Supplement to "Industry Dynamics with Knowledge-Based Competition: A Computational Study of Entry and Exit Patterns"

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Properties of the Technology Landscape

I report below the findings from my computational experiments which replicate the results reported in Kauffman (1993). Let the technology landscape be defined in an $N+1$ dimensional space, where each of the first N dimensions represents the choice of method for that component activity and the last dimension represents the level of overall efficiency for the chosen vector of N methods. A "search" may be viewed as taking a walk on a particular landscape to find a local optimum. For my experiments, I generated 10 random technology landscapes given (N, K). On each landscape, 10,000 initial positions were randomly chosen. From each of these initial positions, search was carried out to find a local optimum over 800 periods. At the end of 800 periods, the final position was tested for optimality. The results I report are the averages over 10 separate landscapes.

First, landscapes were created with $N = 16$ and $K \in \{0, 1, 2, 3, 5, 7, 9, 11, 13, 15\}$. The initial positions were randomly chosen 10,000 times for each landscape. Of those initial positions, some were identical. I confirmed that there were 9,306 distinct initial positions. For every one of the 10,000 separate searches, a local optimum was attained at the end of 800 periods. Comparing among the resulting local optima (attained from 10,000 initial positions), I computed the number of local optima that are *distinct*. Even distinct local optima can sometimes have identical efficiency values and, hence, I also computed the number of local optima with distinct efficiency values. These numbers were obtained for each landscape. Their averages over 10 landscapes are reported below:

K	No. Distinct Init. Positions	Freq. of Optimum	No. Distinct Optima	No. Optima w/ Distinct Efficiency Values
0	9,306	10,000	1	1
1	9,306	10,000	17.3	5.4
2	9,306	10,000	37.5	36.9
3	9,306	10,000	52.6	52.6
5	9,306	10,000	184.8	184.6
7	9,306	10,000	451.4	450.8
9	9,306	10,000	915.1	910.2
11	9,306	10,000	1489.0	1474.5
13	9,306	10,000	2171.8	2139.4
15	9,306	10,000	3184.4	3141.8

Clearly, the number of distinct local optima increases in K.

How does the value of N affect the number of local optima? I report below the results for $K = 5$ and $N \in \{6, 8, 10, 12, 14, 16\}$. An increase in N clearly raises the number of distinct local optima.

N	No. Distinct Init. Positions	Freq. of Optimum	No. Distinct Optima	No. Optima w/ Distinct Values
6	64	10,000	10.7	10.7
8	256	10,000	11.8	11.8
10	1023	10,000	24.3	24.3
12	3729	10,000	56.3	56.3
14	7432	10,000	107.4	107.4
16	9306	10,000	184.8	184.6

The table below then captures the general relationship between (N, K) and the number of local optima:

		N				
		2	4	8	12	16
	0	1	1	1	1	1
	1	1.8	1.9	3.7	2.2	17.3
	2		2.1	4.4	7.1	37.5
K	3		3.6	8.5	17.6	52.6
	5			11.8	56.3	184.8
	7			28.6	107.0	451.4
	11				328.8	1489.0
	15					3184.4

References

[1] Kauffman, S. A., 1993, *The Origins of Order*, Oxford University Press, Oxford.