An Active Disturbance Rejection Approach to Tension and Velocity Regulations in Web Processing Lines

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Abstract—In this paper, a unique disturbance rejection control strategy is proposed for a class of tension and velocity regulation problems found in web process lines. The proposed control system actively estimates and rejects the effects of both dynamic changes in the system and external disturbances. Both open-loop and closed-loop tension regulation schemes are investigated. A tension observer is designed in order to facilitate closed loop tension control in the absence of a tension transducer. The performance of existing schemes and the proposed ones are compared and the results show marked improvements in tracking and disturbance rejection by the proposed solutions.

Index Terms—Web Tension Regulation, Tension Observer, Accumulator, Web Span, Active Disturbance Rejection Control.

I. INTRODUCTION

Web tension regulation is a rather interesting and challenging industrial control problem. Many types of material, such as paper, plastic film, cloth fabrics, and even strip steel are manufactured or processed in a web form. The quality of the end product is often greatly affected by the web tension, making it a crucial variable for feedback control design, together with the velocities at the various stages in the manufacturing process. The ever-increasing demands on the quality and efficiency in industry motivate researchers and engineers alike to explore better methods for tension and velocity control. However, the highly nonlinear nature of the web handling process and changes in operating conditions (temperature, humidity, machine wear, and variations in raw materials) make the control problem both challenging and stimulating.

Accumulators in web processing lines are important elements in web handling machines as they are primarily responsible for continuous operation of web processing lines [1]. For this reason, the study and control of accumulator dynamics is an important class of problems [2, 3], and is, therefore, the focus of this paper. A preliminary study on the modeling of accumulators and the characteristics of an accumulator and its operation are explained and given in [3]. Dynamic behavior and control of the accumulator carriage, web spans, and tension are discussed in [2].

Both open-loop and closed-loop methods are commonly used in web processing industries for tension control purposes [4-8]. In the open-loop control case, the tension in a web span is controlled indirectly by regulating the velocities of the rollers at either end of the web span. An inherent drawback of this method is its dependency on an accurate mathematical description of the web tension as a function of velocities, where such function is generally highly nonlinear and highly sensitive to velocity variations. Still, simplicity of the controller seems to outweigh this drawback in some applications. Closing the tension loop with tension feedback is a straightforward solution to improve accuracy and reduce sensitivity to modeling errors. It requires tension measurement, for example, through a load cell, and the cost and hardware complexity may be justified by the resulting improvements in tension regulation. Some researchers [7, 8] have proposed to use observers in place of tension measurements, which could reduce the hardware complexity and cost. The trade-off is the increased complexity in the control algorithm and its tuning. In addition, the discrepancies between the estimated and the actual tension will likely cause performance degradation. The design of the observer also requires a fairly accurate mathematical model of the tension dynamics, which may not be available. With these considerations in mind, it is not surprising to see that most of today’s tension feedback loops employ tension measurement.

Control systems will unavoidably encounter uncertainties and this is particularly true for tension control applications, where operating conditions change greatly. It is therefore paramount that the tension regulation scheme must be able to deal with unexpected variations in both internal dynamics and external disturbances. This led us to investigate the use of disturbance rejection techniques. One class of disturbance rejection methods is based on the concept of disturbance observer (DOB). Many forms of DOB have been proposed for various disturbance applications [9-13], but the basic idea is to reject external disturbance under the assumption that the internal dynamics is linear and time invariant, and, for the most part, given in a mathematical model.

The objective of the research described in this paper is to find a solution for the web tension and velocity regulation that can deal effectively with the nonlinear, time-varying, nature in most applications. Equally important is such solution should not be dependent on the accurate plant model that most advanced control design assumes, for such
assumption simply does not hold in most real world applications. One such candidate is the Active Disturbance Rejection Control (ADRC), which requires very little information of the plant dynamics, is very easy to tune, and has very good disturbance rejection capability [14-17]. ADRC controllers are inherently robust against plant variations and are effective in a large range of operations [18]. Initial work in the application of ADRC to web tension regulation was evaluated on a linear transfer function model of the tension loop [4]. Good performance was observed but the controller uses many nonlinear gain functions and is difficult to tune, a problem resolved by the parameterization technique described in [16].

