MCE380: Measurements and Instrumentation Lab

Chapter 9: Force, Torque and Strain Measurements

Topics:
Elastic Elements for Force Measurement
Dynamometers and Brakes
Resistance Strain Gages
Holman, Ch. 10

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Elastic Elements for Force Measurement

Material elasticity allows the conversion between force and displacement according to Hooke's law:

$$F = ky$$

The elastic constant k can take various forms according to the geometry of the elastic body, the direction of force application and the direction of the displacement readout. In a straight bar (Fig. 10.3), the constant has the form

$$k = \frac{AE}{L}$$

where E is the Young's modulus, A is the area of the cross section and L is the length. In a cantilever beam with the force at the tip and the displacement at the tip (Fig. 10.4), the constant is

$$k = \frac{3EI}{L^3}$$

where I is the moment of inertia of the cross section. Eq. 10.16 in Holman has a typo.

Elastic Torsion in Torque Measurement

Torsion bars work in an entirely analogous way as linear elastic elements. The basic relationship is

$$M = k\phi$$

where M is torque (N-m) and ϕ is angular deflection (rad). The constant for a hollow cylinder of outer and inner radii r_o and r_i is

$$k = \frac{\pi G(r_o^4 - r_i^4)}{2L}\phi$$

where G is the shear modulus (Pa). The shear modulus is related to the Young's modulus and Poisson ratio μ by

$$G = \frac{E}{2(1+\mu)}$$

If the angular deflection is small, and can be read using strain gages at 45 degrees from the axis (see Fig. 10.7). The strain gages will indicate strains of

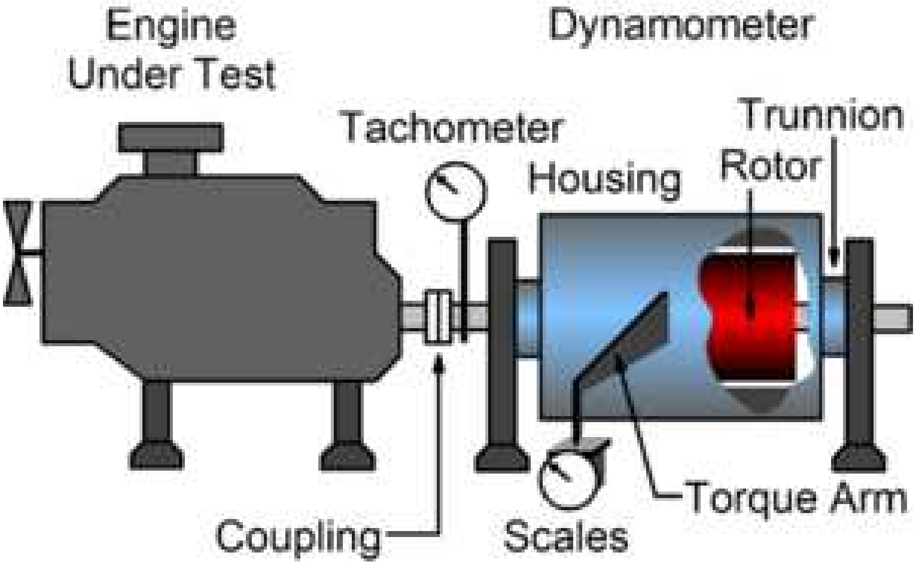
$$\epsilon_{45^\circ} = \pm\frac{Mr_o}{\pi G(r_o^4 - r_i^4)}$$

Rotary Torque Sensor

This device allows the measurement of torque on rotating shafts. We will be using this device in the lab.

Reaction Dynamometer

The rotor of a DC machine is connected to the shaft whose torque is to be determined. The housing of the DC machine is mounted on bearings. An arm attached to the housing rests against a force-measuring device (scale). The torque on the housing equals that of the rotor by action-reaction principle.



Uniaxial Stress and Strain

Consider a uniform elastic bar of area A and length L under the action of an axial force (Fig. 10.10). The stress is given by

$$\sigma = \frac{F}{A}$$

while the strain obeys linear elasticity:

$$\epsilon = \frac{\sigma}{E}$$

ϵ is the *engineering strain*, that is, it doesn't account for the reduction in area that arises as a consequence of elongation.
The engineering strain is also equal to

$$\epsilon = \frac{\Delta L}{L}$$

when Δ is small.

