MCE380: Measurements and Instrumentation Lab

Chapter 2: Non-Statistical Uncertainty Analysis

Topics:
Commercial instrument uncertainty specifications
Worst-case uncertainty derivations
Uncertainty reduction strategies
Reference: Holman, CH 3.

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Accuracy Declarations in Commercial Instruments

Instrument uncertainty is usually expressed as a percent of full-scale. For example, a certain voltmeter can measure voltages from -100 V to 100 V with an accuracy of 0.1 percent of full scale. This means that the true voltage can be as much as 0.2 V away from the true quantity. We often indicate this as 56.3$\pm$0.2 V, for example.

Systematic vs. Random Errors

Systematic errors are due to instrument or operator imperfections of a *non-random* nature. For example, consider the paralax error in reading instrument scales. Another example: turning the micrometer thimble past the point where the spindle touches the measured object. Each operator will have a more or less uniform "feel" for when to stop turning the thimble.

Errors which are significantly less repeatable are classified as random.

Random effects can only be characterized by repeated measurement.

Uncertainty estimation in "one-shot" measurements

Often, we want to estimate a quantity using a formula and measured variables. For example, mechanical power cannot be measured directly. We have to measure force and speed independently and then multiply the measurements together.
What is the uncertainty of our power calculation when the accuracies of the individual measurements are known?
We can address this question in a few ways:

1. We can take repeated measurements and carry a statistical analysis on the calculated power. Holman calls this "multisample data"
2. If cost/time issues prevent repeated measurements, we have to live with "one-shot" readings. Holman calls this "single-sample data".
3. For single-sample data, our uncertainty estimates will be conservative.

Worst-case uncertainty calculations

The most conservative way of estimating uncertainties in computed quantities is to find the combinations of individual uncertainties yielding the maximum and minimum values of the computed quantity. For example, suppose we calculate the density of a cubic piece of material by measuring the length of the sides and weighing the object with a scale.

$$\rho = \frac{m}{l^3}$$

where m is measured in grams using a 0-1 kg scale with an accuracy of 0.01% FS and l is measured with a 0-25 mm micrometer with an accuracy of 0.025 mm. Take the nominal mass to be 250 g and the nominal length to be 10 mm.

Obtain the maximum and minimum densities and express the uncertainty as $\pm$ deviation.

Improved uncertainty calculation

The above is *really* conservative. It can lead to seeking finer instruments when they're not really needed. The problem with the previous method is that not all of the measurements will be at their maximum errors at the same time! A less conservative calculation can be obtained following the method of Kline and Mc.Clintock (see Holman, 3.4). The idea is to take the root mean square (rms) of the perturbations induced by each measurement. For example, take the mechanical power formula with perturbations

$$P + \Delta P = (F + \Delta F)(v + \Delta v) = FV + F\Delta v + v\Delta F = P + F\Delta v + v\Delta F$$

where the term $\Delta F \Delta v$ is neglected (very small). So $\Delta P = F\Delta v + v\Delta F$. Instead of taking the worst cases of Δv and ΔF to calculate ΔP, we use

$$\Delta P = \sqrt{(F\Delta v)^2 + (v\Delta F)^2}$$

Improved uncertainty calculation

The improved method can be generalized to any calculation $R = R(x_1, x_2, ...x_n)$ using

$$w_R(x_1, x_2, ...x_n) = \sqrt{(\frac{\partial R}{\partial x_1} w_1)^2 + (\frac{\partial R}{\partial x_2} w_2)^2 + ... + (\frac{\partial R}{\partial x_n} w_n)^2}$$

where the w's represent the individual uncertainties. Note that $\frac{\partial P}{\partial F} = v$ and $\frac{\partial P}{\partial v} = F$ in the power example.

Another advantage of this is that we don't run into difficulties finding the worst-case combination. Imagine finding the worst-case combination in the formula

$$R(x_1, x_2, x_3) = \frac{(x_1 + x_2)e^{-x_1}}{x_2^2 + x_3^2} + x_1 x_2 - x_2 x_3$$

Apply this method to the density example. At home, please follow example 3.1 in Holman.

Uncertainty Calculation with Numerical Perturbations

Sometimes the computation $R = R(x_1, x_2, ...x_3)$ is very involved, making the computation of the partial derivatives $\frac{\partial R}{\partial x_i}$ a burden.

Or even worse, often there is no explicit formula for R, which is calculated by a program. The program can contain table look-ups and iterations making an analytical determination of these gradients impossible.

In this case, use the approximation

$$\frac{\partial R}{\partial x_i} \approx \frac{R(x + \Delta x_1) - R(x_1)}{\Delta x_1}$$

with small perturbations Δx_i. We can always call the routine computing R with $x_i + \Delta x_i$ as an argument and obtain the value of the derivatives for use in Eq. 3.2 in Holman. Note that $w_i = \Delta x_i$.

Some strategies for uncertainty reduction

- When a formula may be calculated in different ways, the uncertainty in the calculated result will also vary. Pick the more advantageous method.
- Holman, Example 3.3. Calculating DC electrical power with VI or V^2/R.
- Usually, it pays to improve the quality of the less-accurate instruments rather than improving the ones which already have good accuracy. The reason for this is the square power used in Eq. 3.2. Look at page 55 and Example 3.5

Problem

Several options are considered to improve the accuracy of the density estimation described before by a factor of 2. The options are:

1. Only a better scale is used (determine the required accuracy)
2. Only a better length-measuring device is used (determine the required accuracy)
3. A custom method of measuring the volume based on a displaced volume of liquid. It is anticipated that this method will yield an uncertainty of 0.1 percent of the nominal volume. Using this option excludes the use of the scale.

The additional cost associated with the improved scale or the improved length measurement is $75 per each 0.1% FS of accuracy improvement. The third method costs a flat $1200.