In this paper, the ADRC design methodology is applied to a truly nonlinear model of the tension dynamics, which is used as the benchmark for comparison with other methods. The remainder of the paper is organized as follows. The dynamics of the accumulator carriage and the problem formulation are discussed in Section II. Proposed controller design and observer design are given in Section III. Simulation results and the comparisons of four different control schemes can be found in Section IV. Finally, concluding remarks are included in Section V.

II. PROBLEM FORMULATION

The mathematical model of a web process line and the existing control methods are briefly reviewed in this section. The accumulator dynamics, as given in [3], is used as the test bed for the proposed method. A web processing line layout includes an entry section, a process section and an exit section. Operations such as wash and quench on the web are performed in the process section. The entry and exit section are responsible for web unwinding and rewinding operations with the help of accumulators located in each sections.

A. System Dynamics

The dynamics of the carriage tension and the entry/exit rollers in accumulator is shown as [3]:

\[ \dot{i}_c(t) = \frac{AE}{x_c(t)} (v_c(t) + \frac{1}{N} (v_e(t) - v_p(t))) \quad (1) \]

\[ \dot{x}_c(t) = v_c(t) \quad (2) \]

\[ M_c \dot{v}_c(t) = -N t_c(t) - F_d(t) + u_e(t) - M_c g \quad (3) \]

\[ v_c(t) = \frac{1}{J} (-B_1 v_c(t) + R^2 (t - t_c(t)) + \delta_c(t) + RK u_c(t)) \quad (4) \]

\[ \dot{v}_p(t) = \frac{1}{J} (-B_1 v_p(t) + R^2 (t - t_c(t)) - \delta_p(t) + RK u_p(t)) \quad (5) \]

where \( x_c(t) \) is the carriage position, \( t_c(t) \) is the desired web tension in the process line and \( t_e(t) \) is the average web tension. \( u_e(t) \), \( u_c(t) \) and \( u_p(t) \) are the carriage, exit-side and process-side driven roller control inputs, respectively. The disturbance force, \( F_d(t) \), includes friction in the carriage guides, rod seals and other external force on the carriage. \( K_c \) and \( K_p \) are positive gains. \( \delta_c(t) \) and \( \delta_p(t) \) are the tension disturbances on the exit side and process line. The constant coefficients in (1) to (5) are described in the Appendix.

B. Design Objectives

The task at hand is to determine a control law such that \( v_c(t), v_e(t) \) and \( v_p(t) \), which are measured, as well as \( t_c(t) \), which may or may not be measured, all closely follow their desired trajectories or values respectively. It is well known in web transporting system that disturbances can propagate through the whole system. So the challenge is to find a controller that has an inherent disturbance rejection property which enables it to bring all four variables above to their desired values consistently. The problem is challenging because:

a) There is a strong coupling between the carriage dynamics, strip tension dynamics and the roller dynamics.

b) The tension dynamics are highly nonlinear and sensitive to velocity variations.

c) The coefficients in (1) to (5) are highly dependent on the operating conditions and web material characteristics, any changes of which may induce significant variations in system dynamics.

d) There are extensive external disturbances, which propagate through the system which could make the system even unstable in some cases.

C. Motivation

Since the velocities are controlled in open-loop by feed forward and classical PI control method, the industrial controller (IC) needs to retune when the operating conditions are changed and external disturbance appears. In addition, IC has a poor performance in the presence of disturbance. The Lyapunov based controller (LBC) improves upon the industrial controller by adding auxiliary error feedback terms to get better performance and disturbance rejection. However, it is designed specifically to deal with disturbances, which are introduced in the model. And when uncertainties appear in real application, it may require re-design of the controller [2]. Both the IC and LBC solution are given in [2] and they are used as the basis for comparison in this paper.

The imperative in web process regulation is to find a solution that is 1) not overly dependent on plant model; 2) effective in dealing with unknown, nonlinear, and time varying dynamics and external disturbances; and 3) easy to understand and deploy for the field engineers. After careful evaluation of the characteristics of the problem, we believe ADRC offers such a solution as it represents a completely different design paradigm, where the internal dynamics and external disturbances are estimated, not modeled, and compensated for in real time. Therefore, such design is likely to be inherently robust against uncertainties in the real world applications.
III. THE PROPOSED APPROACH

In developing new solutions for this difficult industry problem, performance and simplicity are stressed. That is, the new controller must have a much better performance than the existing ones, and it should also be simple to design, implement, and tune. A key observation in this research is that there are two control problems to consider: velocity and tension. The three velocity loops are very similar in nature and finding a better solution would be a good first step. The tension problem is crucial because of its importance and its nonlinear dynamics. Based on the cost and performance considerations, two solutions will be explored: 1) if the tension model in (1) is reliable, it can be well controlled with fast and accurate velocity loops; 2) industry users are quite willing to install tension sensors for direct tension feedback control in return for better tension performance. The velocity control problem below will be addressed first, followed by the solutions to the tension problem.

