Chapter 9: Column Analysis and Design

Introduction

Columns are usually considered as vertical structural elements, but they can be positioned in any orientation (e.g. diagonal and horizontal compression elements in a truss).

Columns are used as major elements in trusses, building frames, and sub-structure supports for bridges (e.g. piers).
- Columns support compressive loads from roofs, floors, or bridge decks.
- Columns transmit the vertical forces to the foundations and into the subsoil.

The work of a column is simpler than the work of a beam.
- The loads applied to a column are only axial loads.
- Loads on columns are typically applied at the ends of the member, producing axial compressive stresses.
- However, on occasion the loads acting on a column can include axial forces, transverse forces, and bending moments (e.g. beam-columns).

Columns are defined by the length between support ends.
- Short columns (e.g. footing piers).
- Long columns (e.g. bridge and freeway piers).

Virtually every common construction material is used for column construction.
- Steel, timber, concrete (reinforced and pre-stressed), and masonry (brick, block, and stone).

The selection of a particular material may be made based on the following.
- Strength (material) properties (e.g. steel vs. wood).
- Appearance (circular, square, or I-beam).
- Accommodate the connection of other members.
- Local production capabilities (i.e. the shape of the cross section).

Columns are major structural components that significantly affect the building’s overall performance and stability.
- Columns are designed with larger safety factors than other structural components.
• Failure of a joist or beam may be localized and may not severely affect the building’s integrity (e.g. there is redundancy with girders and beams, but not with columns).

• Failure of a strategic column may be catastrophic for a large area of the structure.

• Failure may be due to overstressed, loss of section (deterioration), accident/sabotage (terrorism).

Safety factors for columns are used to account for the following.
• Material irregularities (e.g. out of straightness).
• Support fixity at the column ends.
• Construction inaccuracies (e.g. out of plumbness).
• Workmanship.
• Unavoidable eccentric (off-axis) loading.

9.1 Short and Long Columns – Modes of Failure
Column slenderness and length greatly influence a column’s ability to carry load.
• Very short, stout columns fail by crushing due to material failure.
  - Failure occurs once the stress exceeds the elastic (yield point) limit of the material.
• Long, slender columns fail by buckling – a function of the column’s dimensions and its modulus of elasticity.
  - Buckling is the sudden uncontrolled lateral displacement of a column at which point no additional load can be supported.
  - Failure occurs at a lower stress level than the column’s material strength due to buckling (i.e. lateral instability).

Short columns
Short columns fail by crushing at very high stress levels that are above the elastic limit of the column material.

Compressive stress for short columns is based on the basic stress equation developed at the beginning of Chapter 5.
• If the load and column size (i.e. cross-sectional area) are known, the compressive stress may be computed as
  \[ f_a = \frac{P_{\text{actual}}}{A} \leq F_a \]
where

\[ f_a = \text{actual compressive stress (psi or ksi)} \]
\[ A = \text{cross-sectional area of the column (in}^2\text{)} \]
\[ P_{\text{actual}} = \text{actual load on the column (pounds or kips)} \]
\[ F_a = \text{allowable compressive stress per code (psi or ksi)} \]

- This stress equation can be rewritten into a design form to determine the required short column size when the load and allowable material strength are known.

\[ A_{\text{required}} = \frac{P_{\text{actual}}}{F_a} \]

where

\[ A_{\text{required}} = \text{minimum cross-sectional area of the column} \]

**Long Columns - Euler Buckling**

Long columns fail by buckling at stress levels that are below the elastic limit of the column material.

- Very short column lengths require extremely large loads to cause the member to buckle.
- Large loads result in high stresses that cause crushing rather than buckling.

Buckling in long, slender columns is due to the following.

- Eccentricities in loading.
- Irregularities in the column material.

Buckling can be avoided (theoretically) if the loads were applied absolutely axially, the column material was totally homogeneous with no imperfections, and construction was true and plumb.

A Swiss mathematician named Leonhard Euler (1707 - 1783) was the first to investigate the buckling behavior of slender columns within the elastic limit of the column's material.

- Euler’s equation shows the relationship between the load that causes buckling of a (pinned end) column and the material and stiffness properties of the column.
The critical buckling load can be determined by the following equation.

\[ P_{\text{critical}} = \pi^2 E I_{\text{min}} / L^2 \]

where

- \( P_{\text{critical}} \) = critical axial load that causes buckling in the column (pounds or kips)
- \( E \) = modulus of elasticity of the column material (psi or ksi)
- \( I_{\text{min}} \) = smallest moment of inertia of the column cross-section (in\(^2\))
  (Most sections have \( I_x \) and \( I_y \); angles have \( I_x \), \( I_y \) and \( I_z \).)
- \( L \) = column length between pinned ends (inches)

- As the column length increases, the critical load rapidly decreases (since it is proportional to \( L^2 \)), approaching zero as a limit.
- The critical load at buckling is referred to as Euler's critical buckling load.

Euler's equation is valid only for long, slender columns that fail due to buckling.

- Euler's equation contains no safety factors.
- Euler's equation results in compressive stresses developed in columns that are well below the elastic limit of the material.

**Slenderness Ratios**

The *radius of gyration* is a geometric property of a cross section that was first introduced in Chapter 6.

\[ I = Ar^2 \quad \text{and} \quad r = (I/A)^{1/2} \]

where

- \( r \) = radius of gyration of the column cross section (in)
- \( I \) = least (minimum) moment of inertia (in\(^4\))
- \( A \) = cross-sectional area of the column (in\(^2\))

The radius of gyration is geometric property that is used in the analysis and design of columns.

Using the radius of gyration, the critical stress developed in a long column at buckling can be expressed by the following equation.

\[ f_{\text{critical}} = P_{\text{critical}} / A = \pi^2 E I_{\text{min}} / AL^2 = \pi^2 E (Ar^2) / AL^2 = \pi^2 E / (L/r)^2 \]
The term “L/r” is known as the \textit{slenderness ratio}.

- A higher slenderness ratio means a lower critical stress that will cause buckling.
- Conversely, a lower slenderness ratio results in a higher critical stress (but still within the elastic range of the material).

Column sections with large \( r \)-values are more resistant to buckling.

- \textbf{Compare the difference in} \( r_{\text{min}} \) \textbf{values and slenderness ratios for the three column cross sections shown below}.
  - All three cross sections have relatively equal cross-sectional areas but very different radii of gyration about the critical buckling axis.
  - All three columns are assumed to be 15 feet in length and pin-connected at both ends.

\begin{align*}
\text{W 21 x 50} & : r_x = 8.18'' & r_y = 1.30'' & A = 14.7 \text{ in}^2 \\
\text{W 10 x 49} & : r_x = 4.35'' & r_y = 2.54'' & A = 14.4 \text{ in}^2 \\
\text{12'' Dia. Std. Pipe} & : r_x = 4.39'' & r_y = 4.39'' & A = 13.6 \text{ in}^2
\end{align*}

\[
\frac{L}{r} = \frac{15 \text{ (12''/r)}}{1.30} = 138.5 \quad \frac{L}{r} = \frac{15 \text{ (12''/r)}}{2.54} = 70.9 \quad \frac{L}{r} = \frac{15 \text{ (12''/r)}}{4.39} = 41.1
\]

Comparison of steel cross sections with equivalent areas

The most efficient column sections for axial loads are those with almost equal \( r_x \) and \( r_y \) values.

