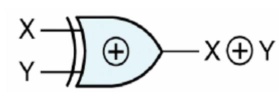
Section 3.2. XOR operation

- XOR (⊕)
  - Exclusive-OR
  - Some interpretations:
    - X not equal to Y
    - X and Y has odd number of 1's

X	Y	X ⊕ Y
0	0	0
0	1	1
1	0	1
1	1	0

$X \oplus Y = X'Y + XY'$



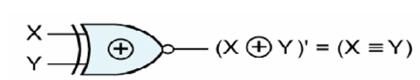
- Theorems:
  - $X \oplus 0 = X$
  - $X \oplus 1 = X'$
  - $X \oplus X = 0$
  - $X \oplus X' = 1$

  - $X \oplus Y = Y \oplus X$
  - $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$
  - $X(Y \oplus Z) = XY \oplus XZ$
  - $(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$

- XNOR (Ξ)
  - Inverted exclusive-OR
  - Ξ notation not used in other text
  - Some interpretations:
    - X equal to Y (i.e., equivalence operator)
    - X and Y has even number of 1's

X	Y	$(X \oplus Y)'$
0	0	1
0	1	0
1	0	0
1	1	1

$(X \oplus Y)' = X'Y' + XY$



- Equivalence is the complement of XOR:

$$
\begin{aligned}
(X \oplus Y)' &= (X'Y + XY')' \\
&= (X + Y')\,(X' + Y) \\
&= X\,(X' + Y) + Y'\,(X' + Y) \\
&= XX' + XY + Y'X' + Y'Y \\
&= XY + X'Y' \\
&= (X \equiv Y)
\end{aligned}
$$

Assignment

- Read Section 3.2
- Problems:
  3.8, 3.9. 3.10

Chap 4. Minterm and Maxterm Expansions

Outline

- Conversion of English Sentence to Boolean Equations
- Combinational Logic Design Using a Truth Table
- Minterm and Maxterm Expansions
- Incompletely Specified Functions
- Examples of Truth Table Construction

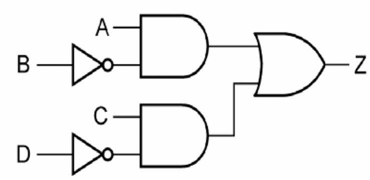
Conversion of English Sentence to Boolean Equations

- Mary watches TV if it is Monday night and she has finished her homework
  F — A — B

  F = A B

- The alarm will ring iff the alarm switch is on and the door is not closed
  Z — A — B’
  or it is after 6pm and the window is not closed
  C — D’

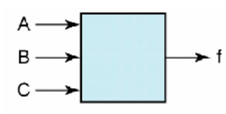
  Z = AB’ + CD’



- Consider A, B, C are three votes, which can be yes (1) or no (0). An output U is 1 iff the vote is unanimous
  - U is 1 iff ABC are all 1’s or all 0’s
  - U = ABC + A’B’C’
- An output M is 1 iff there is a majority vote
  - M = ?
  - Somewhat tedious (and error–prone) to derive the logic expression
  - Truth table is a systematic way to derive the expression

Combinational Logic Design Using a Truth Table

- Switching circuit Logic design using truth table



A B C	f	f'
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	1	0
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

- f is 1 if (row 3 is 1) or (row 4 is 1) or (row 5 is 1) or (row 6 is 1) or (row 7 is 1)
- Logic expression for

  (row 3 is 1): A'BC

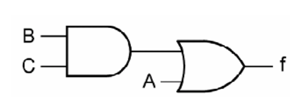
  (row 4 is 1): AB'C'

  (row 5 is 1): AB'C

  (row 6 is 1): ABC'

  (row 7 is 1): ABC
- Derive algebraic expression for *f* = 1

  f = A'BC + AB'C' + AB'C + ABC' + ABC



- f can simplified

  f = A'BC + AB' + AB

  = A'BC + A by Thm 8D: x+yz=(x+y)(x+z)

  = A + BC

- It is easier to think in term of SOP
- It is less intuitive to derive POS expression directly
- Alternative way to construct POS expression
  - Find SOP expression for f'
  - Invert f' to get f
- f'= A'B'C' + A'B'C + A'BC'
- f = (f')'
  = (A'B'C' + A'B'C + A'BC')'
  = (A + B + C)(A + B + C')(A + B' + C)
- f can be simplified
  = (A + B)(A + B' + C)
  = A + B(B' + C)
  = A + BC

A B C	*f*	*f'*
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	1	0
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

- An different interpretation (less intuitive):
  f is 0 if one of the first three rows is 0
  - Row 0 is 0: A=B=C=0; i.e., A + B + C is 0
  - Row 1 is 0: A=B=0 C=1; i.e., A + B + C' is 0
  - Row 2 is 0: A=C=0 B=1; i.e., A + B' + C is 0

- f is 0 if one of the first three rows is 0

  f = (A + B + C)(A + B + C')(A + B' + C)

A B C	*f*	*f'*
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	1	0
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

Minterm and Maxterm Expansions

- A minterm of n variables is a product of n literals in which each variable appears exactly once in true or complemented form, but not in both.
- Sum of minterms is referred to as a minterm expansion or a standard SOP.
- Minterm is written in abbreviated form as m_i where i is the row whose values make m_i=1
- The row is represented as a binary number

3-varaible truth table

- Note that the order of variables is important i.e., the row is (A B C) not (A C B), (C A B) etc.