A. A New Solution to Velocity Regulation

Velocity regulation in a process line is one of the most common control problems in the manufacturing industry. Since most processes are well-behaved, a PID controller is typically tuned by an experienced engineer. Other techniques, such as pole-placement and loop shaping, could potentially improve the performance over PID but require mathematical models of the process. They are also more difficult to tune once they are implemented. By reformulating the problem as that of active disturbance rejection, an alternative to PID and model based controllers such as LBC, is described below.

Notice that the velocity equations (3)-(5) can be rewritten as

\[
\begin{align*}
\dot{v}_c(t) &= f_c(t) + b_c u_c(t) \\
\dot{v}_p(t) &= f_p(t) + b_p u_p(t)
\end{align*}
\]

where

\[
\begin{align*}
f_c(t) &= \frac{1}{M_c} (-N(t) - F_c(t) - M_c g) \\
f_p(t) &= \frac{1}{J} (-B v_p(t) + \dot{\theta}(t) - t(t))
\end{align*}
\]

\(b = [b_c b_p b_p]^T = [1/M_c/RK_c/J RK_p/J]^T\) (9)

The three plants in (6)-(8) are all of the form

\[
\dot{v}(t) = f(t) + bu(t)
\]

where \(v(t)\) is the velocity to be controlled, \(u(t)\) is the control signal, and the value of \(b\) is known, approximately. \(f(t)\) represents the combined effects of internal dynamics and external disturbance.

The key to the control design is to compensate for \(f(t)\), and the job will be much simpler if its value can be determined at any given time. To this end, an extended state observer [14-17] is employed.

Writing the plant in (10) in a state space form

\[
\begin{align*}
\dot{x}_1 &= x_2 + bu \\
\dot{x}_2 &= h \\
y &= x_1
\end{align*}
\]

Let \(x_1 = v, x_2 = f\), where \(x_2\) is the augmented state variable, \(h = f\). The state space of (11) is rewritten as

\[
\begin{align*}
\dot{x} &= Ax + Bu + Eh \\
y &= Cx
\end{align*}
\]

where \(A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\) (12)

Now \(f\) can be estimated using a state observer based on the state space model (12). A state observer, referred to as linear extended state observer (LESO), can be constructed as to estimate both \(x_1 = v\) and \(x_2 = f\):

\[
\begin{align*}
\dot{\hat{x}} &= A\hat{x} + Bu + L(y - \hat{y}) \\
\dot{\hat{y}} &= C\hat{x}
\end{align*}
\]

where \(L = [L_1 L_2]\) is the observer gain vector. By setting \(\lambda(s) = [sI - (A - LC)] = (s + \omega_c)^2\) (14)

that is \(L_1 = \omega_c, L_2 = \omega_c^2\), the observer can be easily tuned with a single tuning parameter, \(\omega_c\), making it attractive as a practical solution.

With a functioning LESO, which results in \(z_1 \rightarrow v\) and \(z_2 \rightarrow f\), the control law will be as

\[
u = (-z_2 + u_0)/b
\]

thus reducing the plant to

\[
\dot{v} = (f - z_2) + u_0 \approx u_0
\]

At this point, an unknown, nonlinear, and time varying plant of (10) is reduced to a linear one with a pole at the origin. The control design is now standardized to that of a simply integral plant, which, for example, can be easily controlled using a proportional term

\[
u_0 = k_1(r - z_1)
\]

Setting \(k_1 = \omega_c\) yields a closed-loop transfer function of

\[
G_{CL}(s) = \frac{\omega_c}{s + \omega_c}\]

where \(\omega_c\) is the desired closed-loop bandwidth and it can be used as the only tuning parameter for the controller.
Remarks 1. Solving (13), (14) and (16) for $z_2$, the transfer function expression of $z_2$ can be shown as

$$z_2 = \frac{(sv(s) - bu(s)) \omega_n^2}{(s + \omega_n)^2}$$  \hspace{1cm} (19)

2. Substituting (15) into (13), the controller is simplified as

$$z_i = L_p(y_i - z_i) + k_p(r_i - z_i)$$

$$u = \frac{1}{b}(k_p(r_i - z_i) - L_p(y_i - z_i))$$  \hspace{1cm} (20)

3. The unknown external disturbance and the internal uncertain dynamics are combined treated as a generalized disturbance. By augmenting the observer an extra state, which can be actively estimated and canceled out the disturbance, thereby achieving active disturbance rejection.