- Circular pipe sections and square tubes are the most effective shapes since the radii of gyration about both axes are the same (\( r_x = r_y \)).
- Circular pipe sections and square tubes are often used as columns for light to moderate loads.
Wide-flange shapes may be preferred despite the structural advantages of closed cross-sectional shapes (like tubes and pipes).

- The practical considerations of wide-flange shapes include the following.
  - Wide-flange sections support heavy loads.
  - Wide-flange sections accommodate beam connections.

Special wide-flange sections are specifically manufactured to provide relatively symmetrical columns ($r_x/r_y$ ratios approaching 1.0) with large load-carrying capability.

- Most of these column sections (generally W8, W10, W12, and W14) have depth and flange widths approximately equal (i.e. a "boxy" configuration).

9.2 End Support Conditions and Lateral Bracing

Previously, each column was assumed to have pinned ends in which the member ends were free to rotate (but not translate) in any direction at their ends.

- When the column buckles, it will do so in one smooth curve.
- The length of this curve is referred to as the effective length.

In practice, a column may not be pinned at the ends.

- The column length free to buckle is greatly influenced by its end support conditions.
- The load-carrying capacity of a column is affected by the end support conditions.
  -Restraining the ends of a column with a fixed support increases the load-carrying capacity of a column.
  -Allowing translation as well as rotation (i.e. free end) at one end of a column generally reduces its load-carrying capacity.

Column design formulas generally assume a condition in which both ends are pinned.

- When other conditions exist, the load-carrying capacity is increased or decreased and the allowable compressive stress is increased or decreased.
- A factor K is used as a multiplier for converting the actual column length to an effective buckling length based on end conditions.

The American Institute of Steel Construction (AISC) provides recommended effective length factors when ideal conditions are approximated.

- The six cases are presented as follows.
Case A: Both ends are pinned.
The structure is adequately braced against lateral forces (e.g. wind and earthquake forces).

Theoretical K-value: $K = 1.0$
Effective length: $L_e = L$

$P_{\text{critical}} = \pi^2EI_{\text{min}}/L^2$

Examples: Timber column nailed top and bottom; steel column with simple clip angle connection top and bottom.

Case B: Both ends are fixed.
The structure is adequately braced against lateral forces (e.g. wind and earthquake forces).

Theoretical K-value: $K = 0.5$
Effective length: $L_e = 0.5L$

$P_{\text{critical}} = \pi^2EI_{\text{min}}/(0.5L)^2 = 4\pi^2EI_{\text{min}}/L^2$

Examples: Concrete column rigidly (monolithically cast) connected to large beams top and bottom; steel column rigidly connected (welded) to large steel beams top and bottom.

Case C: One end is pinned and one end is fixed.
The structure is adequately braced against lateral forces (e.g. wind and earthquake forces).

Theoretical K-value: $K = 0.7$
Effective length: $L_e = 0.707L$

$P_{\text{critical}} = \pi^2EI_{\text{min}}/(0.707L)^2 = 2\pi^2EI_{\text{min}}/L^2$

Examples: Concrete column rigidly connected to concrete slab at the base and attached to light-gauge roofing at the top.
**Case D:** One end is free and one end is fixed.
Lateral translation is possible.

- An eccentric column load is developed.

  Theoretical K-value: \( K = 2.0 \)
  
  Effective length: \( L_e = 2.0 \, L \)
  
  \( P_{\text{critical}} = \frac{\pi^2 EI_{\text{min}}}{(2L)^2} = \frac{\pi^2 EI_{\text{min}}}{4L^2} \)

Examples: Water tank mounted on a simple pipe column; flagpole.

**Case E:** Both ends are fixed with some lateral translation.

  Theoretical K-value: \( K = 1.0 \)
  
  Effective length: \( L_e = 1.0 \, L \)
  
  \( P_{\text{critical}} = \frac{\pi^2 EI_{\text{min}}}{L^2} \)

Examples: Flexible column attached to a rigid beam and supported by a fixed base.

**Case F:** The base is pinned and the top is fixed with some lateral translation.

  Theoretical K-value: \( K = 2.0 \)
  
  Effective length: \( L_e = 2.0 \, L \)
  
  \( P_{\text{critical}} = \frac{\pi^2 EI_{\text{min}}}{(2L)^2} = \frac{\pi^2 EI_{\text{min}}}{4L^2} \)

Examples: Steel column with a rigid connection to a beam above and a simple pin connection at the base.
Intermediate Bracing
End connections affect the buckling capacity of a column.

- Fixed connections are an obvious solution to minimizing column sizes; however, the cost associated with achieving rigid connections is high and such connections are difficult to make.

- Timber columns are generally assumed as pinned-connected because the material strength generally precludes the construction of true rigid joints.

Lateral bracing about the weak axis can increase the strength and stability of a column by reducing the effective length of the column.

- Examples of such lateral bracing include the following.
  - Infill wall panels.
  - Window and door headers.
  - Supports for curtain walls.

Bracing provided in one plane does not provide resistance to buckling in the perpendicular plane.

- Columns must be checked in both directions to determine the critical slenderness ratio to be used in the analysis or design.
Example Problems - Short and Long Columns - Modes of Failure

Problem 9.2 (p. 455)

Given: Two 3-1/2” standard steel pipe sections are strapped together to form a pin-connected column.

\[ L = 24' \]

\[ E = 29,000 \text{ ksi} \]

Find: The critical axial load when buckling occurs.

Solution

Moment of inertia for 3-1/2” standard pipe (ref. Table A5 of the textbook)

\[ I = 4.79 \text{ in}^4 \]

Applicable equation: \( P_{\text{critical}} = \frac{\pi^2EI_{\text{min}}}{L^2} \)

\[ I_x = I_{\text{min}} = 2 \times (4.79) = 9.58 \text{ in}^4 \]

\[ L = 24' \]

\[ P_{\text{critical}} = \frac{\pi^2EI_{\text{min}}}{L^2} \]

\[ = \pi^2 (29,000)(9.58)/(24 \times 12''/')^2 \]

\[ P_{\text{critical}} = 33.1 \text{ kips} \]
Problem 9.4 (p. 455)

Given: An 8" diameter timber pole fixed in a large concrete footing at grade and pinned at the top.

\[ E = 1.0 \times 10^6 \text{ psi} \]

Find: Maximum height of the pole to support a 25 kip load.