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

- f represented in sum-of-minterms format:
  $f(A, B, C) = m_3 + m_4 + m_5 + m_6 + m_7$
  $= \Sigma\, m(3, 4, 5, 6, 7)$
- Again, argument order (A,B,C) is important

A B C	f	f'
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	1	0
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

- Finding sum-of-minterms for a logic expression
  - Truth table
  - Algebraic derivation
- E.g.,
  f(a,b,c,d) = a'(b' + d) + acd'
  f = a'b' + a'd + acd'
  = a'b'(c + c')(d + d') + a'd(b + b')(c + c') + acd'(b + b')
  = a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'b'c'd + a'b'cd
  + a'bc'd + a'bcd + abcd' + ab'cd'
  = a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'bc'd + a'bcd
  + abcd' + ab'cd'

Maxterm

- A maxterm of n variables is the sum of n literals in which each variable appears exactly once in true or complemented form, but not in both.
- product of maxterms is referred to as a maxterm expansion or a standard POS.
- Maxterm is written in abbreviated form as M_i where i is the row whose values make $M_i = 0$;
- Relationship between m_i and M_i
  $(m_i)' = M_i \qquad m_i = (M_i)'$

- f represented in product-of-maxterms format:
  $f(A, B, C) = M_0 M_1 M_2$
  $= \Pi\ M(0, 1, 2)$
- Argument order (A,B,C) is important

A B C	f	f'
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	1	0
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

Sum-of-minterms and Product-of-maxterms conversion

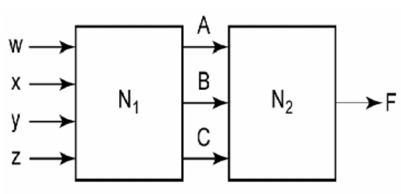
- Procedure
  - Given: function f in sum-of-minterms format
  - Write f' in sum-of-minterms format
    Note that f' consists of all minterms not in f
  - Find (f')', which is in product-of-maxterms format
    Note that f = (f')'
- E.g.,
  $f(A, B, C) = m_3 + m_4 + m_5 + m_6 + m_7$
  $f'(A, B, C) = m_0 + m_1 + m_2$
  $f = (f')' = M_0M_1M_2$

A B C	f	f'
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	1	0
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

Incompletely Specified Functions

- Some input combinations of function may not be used or possible
- e.g., F and B are two signals control a model car to drive forward or backward. An function M is 1 if car in motion and 0 if car is still. The input combination “11” is not possible.
- The condition is known as <u>don’t care</u> (denoted d in truth table).
- When the function is implemented, either 0 or 1 can be assigned
- Good assignment can simplified the final circuit

- E.g.,
  A large digital system would be divided into many subsystems, i.e., N_1 and N_2 as an example.
- Assume N_1 does not generate all possible combinations of values for A, B , C (e.g., 001, 110).
- F = Σ m(0, 3, 7) + Σ d(1, 6)
- F = Π M(2, 4, 5) • Π D(1, 6)



A	B	C	F
0	0	0	1
0	0	1	X
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	X
1	1	1	1

- <u>Alternatives</u>:
  1. Assume d = 0 for both X's:
     F = A'B'C' + A'BC + ABC = A'B'C' + BC
  2. Assume d =1 to the first X and d = 0 for the 2^{nd} X:
     F = A'B'C' + A'B'C + A'BC + ABC = A'B' + BC
  3. Assume d = 1 to both X's:
     F = A'B'C' + A'B'C + A'BC + ABC' + ABC
     = A'B' + BC + AB
- Choice 2 leads to the simplest solution

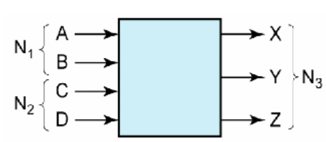
Examples of Truth Table Construction

- Design simple binary adder that adds two 1-bit binary numbers (a, b) and produces 2-bit sum.

- X = AB
- Y = A'B + AB'

A	B	SUM X	SUM Y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

- Design simple binary adder that adds two 2-bit binary numbers (N_1, N_2) and produces 3-bit sum (N_3).



TRUTH TABLE:

N_1: A B	N_2: C D	N_3: X Y Z
0 0	0 0	0 0 0
0 0	0 1	0 0 1
0 0	1 0	0 1 0
0 0	1 1	0 1 1
0 1	0 0	0 0 1
0 1	0 1	0 1 0
0 1	1 0	0 1 1
0 1	1 1	1 0 0

TRUTH TABLE:

N_1: A B	N_2: C D	N_3: X Y Z
1 0	0 0	0 1 0
1 0	0 1	0 1 1
1 0	1 0	1 0 0
1 0	1 1	1 0 1
1 1	0 0	0 1 1
1 1	0 1	1 0 0
1 1	1 0	1 0 1
1 1	1 1	1 1 0

- X(A,B,C,D)= Σ m(7,10,11,13,14,15)
- Y(A,B,C,D)= Σ m(2,3,5,6,8,9,12,15)
- Z(A,B,C,D)= Σ m(1,3,4,6,9,11,12,14)

Assignment

- Read Sections 4.1, 4.2, 4.3, 4.5, 4.6
- Section 4.7 to be covered later
- Problems:
  4.2, 4.3, 4.7, 4.9, 4.13, 4.16, 4.21, 4.23, 4.24