4. The proportional controller in (17) can be replaced with a more advanced design, such as a nonlinear controller, if necessary.

5. The tuning parameters are $\omega_c$ and $\omega_i$. The only parameter needed from the plant is the approximate value of $b$ in (10).

The above controller-observer combination in (13)-(17) is denoted as ADRC. The diagram for ADRC is shown in Figure 1. It is applied separately to all three velocity loops.

![Figure 1 ADRC-Based Velocity Control](image)

B. Tension Control Methods

Both open-loop and closed-loop solutions to tension regulation are discussed here. The former is simple and economic; the latter is more precise but requires an additional observer or a sensing device for tension.

Open-Loop Tension Regulation

High quality velocity regulation allows the tension to be controlled open-loop, if the model of the tension dynamics (1) is accurate. From (1), the tension can be computed as

$$t_c(t) = t_c(0) + \int_0^t AE \frac{x_c(t)}{v_c(t)}(v_c(t) + \frac{1}{N}(v_c(t) - v_p(t)))dt$$  \hspace{1cm} (21)

where $t_c(0)$ is the initial value of tension. For the open-loop control, let the desired velocities $v_c^d, v_p^d$ and $v_p^d$ be carefully chosen so that (21) yields

$$t_c(t) = t_c^d, t \geq t_i$$  \hspace{1cm} (22)

For a given initial condition $t_c(0)$ and a given time constraint, $t_i$. Then, if all three velocity loops are well-behaved, the actual tension should be close to the desired value. This method will be tested in simulation in a later section. Note that, for this purpose, the desired velocities must satisfy the following condition

$$v_c^d(t) = -v_p^d(t) - v_p^d(t), t \geq t_i$$  \hspace{1cm} (23)

The above approach is a low cost, open-loop solution. As the operating condition changes, the tension dynamics (1) could vary, causing variations in tension. For the tension is not measured, such variations go unnoticed until visible effects on the product quality appear. To maintain accurate tension control, industry users usually are willing to install a tension sensor, which regulates the tension in a feedback loop, as discussed below.

Observer-based Closed-loop Tension Regulation

A tension sensor, such as a load cell, can be used for closed-loop tension control. But it increases the hardware complexity and cost. Therefore, implementing tension control without tension sensor would be beneficial from an economic point. For this purpose, a tension observer is designed.

Recall in (3)-(5), tension is coupled in velocity loops, and we use an ADRC controller to decouple the tension from the velocity loops. Actually, tension is thrown into $f(t)$ part, which is estimated and canceled out in LSEO.

Let us look at $f(t)$ in three velocity loops, and we can find out that if the other parts of $f(t)$ are known, tension can be estimated through equation (9) and presented as:

$$\dot{t}_c(t) = \frac{M_c}{N}(f_v(t) + \frac{1}{M_c}(-F_d(t) - M_c g))$$  \hspace{1cm} (24)

$$\dot{v}_c(t) = \frac{1}{R_c}(-Jf_v(t) - B \dot{v}_p(t) + R \ddot{t}_c)$$  \hspace{1cm} (25)

$$\dot{v}_p(t) = \frac{1}{R_p}(Jf_v(t) + B \dot{v}_p(t) + R \ddot{t}_c)$$  \hspace{1cm} (26)

With a proper parameter setting, LSEO can guarantee that $z_1 \rightarrow v$ and $z_2 \rightarrow f$. That is to say, from LSEO, $f_v(t), f_v(t)$ and $f_p(t)$ can be obtained. Since the other parts in $f(t)$ are all known in this problem, tension estimation from three velocity loops can be calculated based on (24)-(26).

![Figure 2 An observer-based tension control system](image)

Finally, the tension observer is obtained from the average of three tension estimations.

$$\hat{t}_c(t) = \frac{1}{3}(\hat{t}_c(t) + \hat{t}_c(t) + \hat{t}_c(t))$$  \hspace{1cm} (27)

The complete block diagram for the velocity and
tension control loops are shown in Figure 2. The simulation results are revealed in the next section, where the proposed method is compared to the two previous methods.

