MCE 380

First Midterm Exam Solution – Spring 2010

Section 51 Exam

Problem 1

$$C = A_c \sqrt{1 + \frac{T_c}{B_c}} \quad , \quad T_c = \frac{5}{9}(T_F - 32)$$

$$C = A_c \sqrt{1 + \frac{5/9(T_F - 32)}{B_c}} = A_c \sqrt{\frac{9B_c + 5T_F - 160}{9B_c}}$$

$$C = A_c \sqrt{\left(\frac{9B_c - 160}{9B_c}\right) + \left(\frac{5}{9B_c}\right) T_F}$$

$$C = A_c \sqrt{\left(\frac{9B_c - 160}{9B_c}\right)\left[1 + \frac{\left(\frac{5}{9B_c}\right) T_F}{\left(\frac{9B_c - 160}{9B_c}\right)}\right]}$$

$$C = A_c \sqrt{\frac{9B_c - 160}{9B_c}} \sqrt{1 + \frac{T_F}{\frac{\frac{9B_c - 160}{9B_c}}{\frac{5}{9B_c}}}}$$

$$\underset{(m/s)}{C} = A_c \sqrt{1 - \frac{160}{9B_c}} \sqrt{1 + \frac{T_F}{(9B_c - 160)/5}}$$

$$\underset{(ft/s)}{C} = \frac{A_c \sqrt{1 - \frac{160}{9B_c}}}{0.3033} \sqrt{1 + \frac{T_F}{\left(\frac{9B_c - 160}{5}\right)}}$$

Use $A_c = A = 331.3$ use $B_c = B = 273.15$

$$\rightarrow \underset{(ft/s)}{C} = 1056.2 \sqrt{1 + \frac{T_F}{459.67}}$$

answer: a)

Variable x: {4.35, 6.12, 8.88, 4.06, 4.83, 5.22, 5.19, 7.62, 5.63, 5.19}

Problem 2)

The median is equal to (5.19+5.22)/2 = 5.205	; The one in the middle.
The mode is equal to 5.19	; The one that repeated most.
The mean is equal to 5.709	; The average (equation 3.3 page 62).
The population standard deviation is equal to 1.414	; Equation 3.7 page 63.
The sample standard deviation is equal to 1.49	; Equation 3.8 page 63.
The sample degrees of freedom is equal to 10-1 = 9	; Degrees of freedom, v = n-1

Choice (a): The median is (5.205/1.414) = 3.68 population standard deviations.

Choice (a) is true.

Choice (c): The average is (5.709/1.49) = 3.83 sample standard deviations.

Choice (c) is false.

The answer is choice (a).

Problem 3)

Choice (a) is wrong because the mean 5.709 is not lie between 2.3776 and 4.0404.

Choice (b): the value of Δ is equal to 5.709 – 5.3776 = 0.3314

$$\Delta = t\sigma/\sqrt{n}$$

$$0.3314 = t*(1.49)/\sqrt{10}$$

$$t = 0.7033$$

From table 3.7 (page 100), the value of t is close to 0.703 which belong to 50% confidence interval. Therefore, choice (b) is true.

Choice (c): is wrong because the mean 5.709 is not lie between 2.3792 and 5.0388.

The answer is (b).

Problem 4)

The function combines of product and additive terms. Use equation 3.2 (page 52).

With 98% confidence interval, t is equal to 2.821 (table 3.7 page 100)

Find Δ for x:

$$\Delta = t\sigma/\sqrt{n}$$

$$= 2.821*(1.49)/\sqrt{10}$$

$$= 1.329$$

Find each term in equation 3.2:

$R = x^2y + xy$

$$\frac{\partial R}{\partial x} = 2xy + y = (2*5.709*1) + 1 = 12.418$$

$$\frac{\partial R}{\partial y} = x^2 + x = 5.709^2 + 5.709 = 38.302$$

Plug all of the values into equation 3.2:

$$w_R = [(12.418 * 1.329)^2 + (38.302 * 0.2)^2]^{1/2}$$

$w_R = 18.19$

The answer is (d).

Problem 5)

For n=10, d_{max}/σ equals to 1.96 (table 3.5 page 79).

Calculate d_i and d_i/σ for each value of x, using $\sigma = 1.49$.

$d_i = \lvert x_i - x_m \rvert$	d_i/σ
1.359	0.912
0.411	0.276
3.171	2.128
1.649	1.107
0.879	0.59
0.489	0.328
0.519	0.348
1.911	1.282
0.079	0.053
0.519	0.348

x = 8.88 has d_i/σ exceeded 1.96, therefore this point can be eliminated.

The answer is (b).

Problem 6)

[4.8449, b] is a p% confidence interval

Choice (a): b = 6.5731

Prove if choice (a) is true:

Lower limit and upper limit must be equal.

