J2)*(Wco2_c/Wco2_s)*(1+Wco2_s*Wlds2*J2/(K21*K22*KT2))
;
% Drive 3
% Speed-control parameters
Kps3=Wco3_s*J3*Fbar3*Fbari3/LIM3;
if  Kps3 > 60
    Kps3 = 60;
    Wco3_s = Kps3*LIM3/(J3*Fbar3*Fbari3);
end

% Tension-control parameters
KT_3 = K31*K32*KT3;
WP_3 = sqrt(KT_3/J3);
Zten3_est = 0;
Design = 1;
if  WP_3 < 50
    if  WP_3 < 5
        Design = 2
    else
        % Assume Design 3 and check zeta
        Wlds = Wco3_s / 10;
        WLDS = Wco3_s/162 * (1 + sqrt(1 + (18*WP_3/Wco3_s)^2));
        if  WLDS < Wlds
            Wlds = WLDS;
        end
        WT = sqrt(WP_3^2 + Wco3_s*Wlds);
        Zten = (Wco3_s + Zten3_est*sqrt(WP_3)) / (2*WT);
        if Zten < 1.1
            Design = 3
        else
            % Making sure zeta > 1 (when using Design 2 eqs)
            Wlds = Wco3_s / 10;
            WLDS = WP_3^2 / (Wco3_s + 2*Zten3_est*sqrt(WP_3));
            if  WLDS < Wlds
                Wlds = WLDS;
            end
            WT = sqrt(WP_3^2 + Wco3_s*Wlds);
            Zten = (Wco3_s + Zten3_est*sqrt(WP_3)) / (2*WT);
            if  Zten > 1
                Design = 2
            else
                Design = 3
            end
        end
    end
end

if Design < 2
% Design 1
% Speed-loop tuning parameters
Design = 1 % Prints out design #
Wlds3 = Wco3_s/10;
WLDS3 = Wco3_s/32*(1+sqrt(1+(8*WP_3/Wco3_s)^2));
if Wlds3 > WLDS3
    Wlds3 = WLDS3;
end
% Tension-loop tuning parameters
WT_3 = WP_3*sqrt(1+Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3));
Wldt3 = WT_3/2;
if Wldt3 > 15
    Wldt3 = 15;
end
Y3 = Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3);
Kpt3 = 0.5*(1+sqrt(1+(8*WP_3/Wco3_s)^2));
end
% Tension-loop tuning parameters
WT_3 = WP_3*sqrt(1+Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3));
Zten3 = (Wco3_s + 2*Zten3_est*sqrt(WP_3))/(2*WT_3);
WA_3 = WT_3*(Zten3 - sqrt(Zten3^2 -1 ));
WB_3 = WT_3*(Zten3 + sqrt(Zten3^2 -1 ));
Wldt3 = WA_3;
if Wldt3 > 15
    Wldt3 = 15;
end
Y3 = Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3);
% Kpt3 = 3*(1+sqrt(1+2*(1+sqrt(1+(8*WP_3/Wco3_s)^2))));
Kpt3 = 0.5*(WA_3+WB_3)*sqrt(Wldt3*(1+Y3))*K32*Tmax*Dbari3/(Kps3*LIM3*PC3/Fbari3);
end
% Feedback lead/lag - set at high freq so not in ckt
Wfldt = 1000;
Wflgt = 1000;
elseif Design < 3
% Design 2
Wlds3 = Wco3_s/10;
WLDS3 = WP_3^2/(Wco3_s + 2*Zten3_est*sqrt(WP_3));
if Wlds3 > WLDS3
    Wlds3 = WLDS3;
end
% Tension-loop tuning parameters
WT_3 = WP_3*sqrt(1+Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3));
Zten3 = (Wco3_s + 2*Zten3_est*sqrt(WP_3))/(2*WT_3);
WA_3 = WT_3*(Zten3 - sqrt(Zten3^2 -1 ));
WB_3 = WT_3*(Zten3 + sqrt(Zten3^2 -1 ));
Wldt3 = WA_3;
if Wldt3 > 15
    Wldt3 = 15;
end
Y3 = Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3);
Kpt3 = 0.5*(WA_3+WB_3)*sqrt(Wldt3*(1+Y3))*K32*Tmax*Dbari3/(Kps3*LIM3*PC3/Fbari3);
end
% Feedback lead/lag - set at high freq so not in ckt
Wfldt = 1000;
Wflgt = 1000;
else
% Design 3
% Speed-loop tuning parameters
Wlds3 = Wco3_s/10;
WLDS3 = Wco3_s/162*(1+sqrt(1+(18*WP_3/Wco3_s)^2));
if  Wlds3 > WLDS3
    Wlds3 = WLDS3;
end

% Tension-loop tuning parameters
WT_3 = WP_3*sqrt(1+Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3));
Wldt3 = WT_3/1.5;
if  Wldt3 > 15
    Wldt3=15;
end
Wco3_t = WT_3/3;
if  Wco3_t > 7
    Wco3_t=7;
end
Y3 = Kps3*LIM3*Wlds3/(KT_3*Fbar3*Fbari3);
Kpt3 = (Wco3_t/Wldt3)*(1+Y3)*K32*Tmax*Dbari3/(Kps3*LIM3*PC3/Fbari3);

% Feedback lead/lag
Wfldt = WT_3/2;
Wflgt = 7*Wfldt;
end

%Tuning values set on lab-line controllers
%Kpt3=3*Dbari3/(Fbari3*WR2i3)
%Wldt3=4;
%Wfldt=15;
%Wflgt=60;
%Wlds3=3.1;