Solution

Applicable equation: \[ P_{\text{critical}} = \frac{2\pi^2 E I_{\text{min}}}{L^2} \]

- Case C: One end pinned and one end fixed.

\[ K = 0.7 \]

\[ L_e = 0.707 L \]

\[ P_{\text{critical}} = \frac{\pi^2 E I_{\text{min}}}{(0.707 L)^2} = \frac{2\pi^2 E I_{\text{min}}}{L^2} \]

\[ I = \pi d^4/64 = \pi (8)^4/64 = 201.1 \text{ in}^4 \]

\[ P_{\text{critical}} = 25 \text{ kips (25,000 lb)} \]

\[ 25,000 = 2\pi^2 (1.0 \times 10^6)(201.1 \text{ in}^4)/L^2 \]

\[ L^2 = 2\pi^2 (1,000,000)(201.1)/25,000 = 158,780 \text{ in}^2 \]

\[ L = 398.5" (33.2') \]
9.3 Axially Loaded Steel Columns

The discussion so far has been limited to short columns that crush and long slender columns that buckle.

- Somewhere in between these two extremes lies a zone where a “short” column transitions into a “long” column.

**Short columns:** Steel (A36) columns with slenderness ratios \( \ell/r \leq 40 \) are defined as “short columns.”
  - The mode of failure is crushing.

**Long columns:** Steel columns with a slenderness ratio of \( 120 \leq K\ell/r \leq 200 \) are defined as “long columns.”
  - The mode of failure is buckling.
  - For structural steel columns: \( K\ell/r \leq 200 \)

**Intermediate columns:** Steel columns with a slenderness ratio of \( 40 \leq K\ell/r \leq 120 \) are defined as “intermediate columns.”
  - The mode of failure is a combination of crushing (yielding) and buckling.

Column classification is based on slenderness.

- Figure 9.4 (Column classification based on slenderness, p. 456 of the textbook) compares Euler’s equation with the American Institute of Steel Construction (AISC) equations.
  - The initially flat portion of the curve (in the short-column range, that is, \( K\ell/r < 40 \)) indicates material yielding with no buckling taking place.
  - On the far right end of the curve (\( K\ell/r > 120 \)), the compressive stresses are relatively low and buckling is the mode of failure.
  - In the intermediate-column range (\( 40 < K\ell/r < 120 \)), failure has aspects of both yielding and buckling.

The load-carrying ability of intermediate-length columns is influenced by both the strength and elastic properties of the column material.

- *Empirical design formulas,* based on extensive testing and research, have been developed to cover the design of columns within the limits of each column category.

Since 1961, the American Institute of Steel Construction (AISC) has adopted a set of column design formulas that incorporate the use of a variable factor of safety, depending on slenderness, for determining allowable compressive stress.
• The AISC formulas recognize only two slenderness categories: short/intermediate and long (ref. Figure 9.15, p. 457 of the textbook).
  - Long steel (A36) columns are defined as those having a $K\ell/r$ exceeding a value called $C_c = 126.1$.
  - Long steel columns for high strength steel ($F_y = 50$ ksi) are defined as those having a $K\ell/r$ exceeding a value called $C_c = 107.0$.

The $C_c$ value represents the theoretical demarcation line between inelastic (i.e. short/intermediate columns) and elastic (i.e. long columns) behavior.

- $C_c = (2\pi^2E/F_y)^{1/2}$

**Allowable compressive stress (AISC)**

- Short/intermediate columns: The AISC allowable compressive stress ($F_a$) for short/intermediate columns ($K\ell/r < C_c$) is computed using AISC Eq. E2-1 (ref. p. 458 of the textbook).
- Long columns: When axially loaded compression members have a slenderness ratio $K\ell/r > C_c$, the allowable compressive stress is computed using AISC Eq. E2-2 (ref. p. 458 of the textbook).

where (for both equations)

- $K\ell/r =$ the largest effective slenderness ratio of any unbraced length of column
- $F_a =$ allowable compressive stress (psi or ksi)

The two preceding equations represent actual design equations that can be used to analyze or design steel columns.

- These equations appear rather intimidating, especially equation E2-1.
- The AISC Manual of Steel Construction has developed a table for $K\ell/r$ from 1 to 200 with the representative allowable stress $F_a$.
- No computations using E2-1 and E2-2 are necessary since the equations have been used in generating these tables (Tables 9.1 and 9.2, pp. 461 - 462 of the textbook).

Pinned end supports are often assumed for design purposes and provide a conservative result.

- Pinned ends are assumed even if the ends of steel columns are typically restrained to some degree at the bottom by being welded to a base plate, which in turn is anchor-bolted to a concrete footing.
Steel pipe columns generally have plates welded at each end, and then bolted to other parts of the structure.
- Such restraints vary greatly and are difficult to evaluate.
- Designers rarely take advantage of the restraint to increase the allowable stress, which therefore adds to the factor of safety of the design.

On the other hand, tests have indicated that, in the case of fixed-end conditions, the "theoretical" $K = 0.5$ values are somewhat *non-conservative* when designing steel columns.
- Since true joint fixity is rarely possible, the AISC recommends the use of *recommended $K$-values* (listed in Figure 9.16, p. 459 of the textbook).

Following are examples of recommended design values for $K$ when ideal conditions are approximated (ref. Figure 9.16, p. 459 of the textbook).

<table>
<thead>
<tr>
<th>Condition</th>
<th>Theoretical K value</th>
<th>Recommended K value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinned-pinned</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Fixed-fixed</td>
<td>0.5</td>
<td>0.65</td>
</tr>
<tr>
<td>Fixed-pinned</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

### Analysis of Steel Columns

*Column analysis means finding the allowable compressive stress $F_a$ on a given column or its allowable load capacity $P$.*
- A simple analysis procedure is outlined below.

**Given:** Column length, support conditions, grade of steel ($F_y$), applied load, and column size.

**Required:** Check the load capacity of the column.
(In other words, is $P_{\text{actual}} < P_{\text{allowable}}$?)

**Procedure:**
- a. Calculate the slenderness ratio $K\ell/r_{\text{min}}$; the largest $K\ell/r$ governs.
- b. Enter the appropriate AISC Table
  (Table 9.1 for $F_y = 36$ ksi or Table 9.2 for $F_y = 50$ ksi)
- c. Pick out the respective $F_a$
- d. Compute $P_{\text{allowable}}$: $P_{\text{allowable}} = F_a \times A$
  where
  $A = \text{cross sectional area of the column (in}^2)$
  $F_a = \text{allowable compressive stress (ksi)}$
e. Check the column adequacy.

\[
\begin{align*}
\text{If } & P_{\text{actual}} < P_{\text{allowable}} \quad \text{then the column is OK.} \\
\text{If } & P_{\text{actual}} > P_{\text{allowable}} \quad \text{then the column is overstressed.}
\end{align*}
\]
Example Problems - Axially Loaded Steel Columns

Problem 9.8 (p. 466)

Given: Two C12 x 20.7 channel sections welded together to form a closed box section.

\[ F_y = 36 \text{ ksi} \]
\[ L = 20' \]

Top and bottom are pinned.

