PROJECTS FOR DIFFERENTIAL GEOMETRY

Information. The following are suggested projects for Differential Geometry. All problems come from the second edition of *Differential Geometry and its Applications*, but some material from my book *The Mathematics of Soap Films* may also be required. Your projects must be written in either Word (or TeX) or with Maple or a combination of these. You must begin with a definition of the thing you are interested in. Do not just list problems and solutions. You may work in groups of no more than *three* people.

Project 1: Developable Surfaces
Developable surfaces are the special cases of ruled surfaces (those having a parametrization $\mathbf{x}(u,v) = \beta(u) + v\,\delta(u)$) having zero Gauss curvature. These surfaces have industrial applications. The project consists of: Exercises 3.2.20 – 3.2.25 and *either* of two choices:

(1) Exercises 5.5.5, 5.5.7, 6.9.1 and the Maple procedure holounroll on p. 332 and Maple material in §5.6.3:
(2) a synopsis of §5.7 together with the Maple work found there and that on p. 111 for the tangent developable. For the synopsis, present the motivation and important points of the discussion as well as the most important calculations.

Project 2: The Gauss Map
The Gauss map is a mapping from a surface to the unit sphere given by the unit normal of the surface. The Gauss map can tell you a great deal about the surface. The project consists of: Exercises 2.3.9 – 2.3.12, 4.8.27, material on p. 118, Proposition 4.8.28, Exercise 4.8.30 and a description of Schwarz's theorem in *The Mathematics of Soap Films* giving a criterior for area minimization. Maple should be used to draw Gauss maps for various surfaces.

Project 3: Minimal Surfaces and Area Minimization
Minimal surfaces are those with mean curvature equal to zero at each point. If a surface minimizes area inside some boundary, it is a minimal surface. So soap films are physical manifestations of minimal surfaces. This project focusses on some aspects of area minimization and minimal surfaces. It consists of: Exercises 2.1.16, 4.2.3, 4.3.5, 4.3.6 (the Maple approach from *The Mathematics of Soap Films*), 4.9.2, 4.9.3, 7.3.8, 7.3.9 and the Maple material in §4.9.5.

Warning: The following two projects do not have as many exercises associated with them, but they require learning about elliptic functions *and* explaining what you learn. So, do not think they are easy.

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Project 4: Unduloids

One-celled organisms sometimes take the shape of unduloids. These are surfaces of revolution that arise from minimizing surface area subject to enclosing a fixed volume (read Theorem 7.7.15). Equivalently, unduloids are examples of surfaces of revolution with constant non-zero mean curvature. The project consists of: deriving the differential equation of Theorem 3.6.1, using the equation to parametrize unduloids via elliptic functions. This material is in §3.7. Explain this material, carry out all Maple calculations and plot unduloids. Furthermore, do Exercise 3.7.4. Finally, plot geodesics on unduloids by carrying out the discussion in §5.6.4 (and using "halfbouncepoint") and do Exercise 5.6.9.

Project 5: The Shape of a Mylar Balloon

Mylar balloons are found at children's birthday parties. They are constructed by sewing together two disks of Mylar and inflating. The "sideways" shape is determined by a variational argument from the calculus of variations. The project consists of: explaining the calculus of variations derivation of the parametrization of the Mylar balloon using elliptic functions. Explain the material on elliptic function in §3.7 (including Exercise 3.7.4) and the material on the Mylar balloon in §7.9 (including the Exercise on p. 408). Finally, create a "halfbouncepoint" procedure for the Mylar balloon similar to the one in §5.6.4. Finally, do Exercise 5.6.9.