Bending Stress and Strain

We will need to use the bending stress and strain formulas for some lab experiments. For pure beam bending, the axial stress due to bending and the moment are related by

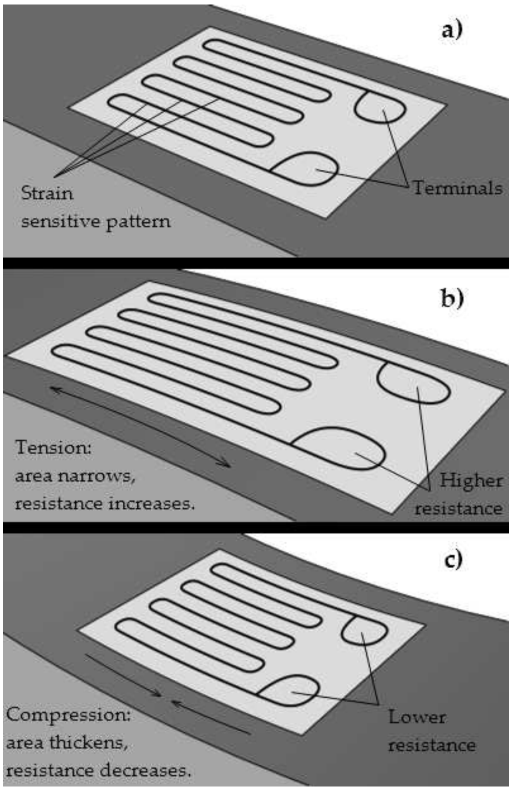
$$\sigma = \frac{My}{I}$$

The strain at the surface of the beam can be obtained as

$$\epsilon = \frac{M\bar{y}}{EI}$$

where $\bar{y}$ is half the thickness of the beam. The bending moment M can be obtained by examining the mounting conditions (cantilever, simply supported, etc.) and the loading conditions.

Resistance Strain Gages



A strain gage is made of a thin wire or foil embedded in an insulating matrix which is very flexible. The wire deforms the same as the surface to which it's bonded, changing its resistance. Once more, a Wheatstone bridge can be used to find the resistance and, therefore, the strain. A *strain gage reader* is a signal-conditioning unit containing the bridge and other necessary electronic elements allowing a direct readout of strain.

The Gage Factor

The resistance is related to length as follows:

$$R = \rho \frac{L}{A}$$

where L, A and ρ are the length, cross-sectional area and resistivity of the wire. Taking differentials, the following can be derived:

$$\frac{dR}{R} = \epsilon(1 + 2\mu) + \frac{d\rho}{\rho}$$

The gage factor F is defined by

$$F = \frac{\frac{dR}{R}}{\epsilon}$$

We can now express the strain as

$$\epsilon = \frac{1}{F} \frac{\Delta R}{R}$$

That is, the gage factor simply expresses proportionality between percent elongation (strain) and percent change in resistance.

Strain Gage Bridges

Suppose the strain gage of resistance R_1 is one of the branches of a Wheatstone bridge. The voltmeter voltage is ΔE_D, while the excitation voltage is E (see Fig. 4.25).
When strain is applied and the resistance of the gage changes to $R_1 + \Delta R_1$, the voltage detected at the voltmeter is:

$$\frac{\Delta E_D}{E} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4} - \frac{R_2}{R_2 + R_3}$$

The resistance change can be obtained from this equation as

$$\frac{\Delta R_1}{R_1} = \frac{\frac{R_4}{R_1}\left(\frac{\Delta E_D}{E} + \frac{R_2}{R_2+R_3}\right)}{1 - \frac{\Delta E_D}{E} - \frac{R_2}{R_2+R_3}} - 1$$

Example

A strain gage is bonded 10mm from the fixed end of a cantilever beam of $L = 100$mm. The beam is made of steel (E=210 GPa) and has a cross section of 1 by 10 mm. The gage factor is 2.05 and the resistance is 120 Ω. A Wheatstone bridge with $R_2 = R_3 = R_4 = 120\Omega$ is used with an excitation voltage of $E = 10$V. An unknown deflection is applied at the tip of the beam. The voltage at the terminals of the Wheatstone bridge is $\Delta E_D = 3.5$mV. Determine the deflection and the strain.