Find: Allowable axial load \( P_a \).

Solution

Find the following geometric properties for the channel section (C12 x 20.7) from Table A4 (p. 573 of the textbook).

\[ A = 6.09 \text{ in}^2, b_f = 2.942", I_x = 129 \text{ in}^4, r_x = 4.61" \]
\[ I_y = 3.88 \text{ in}^4, r_y = 0.799", \bar{x} = 0.698" \]

Calculate the moment of inertia and radius of gyration for each axis.

\[ A = 2 (6.09) = 12.18 \text{ in}^2 \]
\[ I_x = 2 (129) = 258 \text{ in}^4 \]
\[ r_x = (I_x/A)^{1/2} = (258/12.18)^{1/2} = 4.61" \text{ (same as } r_x \text{ for a single channel)} \]
\[ I_y = 2 [3.88 + (6.09)(2.942 - 0.698)^2] = 2 (34.55) = 69.10 \text{ in}^4 \]
\[ r_y = (I_y/A)^{1/2} = (69.10/12.18)^{1/2} = 2.38" \]

Calculate the slenderness ratios (\( K = 1.0 \)).

\[ KL/r_x = (1.0) 20 (12"/')/4.61 = 52.1 \]
\[ KL/r_y = (1.0) 20 (12"/')/2.38 = 100.8 \text{ \hspace{1cm} Governs} \]
Find \( F_a \) (i.e. the allowable compressive stress).

- Use a slenderness ratio \( \frac{KL}{r_y} = 100.8 \) (ref, Table 9.1, p. 461 of the textbook).
  
  \( F_a = 12.88 \) ksi (by interpolation)

**Calculate the allowable axial load.**

\[
P_{allow} = F_a (A) = 12.88 \times 12.18 = 156.9 \text{ k}
\]

\( P_{allow} = 156.9 \) k

**Compare \( P_{allow} \) with \( P_{critical} \) calculated using the Euler equation.**

- Recall that \( P_{critical} \) has no factor of safety.

\[
P_{critical} = \pi^2 EI_{min}/L^2
\]

\[
I_y = I_{min} = 69.10 \text{ in}^4
\]

\[
E = 29,000 \text{ ksi}
\]

\[
P_{critical} = \pi^2 (29,000)(69.10)/(20 \times 12')^2 = 343.2 \text{ kips}
\]

\( P_{critical} = 343.2 \) kips
Problem 9.10 (p. 466)

Given: 5” dia standard steel pipe column (A36 steel) with an applied load of 60 kips. Top is pinned and the base is fixed.

Find: Maximum allowable height of column.

Solution

From Table A5 (p. 607 of the textbook) find the following geometric properties for the 5” dia. standard steel pipe section.

\[ A = 4.30 \text{ in}^2, \ r = 1.88” \]

Calculate the compressive stress in the column.
\[ F_a = \frac{P}{A} = \frac{60}{4.30} = 13.95 \text{ ksi} \]

Determine the slenderness ratio (ref. Table 9.1, p. 486 of the textbook).
\[ KL/r = 92.15 \text{ for } F_a = 13.95 \text{ ksi} \]
(by interpolating)

Select the recommended design value K.

- K = 0.80 Case (b): Pinned-fixed (ref. Figure 9.16, p. 459 of the textbook).
\[ KL/r = 0.8 (L)/1.88 = 92.15 \]
\[ L = 92.15 (1.88)/0.8 = 216.6” (18.0’) \]
Thus, maximum height is \( L = 18.0’ \)
Design of Steel Columns
The design of axially loaded steel columns involves the selection of an appropriate column size.

- Accomplished by using specialized column tables such as those contained in the American Institute of Steel Construction’s Manual of Steel Construction - Allowable Stress Design, Ninth Edition.

- Structural design varies from analysis in that there are several possible answers to a problem (e.g. different sizes, different shapes).

The selection of a column size is dependent on the following.
1. Strength and safety requirements.
2. Architectural issues (e.g. shape, appearance) and construction issues (e.g. beam connections) may influence the final selection.

Since the AISC Column Design Tables are not available (that would require the purchase of the AISC manual), steel column design will involve an iterative trial-and-error process.

A few guidelines for selecting a column cross section follow.

- Efficient column cross sections for axial loads often use circular or “boxier” wide-flange members.

- Along with spatial and construction concerns, relative maximum or minimum sizes may be specified by the architect, thus limiting the choices.

- Smaller scale steel structures may use 8” and 10” nominal size wide-flange columns, while larger buildings with heavier loads will often use 12” and 14” nominal sizes.
  - These sections are the “boxier” or square sizes, with the depth and flange width of the cross section being approximately equal.

The trial-and-error procedure may be outlined as follows.

**Given:** Column length, support conditions, grade of steel (F_y), applied load (P_{actual}).

**Required:** Column size to safely support the load.
Procedure:

a. **Guess at a size.**
   - For a smaller scale building, maybe try a square W8 or W10 in the middle of the weight grouping.
   - A similar trial using larger sections is appropriate for heavier loads.

b. Once the trial size has been selected, cross-sectional properties are known.
   - Compute the critical slenderness ratio, taking into account the end conditions and intermediate bracing.

c. Using the larger $K\ell/r$ value, enter Table 9.1 (for $F_y = 36$ ksi, p. 461 of the textbook) or Table 9.2 (for $F_y = 50$ ksi, p. 462 of the textbook) and obtain the respective $F_a$ (i.e. the allowable compressive stress).

d. Calculate the $P_{\text{allowable}} = F_a \times A$ of the trial section.

e. **Compare $P_{\text{actual}}$ to $P_{\text{allowable}}$.**
   - If $P_{\text{actual}} > P_{\text{allowable}}$, then the column is overstressed and a larger section should be selected next.
   - If the trial section is too strong (i.e. $P_{\text{actual}}$ much less than $P_{\text{allowable}}$), try again with a smaller column size.

One way to check the relative efficiency of the cross section is to examine its percent of stress level.

$$\text{Percent of stress} = \frac{P_{\text{actual}}}{P_{\text{allowable}}} \times 100\%$$

A percent of stress in the 90 - 100 percent level is very efficient.

f. Repeat this process until an adequate but efficient section is obtained.

**Note:** Steps (b) through (e) are essentially the procedure used previously in the analysis of steel columns.

Alternative trial and error procedures:

1. **Assume a design stress ($F_a$).**
   - Divide the column load by the compression stress for an estimated column area (i.e. $A = P/F_a$).
   - Select a trial column section with approximately that area.
   - Then, continue the procedure following steps b through f, as outlined above.
2. Assume a slenderness ratio $KL/r$.
   - Determine the corresponding allowable compressive stress $F_a$ from Table 9.1 (for $F_y = 36$ ksi, p. 461 of the textbook) or Table 9.2 (for $F_y = 50$ ksi, p. 462 of the textbook).
   - Divide the column load by the compressive stress for an estimated column area (i.e. $A = P/F_a$).
   - Select a trial column section with approximately that area.
   - Then, continue the procedure following steps b through f, as outlined above.