IV. SIMULATION AND COMPARISON

In this section, four types of control systems are compared via simulations, including: 1) the commonly used industrial controller (IC); 2) the improved LBC 3) the three ADRC controllers, described in (13)-(17), for the velocity loops with tension regulated in open-loop (ADRC1); and 4) the same ADRC velocity controllers with an additional ADRC controller for the tension loop (ADRC2).

Note that in IC and ADRC1, the tension is controlled open-loop, while ADRC2 closes the tension loop with a tension feedback. LBC relies on the tension estimator for its closed-loop tension control.

The comparison of these controllers is carried out in the presence of disturbances. In addition, to demonstrate the feasibility of the proposed methods, they are implemented in discrete-time form with a sampling period of 10 ms.

A. Simulation Setup

Three control schemes are investigated by conducting simulations on an industrial continuous web process line. The desired tension in the web span is 5180 N. The desired process speed is 3.3 m/s. A typical scenario of the exit speed and the carriage speed during a rewind roll change is depicted in Figure 3.

The objective of control design is to make the carriage, exit velocity, and process velocities closely track their desired trajectories, while maintaining the desired average web tension level.

To make the simulation results realistic, three sinusoidal tension disturbances are injected in the velocities loops. For the carriage velocity loop, \( F_d(t) \) in (3) is a sinusoidal disturbance with the frequency of 0.5 Hz and amplitude of 44 N, and is applied only in three short specific time intervals: 20:30 seconds, 106:126 seconds, and 318:328 seconds as shown in Figure 4. For exit roller velocity and process velocity loops, \( \delta_v(t) \) and \( \delta_p(t) \), in equation (4) and (5), respectively are the tension disturbances added on these two velocities loops. They are also sinusoidal functions with the frequency of 0.2 Hz and the amplitude of 44 N, and are applied throughout the simulation.

\[
\omega \approx 318 \text{ Hz}
\]

\[
\text{Sinusoid Disturbance to Carriage with Intervals}
\]

B. Parameterization setup and Tuning Procedures

Following the parameterization and design procedure described above, \( \omega_c \) and \( \omega_p \) are the two parameters need to be tuned. As discussed in [16], relationship between \( \omega_c \) and \( \omega_p \) is \( \omega_p \approx 3 - 5 \omega_c \). So we only have one parameter to tune, which is \( \omega_c \).

The other important parameter needed is the approximate value of \( b \) in (9). For this problem, the best estimate of \( b \) in (9) is as follows:

\[
b = [b_c, b_e, b_p] = [1/M, RK_e/J, RK_p/J] = [1.36, 0.707, 0.707]^T, b = AE/5 = 3.76 \times 10^6.
\]

A cohesive ADRC design and optimization procedure is given as follows:

**Step 1**: Design parameterized LESO and controller where \( \omega_c \) and \( \omega_p \) are design parameters;

**Step 2**: Choose an approximate value of \( b \) in different plant, such as \( b_c, b_e, b_p \),

**Step 3**: Set \( \omega_c = 5 \omega_p \) and simulate/test the ADRC in the simulation or a hardware set-up;

**Step 4**: Incrementally increase \( \omega_p \) until the noise levels and/or oscillations in the control signal and output exceed the tolerance;

**Step 5**: If necessary, slightly increase or decrease the ratio of \( \omega_c \) and \( \omega_p \).

The parameters of the four controllers are shown in Table I.

**TABLE I: VALUES OF THE GAINS USED IN THE SIMULATION**

<table>
<thead>
<tr>
<th></th>
<th>Velocity Loops</th>
<th>Tension loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC</td>
<td>( k_c = 1 )</td>
<td>100, ( k_{\text{tg}} = 100 )</td>
</tr>
<tr>
<td>LBC</td>
<td>( \gamma_c = 100 )</td>
<td>( \gamma_e = 100 ), ( \gamma_p = 100 )</td>
</tr>
<tr>
<td>ADRC1</td>
<td>( \alpha_c = 15 ), ( \alpha_e = 40 ), ( \alpha_p = 40 )</td>
<td></td>
</tr>
<tr>
<td>ADRC2</td>
<td>Same as ADRC1</td>
<td>( \omega_p = 12 )</td>
</tr>
</tbody>
</table>

