II. Control Design Practice

- Modeling
- System Configurations
- Classical Design Techniques
- Advanced Techniques
- Simulation and Evaluation
Modeling

- Intuitive Model
  - Understanding of the cause-effect relationship
  - Fly ball governor, first industrial controller, speed regulation of steam engine, 1769
  - Error model: GNC in early years

- Mathematical Model
  - Differential Equation
  - Transfer Function
  - State Space
Open Loop

\[ G_c(s) \quad \rightarrow \quad G_p(s) \]

reference input \rightarrow \quad output

\[ r \quad \rightarrow \quad u \quad \rightarrow \quad y \]
Example: DC-DC Converter Circuit

![DC-DC Converter Circuit Diagram]

- **V1**: 120Vdc
- **R1**: 87.33k
- **C1**: 4u
- **L1**: 0.028mh
- **R2**: 31.124k
- **C2**: 4u
- **L2**: 0.026mh
- **C3**: 3000u
- **L3**: 0.027mh
- **D1**: D1N4002
- **R3**: 35.46
- **C4**: 2.2n
- **R4**: 35.46
- **C5**: 2.2n
- **S1**: VON = 1.0V, VOFF = 0.0V
- **S2**: VON = 1.0V, VOFF = 0.0V
- **S3**: VON = 1.0V, VOFF = 0.0V
- **S4**: VON = 1.0V, VOFF = 0.0V
- **L4**: 0.078m
- **D2**: D1N4002
- **R5**: 10.42
- **C6**: 15n
- **C7**: 15n
- **R6**: 10.42
- **C8**: 3290u
- **C9**: 1.075
- **V2**: V1 = 0V, V2 = 1V, TD = 23us, TR = 1ns, TF = 1ns, PW = 23us, PER = 46us
- **V3**: V1 = 0V, V2 = 1V, TD = 0us, TR = 1ns, TF = 1ns, PW = 23us, PER = 46us
- **CLK_1**: 30:1
- **CLK_2**: 30:1
Basis of Open Loop Control

\[
V_{out} = \frac{V_{in}}{3 \cdot 255} \left( Pulse\ Count \right) - 0.8 - (0.075 \cdot I_{load})
\]
Feedback Control Block Diagram

Diagram shows a feedback control system with:
- Reference input labeled as $r$
- Error signal labeled as $e$
- Controller block labeled as $G_c(s)$
- Plant block labeled as $G_p(s)$
- Output $y$
Feedforward

\[ G_{ff}(s) \]

\[ G_c(s) \]

\[ G_p(s) \]

Reference input

Output
What really happens

Digital Controller

input disturbance

output disturbance

sensor noise

output

G_p(s)

Transient Profile Generator

reference input

r
Transient Profile

- Known as Motion Profile in industry
- Provides the desired output trajectory
  - Energy
  - Max speed required
  - Max acceleration (torque) required
  - Smoothness (max Jerk)
    - Reduce mechanical wear and tear
    - Avoid exciting the resonant modes
- Keep the error small and controller aggressive
- Not seen in control textbooks
Motion Profile Examples

- **Parabola**
  - Position: Smooth curve
  - Velocity: Smooth curve
  - Acceleration: Linear change
  - Jerk: Constant

- **Triangular Velocity**
  - Position: Triangular
  - Velocity: Triangular
  - Acceleration: Linear
  - Jerk: Step change

- **Triangular Acceleration**
  - Position: Triangular
  - Velocity: Triangular
  - Acceleration: Triangular
  - Jerk: Step change

- **Sinusoid**
  - Position: Sinusoidal
  - Velocity: Sinusoidal
  - Acceleration: Sinusoidal
  - Jerk: Sinusoidal

**Time (second)**
Motion Profile Examples (w/ slew)
Torque-Speed Curves

torque-speed plot of the existing profiles

Torque

Speed
Disturbances in the conveter

Load Current (Amps)

Voltage Input Change

% Pulse Width

“Duty Ratio”

(Pulse Count)

Output Voltage

\[
\begin{align*}
\text{Voltage Input Change} &= X \\
\text{% Pulse Width} &= \div 255 \\
\text{“Duty Ratio”} &= \div 255 \\
\text{(Pulse Count)} &= 0.075 \left[ \frac{S}{1562} + 1 \right] \\
&- \frac{S^2}{(2\pi \cdot 469.32)^2} + \frac{2(0.2477)S}{(2\pi \cdot 479.32) + 1} \\
&+ \frac{1}{3} \left[ \frac{S}{1500} + 1 \right] \\
&- \left[ \frac{S}{1700} + 1 \right] \left[ \frac{S^2}{(2\pi \cdot 270.56)^2} + \frac{2(0.13)S}{(2\pi \cdot 270.56) + 1} \right] \\
&+ \frac{120}{3 \cdot 255} \\
&- \frac{S^2}{(2\pi \cdot 448.37)^2} + \frac{2(0.2477)S}{(2\pi \cdot 448.37) + 1}
\end{align*}
\]
Digital Control
Digitally Controlled Power Converter