The following suggestions allow an inexperienced designer to make good initial assumptions for the slenderness ratio $KL/r$.
   - The slenderness ratio $KL/r$ for an average column of 10 to 15-feet length will generally fall between 40 and 60.
   - If the column is longer than 15 feet, assume the slenderness ratio to be a little higher than 40 to 60.
   - If the column carries a very heavy load (e.g. 750 – 1000 kips), a larger column will be required and the designer may estimate a smaller value for the slenderness ratio.
   - For lightly loaded bracing members, the designer may estimate high slenderness ratios of 100 or more.
Example Problem - Design of Steel Columns

Problem 9.15 (p. 473)

Given: Six-story building.
   - Tributary floor area = 20' x 25' = 500 SF
   - Roof load: 80 psf (dead load)
     40 psf (snow load)
   - Floor load: 125 psf (live load)
     100 psf (dead load)
   - L_{first floor} = 20'
   - L_{third floor} = 16'
   - F_y = 36 ksi

Find: Design a typical interior third-floor column and first-floor column using the most economical W12* section at each level.

(*Note: There are several possible solutions using W10 and W14; specifying a W12 limits the number of solutions.)

Solution

Third floor column
Determine load on third-floor column.
\[ P_{actual} = P_{roof} + P_{floors} \text{ (4th, 5th, and 6th floor loadings)} \]
\[ P_{actual} = 500 \text{ SF} \times (80 \text{ psf} + 40 \text{ psf}) + 3 \times [500 \text{ SF} \times (125 \text{ psf} + 100 \text{ psf})] \]
\[ = 60,000 + 3 \times (112,500) = 397,500 \text{ pounds} \]
\[ P_{actual} = 397.5 \text{ k (third floor column)} \]

First try

a. Select W12 section for third floor column.
   Try W12 x 40 (A = 11.8 \text{ in}^2, d = 11.94", r_x = 5.13", r_y = 1.93")

b. Compute the critical slenderness ratio (K = 1.0).
   \[ KL/r_{min} = 1.0 \times (16"/')/1.93 = 99.5 \]
c. Using Table 9.1 (p. 461 of the textbook), find $F_a$ (i.e. the allowable compressive stress) using $KL/r_{\text{min}} = 99.5$.

$$F_a = 13.04 \text{ ksi (by interpolating)}$$

d. Calculate the allowable axial load.

$$P_{\text{allowable}} = F_a (A) = 13.04 \times (11.8) = 153.87 \text{ k}$$

e. Compare $P_{\text{actual}}$ to $P_{\text{allowable}}$.

$$P_{\text{allowable}} = 153.87 \text{ k} < 397.5 \text{ k} \quad \text{NG}$$

Second try
a. Select W12 section for third floor column.

Try W12 x 96 ($A = 28.2 \text{ in}^2$, $d = 12.71\text{"}$, $r_x = 5.44\text{"}$, $r_y = 3.09\text{"}$)

b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{\text{min}} = 1.0 \times (16)(12\text{")}/3.09 = 62.1$$

c. Using Table 9.1 (p. 461 of the textbook), find $F_a$ (i.e. the allowable compressive stress) using $KL/r_{\text{min}} = 62.1$.

$$F_a = 17.23 \text{ ksi (by interpolating)}$$

d. Calculate the allowable axial load.

$$P_{\text{allowable}} = F_a (A) = 17.23 \times (28.2) = 485.9 \text{ k}$$

e. Compare $P_{\text{actual}}$ to $P_{\text{allowable}}$.

$$P_{\text{allowable}} = 485.9 \text{ k} > 397.5 \text{ k} \quad \text{OK (but over-designed)}$$

Third try
a. Select W12 section for third floor column.

Try W12 x 72 ($A = 21.1 \text{ in}^2$, $d = 12.25\text{"}$, $r_x = 5.31\text{"}$, $r_y = 3.04\text{"}$)

b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{\text{min}} = 1.0 \times (16)(12\text{")}/3.04 = 63.2$$
c. Using Table 9.1 (p. 461 of the textbook), find \( F_a \) (i.e. the allowable compressive stress) using \( KL/r_{\text{min}} = 63.2 \).

\[ F_a = 17.12 \text{ ksi (by interpolating)} \]

d. Calculate the allowable axial load.

\[ P_{\text{allowable}} = F_a (A) = 17.12 \times (21.1) = 361.2 \text{ k} \]

e. Compare \( P_{\text{actual}} \) to \( P_{\text{allowable}} \).

\[ P_{\text{allowable}} = 361.2 \text{ k} < 397.5 \text{ k} \quad \text{NG} \]

Fourth try

a. Select W12 section for third floor column.

Try W12 x 79 (\( A = 23.2 \text{ in}^2 \), \( d = 12.38" \), \( r_x = 5.34" \), \( r_y = 3.05" \))

b. Compute the critical slenderness ratio (\( K = 1.0 \)).

\[ KL/r_{\text{min}} = 1.0 \times (16)(12")/3.05 = 62.95 \]

c. Using Table 9.1 (p. 461 of the textbook), find \( F_a \) (i.e. the allowable compressive stress) using \( KL/r_{\text{min}} = 62.95 \).

\[ F_a = 17.14 \text{ ksi (by interpolating)} \]

d. Calculate the allowable axial load.

\[ P_{\text{allowable}} = F_a (A) = 17.14 \times (23.2) = 397.6 \text{ k} \]

e. Compare \( P_{\text{actual}} \) to \( P_{\text{allowable}} \).

\[ P_{\text{allowable}} = 397.6 \text{ k} > 397.5 \text{ k} \quad \text{OK} \]

f. Check efficiency.

\[ \text{Efficiency} = P_{\text{actual}}/P_{\text{allowable}} \times 100\% = 397.5/397.6 \times 100\% = 99.97\% \]

Select: W12 x 79
First floor column
Determine load on first-floor column.

\[ P_{\text{actual}} = P_{\text{roof}} + P_{\text{floors}} (2^{\text{nd}}, 3^{\text{rd}}, 4^{\text{th}}, 5^{\text{th}}, \text{and 6}^{\text{th}} \text{ floor loadings}) \]

\[ P_{\text{actual}} = 500 \text{ SF} (80 \text{ psf} + 40 \text{ psf}) + 5 [500 \text{ SF} (125 \text{ psf} + 100 \text{ psf})] \]
\[ = 60,000 + 5(112,500) = 622,500 \text{ pounds} \]

\[ P_{\text{actual}} = 622.5 \text{ k} \text{ (first floor column)} \]

Recall from previous work: \( W12 \times 96 \ P_a = 485.9 \) for 16’ column. So a heavier column will be needed for a longer column – 20' – carrying a greater \( P_{\text{actual}} \).