Here \( k_c, k_{\text{tg}}, \alpha_c \), and \( \alpha_p \) are the gains for the IC. \( \gamma_c, \gamma_e \), and \( \gamma_p \) are the gains for the LBC. \( b_c, b_e, \) and \( b_p \) are specific
values of $b$ in (9) for the carriage, exit, and process velocity loops, respectively. Similarly, $\omega_{oc}$, $\omega_{oe}$ and $\omega_{op}$ are the observer gains in equation (14); and $\omega_{cc}$, $\omega_{ce}$ and $\omega_{cp}$ are the controller gains ($k_p$) in equation (28). $b_c$, $b_e$, and $b_p$ are the corresponding ADRC parameters for the tension plant in (1).

C. Simulation Results and Comparison

The velocity and tension tracking errors resulting from ADRC1 are shown in Figure 5. Obviously, the velocity and tension tracking errors are quite small, despite the fact that the controller is design not based on the complete mathematical model of the plant and there are significant disturbances in the process. All the ratios of tracking errors to set point are below 0.1%.

![Figure 5 Tracking errors for Carriage Roller by ADRC1](image)

Similar characteristics are also found in the exit and process velocity loops. For the sake of the length of this paper, those results are not included here.

Due to the poor results of IC, only LBC, ADRC1, ADRC2 are compared in the tension control results in Figure 8. Note that, with an open-loop tension control scheme, the steady state error of ADRC1 is caused by the constant sinusoid disturbances added in the three velocity loops, which enter into the tension loop. With a direct tension measurement and feedback, the closed-loop ADRC2 tension control can result in negligible tension errors. Furthermore, even in an open-loop control, ADRC1 has a smaller error than LBC. This can be attributed to the high quality velocity controllers in ADRC1.

![Figure 7 Control Signal for Carriage Roller by IC, LBC and ADRC1](image)

The velocity and tension errors of all four control systems are summarized in Table II. Overall, these results reveal that the proposed ADRC controllers have a distinct advantage in the presence of sinusoidal disturbances and a much better performance in tension control.
TABLE II SIMULATION COMPARISON

<table>
<thead>
<tr>
<th>Maximum Error</th>
<th>Root Mean Square Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_c ) ( (m/s) )</td>
<td>( v_e ) ( (m/s) )</td>
</tr>
<tr>
<td>IC</td>
<td>5.0E-4</td>
</tr>
<tr>
<td>LBC1</td>
<td>1.2E-4</td>
</tr>
<tr>
<td>ADRC1</td>
<td>8.0E-3</td>
</tr>
<tr>
<td>ADRC2</td>
<td>7.0E-3</td>
</tr>
</tbody>
</table>

V. CONCLUDING REMARKS

A new control strategy is proposed for web processing applications, based on the active disturbance rejection concept. It is applied to both velocity and tension regulation problems. Although only one section of the process, including the carriage, the exit, and the process stages, is included in this study, the proposed method applies to both the upstream and downstream sections, covering the entire web line. Simulation results, based on a full nonlinear model of the plant, demonstrate that the proposed control algorithm results in not only better velocity control but also significantly less web tension variation. The proposed method is promising because: 1) no detailed mathematical model is required; 2) zero steady state error is achieved without using the integrator term in the controller; 3) much better command following is demonstrated during the transient stage; and finally 4) excellent disturbance rejection was achieved.

Appendix I: Plant Coefficients

<table>
<thead>
<tr>
<th>Values</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_c )</td>
<td>7310 kg Mass of the carriage</td>
</tr>
<tr>
<td>( A )</td>
<td>3.27×10(^{-1} ) m(^2 ) Cross sectional area of web</td>
</tr>
<tr>
<td>( E )</td>
<td>6.90×10(^{10} ) N/m(^2 ) Modulus of elasticity</td>
</tr>
<tr>
<td>( R )</td>
<td>0.1524 m Radius of roller</td>
</tr>
<tr>
<td>( N )</td>
<td>34 Number of web spans</td>
</tr>
<tr>
<td>( J )</td>
<td>2.1542 kg-m(^2 ) Moment of inertia</td>
</tr>
<tr>
<td>( v_f )</td>
<td>35.037×10(^{4} ) N-s/m Viscous friction coefficient</td>
</tr>
<tr>
<td>( B_f )</td>
<td>2.25×10(^{-3} ) N-m-s Bearing friction coefficient</td>
</tr>
</tbody>
</table>

REFERENCES