

Chapter 6: Basic Electrical Measurements

Topics:

Resistivity and Resistance

Temperature Dependence of Resistivity

Inductance and Inductors

Phase Lag and Power in AC circuits

Power Factor Correction

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Resistivity

The *resistivity* is a material property. It is a measure of opposition to electric current along a particular direction within the material. The resistivity is an inherent material property, independent of geometry. The resistivity of a material is defined as

$$\rho = R \frac{A}{l}$$

where ρ is the resistivity in ohm-meters and R is the resistance of a constant-section piece of material having area A and length l .

The resistivity of copper (a very good conductor) at 20°C is $1.7 \times 10^{-8} \Omega\text{m}$

The resistivity of Teflon (a very good insulator) at 20°C is $1 \times 10^{24} \Omega\text{m}$.

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Resistivity: Temperature Dependence

The resistivity of a material is a function of temperature. In some cases, we seek materials with a strong temperature dependence (for making temperature sensors). In other cases, we need materials whose ρ changes only a little with temperature (for making sensors with good temperature stability: strain gauges).

Metals generally increase their resistivity with temperature.. The temperature dependence is described by the equation:

$$\rho(T) = \rho(T_o)(1 + \alpha(T - T_o))$$

where T_o is a reference temperature and α is the *temperature coefficient*.

The higher α , the more sensitive the material to temperature. The dimensions of α are 1/temperature.

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Temp-Sensitive and Temp-Stable Materials

Constantan is an alloy made with 55% Cu and 45% Ni. It has a temperature coefficient of 0.00001. For this reason, it is used for making strain gages. (The strain readout is not affected by temperature).

Silicon has a temperature coefficient of -0.075, while Germanium has $\alpha = -0.048$. Semiconductors usually reduce their resistivity with increasing temperature. Their dependence is nonlinear (logarithmic). Semiconductors are used to make *thermistors*, a kind of temperature sensor (more later).

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Problem

A copper wire having a diameter of 0.1 mm and a length of 5mm is used as a stress sensor by measuring the current flowing through it when a constant voltage of 5V is applied. As the wire is stretched, the cross-section and the length of the wire change, creating a change in resistance and therefore, a change in current. The Young's modulus of copper is 130 GPa and the temperature coefficient is $\alpha = 0.0039$. The yield strength of annealed copper is 10ksi. Calculate:

1. The sensitivity of the sensor (amperes per Newton) at 20°C
2. The sensitivity of the sensor (amperes per Newton) at 100°C
3. The current, resistance and elongation at the yield point, for the above two temperatures.

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Inductance and Coils

An inductor can be obtained by winding a conductor in a helical shape. The value of the inductance depends on various factors, among them the material used for the core. You can calculate the inductance with:

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

where L is the inductance in Henries, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m), μ_r is the relative permeability of the core material, N is the number of turns, A is the area of the cross-section of the coil in m^2 and l is the length of the coil in m.

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Example

Estimate the inductance of a coil made by winding 1000 turns of #30 AWG copper wire in one layer on a 24mm-diameter ferrite cylinder. Ferrite has a relative permeability of 3000. What is the required length of copper wire and the resulting length of the inductor? What is the resistance of the inductor at 20°C? (assume circular loops for length calculation)

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Inductive Reactance

Suppose a sinusoidal voltage $V(t) = V_o \sin(\omega t)$ is applied to an inductor with inductance L .

Since

$$V(t) = L \frac{di(t)}{dt}$$

we have

$$i = \frac{1}{L} \int V_o \sin(\omega t) dt = -\frac{V_o}{\omega L} \cos(\omega t) = -\frac{V_o}{\omega L} \sin(\omega t + \pi/2)$$

That is, the current *lags* the voltage by a quarter of a cycle. If we look at the magnitudes of the voltage and current we see that they are proportional:

$$|i| = \frac{|V|}{\omega L}$$

Due to the similarity with Ohm's Law, the quantity ωL is called *inductive reactance* and it is measured in ohms.

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Capacitive Reactance

Suppose a sinusoidal voltage $V(t) = V_o \sin(\omega t)$ is applied to a capacitor with capacitance C .

Since

$$i(t) = C \frac{dV(t)}{dt}$$

we have

$$i = C\omega \cos(\omega t) dt = C\omega \sin(\omega t - \pi/2)$$

That is, the current *leads* the voltage by a quarter of a cycle. If we look at the magnitudes of the voltage and current we see that they are proportional:

$$|i| = C\omega |V|$$

Due to the similarity with Ohm's Law, the quantity $\frac{1}{\omega C}$ is called *capacitive reactance* and it is measured in ohms.

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Power in AC Circuits

- An AC load may be reducible to a combination of resistance and reactance. The current phase lag will differ from 90 degrees and may be lagging or leading.
- Suppose the load voltage is $V \sin(\omega t + \phi + \theta)$, while the current is $I \sin(\omega t + \phi)$, that is, θ is the angle between voltage and current.
- The instantaneous power flowing across the terminals is $p(t) = VI$. The average value over a period is given by

$$P = \frac{1}{T} \int_0^T VI \sin(\omega t + \phi + \theta) \sin(\omega t + \phi) dt$$

Using the trig identity $\sin A \sin B = \frac{1}{2} \cos(A - B) - \frac{1}{2} \cos(A + B)$ we obtain

$$P = \frac{VI}{2T} \left[\int_0^T \cos \theta dt - \int_0^T \cos(2\omega t + 2\phi + \theta) dt \right]$$

The second integral vanishes, while the second gives

$$P = \frac{VI}{2} \cos \theta$$

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Power in AC Circuits...

- Only a fraction of VI is power dissipated at the load. The term $\cos \theta$ is known as *power factor*.
- The quantity VI is known as *apparent power*, measured in VA, while $\frac{VI}{2} \cos \theta$ is the *active power*, measured in Watts.
- Resistive networks draw only active power ($\theta = 0$), while capacitors and inductors introduce reactive power.
- *Reactive power is only exchanged between the source and reactive elements in the load*

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Power in AC Circuits...

- Resistive networks draw only active power ($\theta = 0$), while capacitors and inductors introduce reactive power.
- *Reactive power is only exchanged between the source and reactive elements in the load.*
- Active power can be also expressed in terms of rms voltage and current:

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T I^2 dt}, \quad V_{rms} = \sqrt{\frac{1}{T} \int_0^T V^2 dt}$$

For sinusoids, $I_{rms}/I = V_{rms}/V = 1/\sqrt{2}$, giving

$$P = V_{rms} I_{rms} \cos \theta$$

- The rms values are also called *effective*, since the heat produced by a resistor is $I_{rms} V_{rms}$ and not IV .

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Power Factor Correction

Utility companies charge industrial users by the kVA, not by the kW. That is, they charge for apparent power, not active power.

Plants usually have AC motors and other equipment that contains inductors, resulting in a poor power factor.

Power factor correction is used to significantly reduce electricity costs.

Correction equipment consists in large banks of capacitors which eliminate or reduce the reactive component of power.