First try
a. Select \( W12 \) section for first floor column.
   Try \( W12 \times 120 \) (\( A = 35.3 \text{ in}^2, d = 13.12", r_x = 5.51", r_y = 3.13" \))

b. Compute the critical slenderness ratio \((K = 1.0)\).
   \[ KL/r_{\text{min}} = 1.0 \frac{(20)(12'')}{3.13} = 76.7 \]

c. Using Table 9.1 (p. 461 of the textbook), find \( F_a \) (i.e. the allowable compressive stress) using \( KL/r_{\text{min}} = 76.7 \).
   \[ F_a = 15.72 \text{ ksi (by interpolating)} \]

d. Calculate the allowable axial load.
   \[ P_{\text{allowable}} = F_a (A) = 15.72 (35.3) = 554.9 \text{ k} \]

e. Compare \( P_{\text{actual}} \) to \( P_{\text{allowable}} \).
   \[ P_{\text{allowable}} = 554.9 \text{ k} \times 622.5 \text{ k} \quad \text{NG} \]

Second try
a. Select \( W12 \) section for first floor column.
   Try \( W12 \times 152 \) (\( A = 44.7 \text{ in}^2, r_x = 5.66", r_y = 3.19" \))

b. Compute the critical slenderness ratio \((K = 1.0)\).
   \[ KL/r_{\text{min}} = 1.0 \frac{(20)(12'')}{3.19} = 75.2 \]
c. Using Table 9.1 (p. 461 of the textbook), find $F_a$ (i.e. the allowable compressive stress) using $KL/r_{\text{min}} = 75.2$.
   \[
   F_a = 15.88 \text{ ksi (by interpolating)}
   \]

d. Calculate the allowable axial load.
   \[
   P_{\text{allowable}} = F_a (A) = 15.88 (44.7) = 709.8 \text{ k}
   \]
e. Compare $P_{\text{actual}}$ to $P_{\text{allowable}}$.
   \[
   P_{\text{allowable}} = 709.8 \text{ k} > 622.5 \text{ k} \quad \text{OK (but over-designed)}
   \]

Third try
a. Select W12 section for first floor column.
   
   Try W12 x 136 ($A = 39.9 \text{ in}^2$, $d = 13.41"$, $r_x = 5.58"$, $r_y = 3.16"$) - only section remaining between the W12 x 120 and W12 x 152.

b. Compute the critical slenderness ratio ($K = 1.0$).
   \[
   KL/r_{\text{min}} = 1.0 (20)(12")/3.16 = 75.9
   \]
c. Using Table 9.1 (p. 461 of the textbook), find $F_a$ (i.e. the allowable compressive stress) using $KL/r_{\text{min}} = 75.9$.
   \[
   F_a = 15.80 \text{ ksi (by interpolating)}
   \]
d. Calculate the allowable axial load.
   \[
   P_{\text{allowable}} = F_a (A) = 15.80 (39.9) = 630.4 \text{ k}
   \]
e. Compare $P_{\text{actual}}$ to $P_{\text{allowable}}$.
   \[
   P_{\text{allowable}} = 630.4 \text{ k} > 622.5 \text{ k} \quad \text{OK}
   \]
f. Check efficiency.
   \[
   \text{Efficiency} = P_{\text{actual}}/P_{\text{allowable}} \times 100\% = 622.5/630.4 \times 100\% = 98.75\% 
   \]

Select: W12 x 136
9.4 Axially Loaded Wood Columns

Wood columns support beams and girders that, in turn, support tributary areas of roof and floor loads.

Other structural members that are in compression are designed using the same methods that are utilized for the design of building columns.

• Bridge piers
• Compression chords of a truss
• Studs in a load-bearing wall

Modes of failure
• Long columns tend to buckle under critical load.
• Short columns fail by the crushing of the wood fibers.

Slenderness ratio
• For wood columns, the ratio of the column length to its width is just as important as the slenderness ratio is for steel columns.
• In wood columns, the slenderness ratio is defined as the laterally unsupported length in inches divided by the least (minimum) dimension of the column.

\[
\text{Slenderness ratio} = \frac{L}{d_{\text{min}}} = \frac{L}{d_1}
\]

where \(d_1 < d_2\)

• Wood columns are restricted to a maximum slenderness ratio of \(\ell_e/d \leq 50\).
  - This slenderness ratio is analogous to the limiting slenderness ratio of \(KL/r_{\text{min}} \leq 200\) used for steel columns.

Effective length
• The effective length of steel columns was determined by applying a K factor to the unsupported length of the column to adjust for the end support.
• Similar effective length factors, called \(K_e\) in wood columns, are used to adjust for the various end conditions.
• Recommended \(K_e\) values (ref. Figure 9.20, p. 474 of the textbook) are identical to those of steel columns.
**Typical \( K_e \) values**

Most wood construction is detailed such that translation (sidesway) is restrained but the ends of the column are free to rotate (i.e. pin connection).

- The \( K_e \) value is generally taken as 1.0, and the effective length is equal to the actual unsupported length.
- Even if some fixity exists in the top or bottom connection, it is difficult to evaluate the degree of fixity to assume in design.
- \( K_e = 1.0 \) is an acceptable assumption that is usually a bit conservative in some connection conditions.

**Wood columns**

- Wood columns can be solid members or rectangular, round, or other shapes, or spaced columns built up from two or more individual solid members separated by blocking.
- The majority of all wood columns in buildings are solid rectangular sections. The analysis and design methods examined in this section will be limited to these types.


- The new standard incorporated a new method and new equations for the design of wood elements and connections.
- Previous categorizing of wood columns into the short-, intermediate-, or long-column range resulted in three different equations for each respective slenderness range.
- The NDS-91 now utilizes a single equation, providing a continuous curve over the entire range of slenderness ratios.

The compressive stress for an axially loaded wood column of known size is expressed as follows.

\[
f_c = \frac{P}{A} \leq F_c'
\]

where

- \( f_c \) = actual compressive stress parallel to grain
- \( P \) = axial compressive force in the member
- \( A \) = cross-sectional area of the column
- \( F_c' \) = allowable compressive stress parallel to grain
The allowable load for an axially loaded wood column is expressed as follows.

\[ P_{\text{allow}} = F'_c A \text{ and } P_{\text{actual}} \leq P_{\text{allow}} \]

To obtain the allowable compressive stress \( F'_c \), many adjustments to the tabulated base (reference) compressive stress \( F_c \) are necessary.

The NDS-91 defines the \( F'_c \) as follows.

\[ F'_c = F_c (CD) (CM) (CT) (CF) (C_p) \]

where

- \( F'_c \) = allowable compressive stress parallel to grain
- \( F_c \) = reference compressive stress parallel to grain (found in building code tables, NDS tables, and wood design handbooks, such as Table 5.2, p. 278 of the textbook)
- \( CD \) = load duration factor (ref. Figure 9.23, p. 477 of the textbook)
- \( CM \) = wet service factor (accounts for moisture content in the wood)
  - = 1.0 for dry service conditions as in most covered structures, dry service condition defined as follows:
    - Moisture content \( \leq 19\% \) for sawn lumber
    - Moisture content \( \leq 16\% \) for glu-lams
- \( CT \) = temperature factor (usually taken as 1.0 for normal temperature conditions)
- \( CF \) = size factor (an adjustment based on member sizes used)
- \( C_p \) = column stability factor (accounts for buckling and is directly affected by the slenderness ratio)

The objective of this course is to analyze and design structural elements in a preliminary way (rather than the full complement of equations and checks performed by a structural engineer).