- Input Voltage: 110~150v (DC)
- EMI/RFI Filter
- H-Bridge MOSFETS Switches
- CPLD (PWM generator)
- DSP (Digital controller)
- Sensor / feedback
- Transformer
- Full wave rectification
- EMI/RFI Filter
- OutPut: 28V (DC)
Classical Design Techniques

- Proportional-Integral-Derivative Controller
- Root Locus (Pole/Eigenvalue Assignment)
- Lead-Lag Compensator (Frequency Response, and later, Loop-Shaping)
- State Feedback and Observer Based Design

http://www.engin.umich.edu/group/ctm/
Closing the loop with a Constant Gain

Over a thousand years old
Fly ball governor, etc
Closing the loop with a PID Controller (Proportional-Integral-Derivative)

\[ u = k_p e + k_i \int e + k_d \dot{e} \]

\[ \frac{U(s)}{E(s)} = k_p + k_i \frac{1}{s} + k_d s \]

Norm Minorsky, 1922

Used in >90% industrial applications
Dissecting PID

- **Proportional Control**
  - The essential feedback control means
  - Used over 2000 years ago
  - Effects disappear when error is small

- **Integral Control**
  - Mainly for reducing steady state
  - Introduces lag
  - May lead to overshoot/instability

- **Differential Control**
  - Predicative, overcomes lag
  - Noise issue unresolved
Given $G_c(s)$ and $G_p(s)$,
plot all closed-loop poles (root locus) for $K: 0 \rightarrow \infty$
Root Locus

(a) Schematic of the system showing the motor, position sensor, controller, and the actual position, $x_A(t)$.

(b) Control diagram with the controller transfer function $\frac{K}{(s + 500)(s + 800)}$ and the motor and load transfer function $\frac{20,000}{s(s + 100)}$.

Graph showing the root locus plot with real and imaginary axes.

II-21
State Feedback

- Plant
  \[
  \dot{x} = Ax + Bu \\
y = Cx + Du
  \]

- Control Law
  \[
  u = r - Kx
  \]

- Closed Loop
  \[
  \dot{x} = (A - BK)x + Br \\
y = Cx + Du
  \]

- Design Criteria
  \[
  \text{eig}(A-BK) \text{ assignment}
  \]
State Feedback with an Observer

- **Plant**
  \[ \dot{x} = Ax + Bu \]
  \[ y = Cx + Du \]

- **Control Law**
  \[ u = r - K\hat{x} \]

- **Closed Loop**
  \[ \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \]
  \[ \hat{y} = C\hat{x} + Du \]

- **Separation Principle**
  Independence in Control Law and Observer Designs
Practicality

- How to Choose Closed-loop Eigenvalues/poles?
- Observer Eigenvalue Selection?
- Disturbance Rejection?
- Sensor Noise?
- Sensitivity to Plant Changes?
Loop Shaping Design

Loop Gain Frequency Response: $L(j\omega) = G_c(j\omega)G_p(j\omega)$

- Performance Specs to Loop Gain Constraints
- Loop Gain Shaping by Lead-Lag Compensators
Loop Shaping Design

\[ G_p(j\omega)G_c(j\omega) \]

-20 dB/Dec

- Command Following
- Disturbance Rejection

Sensor Noise
Unknown Dynamics
Advanced Design Techniques

- Nonlinear PID
- Model Independent Methodology
- Parameterization and Practical Optimization
- Wavelet denoising
- Fuzzy Logic
- Neural Networks
- Genetic Algorithms
- ...
Misconceptions

- Control Design Is About Pole Placement
- “Optimal” Control
- Step Command
- Must Have a Math Model
- PID is Art / Control Theory is Science
Robustness of PID

- **Plant:** \( \dot{y} = -a_1 y - a_2 \dot{y} + u \quad a_1 = 1, a_2 = 1 \)

- **Controller:** \( k_p = 3, k_i = 1, \text{ and } k_d = 2 \) \( (T_{\text{settle}} < 3 \text{ sec}) \)
Transient Profile and Robustness

- Controller: $k_p = 300$, $k_i = 100$, and $k_d = 200$
Simulating a feedback control system

- Introduction to Matlab/Simulink
  - Matlab Basics
  - Matlab GUI
  - Simulink

- PID Design and Evaluation
  - Simulation Model Setup
  - PID Gain Selection
  - Evaluation of Performance
    - Command Following: how fast and accurate
    - 10% Input Disturbance Test
    - 1% White Noise Test
    - Time Delay Test
    - Smoothness of Control Signal