5.709 - 4.8449 =? 6.5731 – 5.709

0.8641 = 0.8641

Choice (a) is true.

Choice (b): t = 1.833

Δ = 5.709 – 4.8449 = 0.8641

$\Delta = t\sigma/\sqrt{n}$

$0.8641 = t*(1.49)/\sqrt{10}$

$t = 1.834$

Choice (b) is true.

Choice (c): $p = 95\%$

From table 3.7 page 100, with $t = 1.833$, $p = 90$. Choice (c) is false.

The answer is (e).

Problem 7)

As discussed in class, it is generally better to improve the accuracy of the more inaccurate instruments first, since the upgrade costs are lower and the contribution of the rough instruments to the overall error is usually the largest. Therefore a) is false. Statement b) is true and the rest are false.

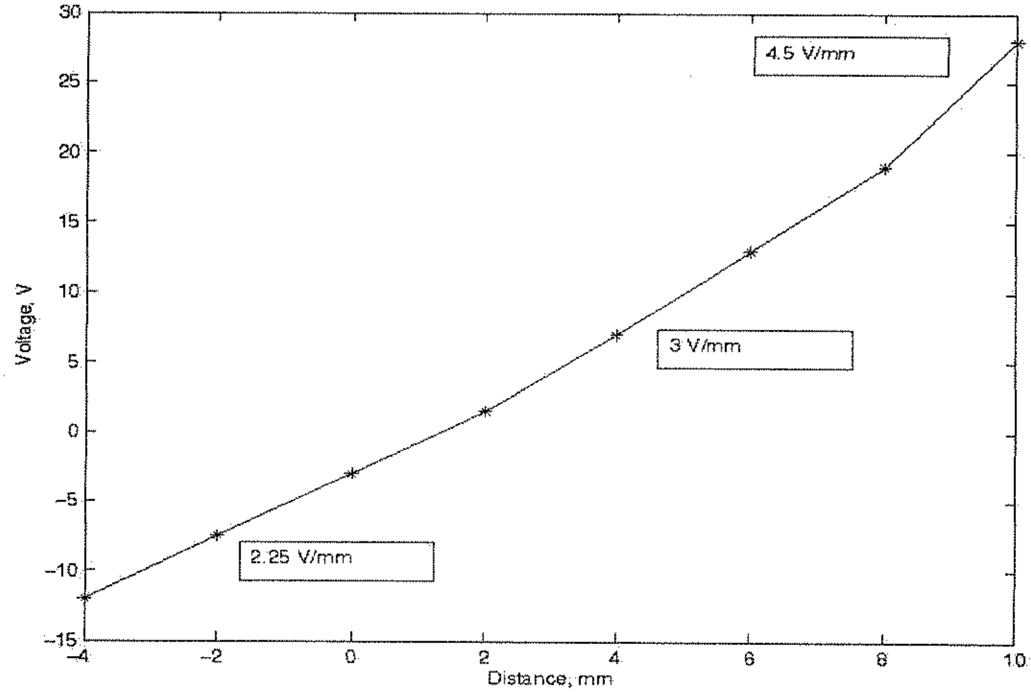
The answer is (b).

Problem 8)

A quick sketch and slope (sensitivity) calculations show that:

- The transducer has variable sensitivity (nonlinear)
- The sensitivity is 4.5 V/mm in the high range
- The sensitivity is always positive
- The sensitivity increases in the high range of distances

The answer is (e): b) and d) are true.



Problem 9)

P = mRT/V = 0.002*348*287.06/(1e-4) = 1.998 MPa

To get the worst combination, use maximum value of the numerator and minimum value of the denominator.

P_{worst} = 0.00201*287.06*349/(9.9e-5) = 2.034 MPa

The deviation is 2.034 – 1.998 = 0.036 MPa

The answer is (c).

Problem 10)

Interpolation for 85% confidence interval:

$$\frac{y-y1}{x-x1} = \frac{y2-y1}{x2-x1}$$

y = 1.638 + (85-80)*(2.353-1.638)/(90-80) ; Values from table 3.7 page 100.

$y = 1.9955$

The pressure values are 2.0088, 2.0857, 2.0853, and 2.1219 MPa.

The mean is equal to 2.0754 MPa. ; Equation 3.3 page 62.

The sample standard deviation is equal to 0.0476 ; Equation 3.8 page 63.

$$\Delta = t\sigma/\sqrt{n}$$

$$= 1.9955*(0.0476)/\sqrt{4}$$

$$= 0.04749$$

$$X = X_m \pm \Delta$$

$$= 2.0754 \pm 0.04749$$

The 85% confidence interval for the mean pressure is [2.02791, 2.1229] MPa.

The answer is choice (c).