- Accordingly, the preceding allowable compressive stress equation is simplified as follows.

\[ F'_c = F_c * C_p \]

where

- \( F_c * = F_c (CD) (CM) (CT) (CF) \approx F_c CD \) (for preliminary column design)
- \( C_p \) = column stability factor (ref. Table 9.3)
\( F_c = \text{tabulated compressive stress parallel to grain} \) (found in building code tables, NDS tables, and wood design handbooks, such as Table 5.2, p. 278 of the textbook)

\( C_D = \text{load duration factor} \)

\( C_M, C_t, \text{and } C_F \) are all assumed to be equal to 1.0, which is generally the case for a majority of wood columns.

Load duration factor \( C_D \) (ref. Figure 9.23, p. 477 of the textbook)

- Wood has a unique structural property in which it can support higher stresses if the applied loads are for a short period of time.
- All tabulated stress values contained in building codes, NDS, or wood design manuals apply to “normal” load duration and dry service conditions.
- The \( C_D \) value adjusts tabulated stresses to allowable values based on the duration (time) of loading.
  a) “Normal” duration is taken as 10 years and \( C_D = 1.0 \).
  b) For permanent dead load, \( C_D = 0.9 \).
  c) Short-duration loading from wind, earthquake, snow, or impact allows \( C_D \) values higher than 1.0 but less than 2.0.

The column stability factor \( C_p \) multiplied by \( F_c \) essentially defines the column curve (equation).

- This equation, originally developed by Ylinen, explains the behavior of wood columns as the interaction of the two modes of failure: buckling and crushing.
- An equation for computing \( C_p \) is provided on page 477 of the textbook.
  - Terms in the equation for \( C_p \) include the following.
    \[
    F_{cE} = \text{Euler critical buckling stress for columns} \\
    F_{cE} = \frac{K_{cE}E'}{(\ell_e/d)^2} \approx \frac{K_{cE}E}{(\ell_e/d)^2} \\
    F_c^* \approx F_c C_D \\
    c = \text{buckling and crushing interaction factor for columns} \\
    = 0.8 \text{ for sawn lumber columns} \\
    = 0.9 \text{ for glu-lam columns} \\
    \]
  - Terms used in calculating \( F_{cE} \) include the following.
    \( K_{cE} = 0.30 \text{ for sawn lumber} \)
    \( = 0.418 \text{ for glu-lams} \)
Table 9.3 (p. 478 of the textbook) is provided to simplify the computations for preliminary column analysis/design.

- This table (similar to Tables 9.1 and 9.2 for steel) was developed by inputting slenderness ratios between 1 and 50 to determine $F_{CE}$ values for sawn and glu-lam members.
- By dividing $F_{CE}$ values by $F_c^*$ and generating ratios of $a (F_{CE}/F_c^*)$, corresponding $C_p$ values can be taken from the table.
- Table 9.3 eliminates the necessity of laborious computations for $C_p$ using the equation developed by Ylinen.

Analysis of Wood Columns

A simple procedure can be adopted for checking the adequacy or capacity of wooden columns.

- This methodology is for approximate analysis and assumes the simplifications discussed previously.

Given: Column size, column length, grade and species of lumber, and end conditions.

Required: The allowable capacity of a column or the adequacy of a given column.

Procedure:

a. Calculate the $(l_e/d_{\text{min}})$.

b. Compute $F_{CE}$ (i.e. Euler’s critical buckling stress).

$$F_{CE} = \frac{K_{CE} E}{(l_e/d)^2}$$

$K_{CE} = 0.3$ for sawn lumber
$K_{CE} = 0.418$ for glu-lams

$E = \text{reference modulus of elasticity (ref. Table 5.2, p. 278 of the textbook)}$
c. Compute $F_c^* \approx F_c C_D$

$F_c$ = reference compressive stress parallel to grain (found in building code tables, NDS tables, and wood design handbooks, such as Table 5.2, p. 278 of the textbook)

$C_D$ = load duration factor (ref. p. 477 of the textbook)

d. Calculate the ratio: $F_{cE}/F_c^*$

e. Enter Table 9.3 (p. 478); obtain respective $C_p$.

f. Calculate the allowable compressive stress $F_c'$.

$F_c' = F_c^* C_p$

g. Calculate the allowable load and compare the allowable load with the actual load.

$P_{\text{allowable}} = F_c' \times A \leq P_{\text{actual}}$

where

$A =$ cross-sectional area of column
Problem 9.16 (p. 485)

Given: 6x6 S4S Southern Pine Dense No. 1 column.

Assume 7-day duration roof live load.

Pin connections top and bottom.

\[ L = 14' - 0'' \]

\[ F_c = 975 \text{ psi (Table 5.2, p. 278 of the textbook)} \]

\[ E = 1.6 \times 10^6 \text{ psi (Table 5.2, p. 278 of the textbook)} \]

Find: Capacity of column.

Solution

Section properties for the 6 x 6 S4S column.

Actual dimensions: 5.5” x 5.5”

\[ A = 30.25 \text{ in}^2 \text{ (Table A1-b, p. 567 of the textbook)} \]

a. Calculate the slenderness ratio \( \ell_e/d_{\text{min}} \).

\[ \ell_e/d_{\text{min}} = 14(12'')/5.5 = 30.55 \]

b. Compute \( F_{ce} \) (i.e. Euler's critical buckling stress).

\[ F_{ce} = K_{ce} E/\ell_e/d_{\text{min}}^2 = 0.30(1.6 \times 10^6)/(30.55)^2 = 514.3 \]

where \( K_{ce} = 0.30 \) for visually graded lumber (p. 502)

c. Compute \( F_c^* \approx F_c C_D \quad (C_M, C_t, C_F = 1.0) \)

\[ F_c^* = 975 (1.25) = 1,218.75 \]

where \( C_D = 1.25 \) for 7-day roof live load (Figure 9.23, p. 477)

d. Calculate the ratio \( F_{ce}/F_c^* \)

\[ F_{ce}/F_c^* = 514.3/1,218.75 = 0.422 \]

e. Enter Table 9.3 (p. 478) and obtain \( C_p \).

\[ C_p = 0.377 \text{ (for sawn lumber, by interpolating)} \]
f. Calculate the allowable compressive stress $F_c'$.

$$F_c' = F_c \cdot C_p = 1,218.75 \times 0.377$$

$$F_c' = 459.5 \text{ psi}$$

g. Calculate the allowable load (i.e. the capacity of the column).

$$P_{\text{allowable}} = F_c' \times A = 459.5 \times 30.25 = 13,900 \text{ lb}$$

$$P_{\text{allowable}} = 13.9 \text{ k}$$
Problem 9.18 (p. 485)

Given: 6-3/4” x 10-1/2” glu-lam column
   Lateral bracing about the weak axis at mid height.
   Pin connections top and bottom.
   \( A = 70.88 \text{ in}^2 \)
   \( F_c = 1,650 \text{ psi} \) (Table 5.2, p. 278 of the textbook)
   \( E = 1.8 \times 10^6 \text{ psi} \) (Table 5.2, p. 278 of the textbook)

Find: Load capacity of the column.

Solution

a. Calculate the slenderness ratios \( \ell_e/d \) for each axis of bending and determine the critical value.
   \[(\ell_e/d)_x = 22(12”)/10.5 = 25.14 \quad \text{Governs}\]
   \[(\ell_e/d)_y = 11(12”)/6.75 = 19.56\]

b. Compute \( F_{cE} \) (i.e. Euler’s critical buckling stress).
   \[F_{cE} = K_{cE} E/(\ell_e/d)^2 = 0.418(1.8 \times 10^6)/(25.14)^2 = 1,190.5\]
   where \( K_{cE} = 0.418 \) for glu-lams (p. 477)

c. Compute \( F_{c*} \approx F_c C_D \) \( (C_M, C_t, C_F = 1.0) \)
   \[F_{c*} = 1,650 (1.00) = 1,650\]
   where \( C_D = 1.00 \) for 10-year design (Figure 9.23, p. 477)

d. Calculate the ratio \( F_{cE}/F_{c*} \)
   \[F_{cE}/F_{c*} = 1,190.5/1,650 = 0.722\]

e. Enter Table 9.3 (p. 478) and obtain \( C_p \).
   \( C_p = 0.620 \) (for glu-lam, by interpolating)

f. Calculate the allowable compressive stress \( F_{c'} \).
   \[F_{c'} = F_{c*} C_p = 1,650 (0.620)\]
   \[F_{c'} = 1,023.0 \text{ psi}\]
g. Calculate the allowable load (i.e. the capacity of the column).

\[ P_{\text{allowable}} = F'_c \times A = 1,023.0 \times (70.88) = 72,510 \text{ lb} \]

\[ P_{\text{allowable}} = 72.5 \text{ k} \]
Design of Wood Columns

Column design in wood is a trial-and-error process.

- Start by making an estimate on size (try out your intuition) and check out the adequacy or inadequacy by following the analysis procedure given in the previous section.
- Axially loaded wood columns without mid height bracing are generally square in cross section, or in some cases just slightly rectangular.
- Fortunately, there are fewer possible wood sections to choose from compared with the wide array of sizes available in steel.

One design procedure using the trial-and-error method could be:

Given: Column length, column load, grade and species of lumber to be used, and end conditions.

Required: An economical column size.

Procedure:

a. Guess at a trial size.
   - Try and select a square or almost square cross section unless the column’s weak axis is braced.

b. Follow the same steps used in the analysis procedure in the previous section.

c. If $P_{\text{allowable}} \geq P_{\text{actual}}$, then OK.

d. If $P_{\text{allowable}} \leq P_{\text{actual}}$, pick a larger size and cycle through the analysis procedure again.
9.5 Columns Subjected to Combined Loading or Eccentricity

Previous sections have assumed that the compression members are subjected to concentric loading (loads acting through the centroid of the column section).

- In practice, however, concentric loading is rarely the case.

Many columns are subjected to bending in combination with axial compression loads.

- A load may not act through the centroid of a column cross section due to
  - Non-uniform bearing.
  - Misalignment of the framing.
  - The crookedness of a member.
- Compression members carrying bending moment due to eccentricity or side loading in addition to compression are referred to as beam columns.

Axially loaded columns have a relatively uniform distribution of stress over the cross-sectional area.

Bending stress, which involves tension and compression stresses, must be added algebraically to the compressive stress due to the axial load.

- Small eccentricities (e.g. flexible beam connected to a rigid column) alter the stress distribution.
  - The cross section remains in compression, although non-uniform as shown at the right.
If large eccentricities (e.g. rigid beam connected to a less rigid column) exist, tensile stresses may develop over part of the cross section, as shown in the figure at the right.

**Tensile stress**

- The tension stresses that developed in masonry construction of the past were formerly of great concern.
- Tension stresses are of little significance for the building systems and materials used today in contemporary buildings.
- Timber, steel, pre-stressed concrete, and reinforced concrete all possess good tension capability.

*Beam columns* are evaluated using an *interaction equation* that incorporates the bending stress with the compressive stress.

- The general interaction equation is expressed mathematically as follows.

\[
\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0
\]

where

- \( f_a = \frac{P}{A} \) (the actual compressive - axial - stress)
- \( F_a \) = allowable compressive stress (based on \( KL/r \) for steel or \( l_e/d \) for timber)
- \( f_b = \frac{Mc}{I} = \frac{M}{S} \) (actual bending stress)
- \( M = P \times e \) for eccentrically loaded members
- \( M = \) bending moment due to side load or rigid frame action
- \( e = \) eccentricity
- \( F_b = \) allowable bending stress

If a member is subjected to axial compression and bending about both the x- and y-axes, the interaction formula is adapted to incorporate the biaxial bending.

- Therefore, the most generalized form of the equation is expressed mathematically as follows.

\[
\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \text{ (for biaxial bending)}
\]
where
\[
f_{bx} = M/S_x = \text{actual bending stress about the } x\text{-axis}
\]
\[
f_{by} = M/S_y = \text{actual bending stress about the } y\text{-axis}
\]

An interaction curve, shown as Figure 9.33 (p. 490 of the textbook), illustrates the theoretical combining of the axial compressive and bending stresses.

\textbf{P-\Delta effect}

Bending moments in columns may result from the following.

\begin{itemize}
  \item Lateral forces
  \item Applied moments
  \item Eccentricity of the end loads
\end{itemize}

These bending moments can cause a member to deflect laterally, resulting in additional bending moment due to the P - \Delta effect.

\begin{itemize}
  \item The lateral displacement generates an eccentricity for the load P.
  \item The eccentricity results in the creation of additional moment at the mid-height of the column equal to P \times \Delta (known as a second-order bending moment).
  \item Slender columns are particularly sensitive to this P-\Delta effect and must be accounted for in the interaction equation.
\end{itemize}

The AISC (steel) and NDS (timber) manuals have introduced a magnification factor to incorporate the P-\Delta effect.

\begin{itemize}
  \item A generalized interaction equation for both steel and wood is expressed mathematically as follows.
    \[
    f_a/F_a + [f_b \times (\text{Magnification Factor})]/F_b \leq 1.0
    \]
  \item The actual analysis/design equations for steel and wood are noted in the textbook (pp. 490 - 491).
\end{itemize}

Analyzing and designing beam columns using the AISC and NDS equations are more appropriately done in follow-up courses dealing specifically with steel and wood design.