MCE380 1st MIDTERM SOLUTION. SPRING 09

1) Re and Pr are dimensionless :

$$Re = \frac{\rho V D}{\mu}, \quad [Re] = \frac{kg\, m^{-3}\, m\, s^{-1}\, m}{N\, s\, m^{-2}} = \frac{\cancel{kg\, m^{-1}\, s^{-1}}}{\cancel{kg\, m\, s^{-2}\, s\, m^{-2}}}$$

$$[Re] = \text{—} \quad (\text{no units})$$

$$Pr = \frac{c_p \mu}{k} = \frac{J\, \cancel{kg^{-1}}\, \cancel{deg^{-1}}\, \cancel{kg}\, \cancel{m}\, s^{-2}\, s\, \cancel{m^{-2}}}{W\, \cancel{m^{-1}}\, \cancel{deg^{-1}}} = \frac{\cancel{J\, s^{-1}}}{\cancel{J\, s^{-1}}} = \text{—}$$

(no units)

Then Pr^x is also dimensionless, and so is $Re^{1/2} Pr^x$.

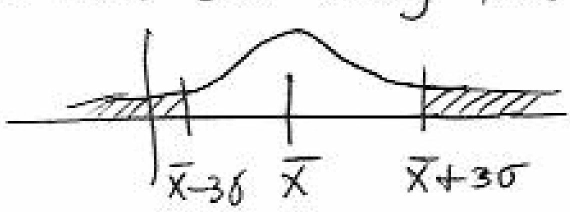
The value of x is unrelated to dimensions, it is just obtained from data fit. (Correlation). Answer: e)

2) $\bar{X} = 89.03$; median $= 88.63$; mode $= 88.13$

so the mean is not between the other two. a) is false

The population mean as estimated from the sample is $\bar{X} = 89.03$
and the std " " " " " is $\sigma = 2.0248$
(normalized by n−1). b) is true

c) According to table 3.3, the probability of a sample to be more than 3σ away from the mean is the shaded area



$$P = \frac{1}{370} \neq \frac{1}{22}$$ (c false)

since c) is false, the answer is b)

3) t- values for $\nu = 10-1 = 9$

$t_{80} = 1.383 \qquad t_{99.9} = 4.781$

Intervals: At 80% $\qquad \bar{x} \pm \frac{t\sigma}{\sqrt{10}} = [88.1444, 89.9156]$

At 99.9% $\qquad \bar{x} \pm \frac{t\sigma}{\sqrt{10}} = [85.9687, 92.0913]$

Answer: b)

4) $f(x) = \frac{x^3}{1000} - \sin(x)$

$$\Delta f = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \Delta x = \underbrace{\left.\left(\frac{3x^2}{1000} - \cos x\right)\right|_{\bar{x}}}_{22.9040} \Delta x$$

$\Delta x = \frac{t\sigma}{\sqrt{10}}$ using $t = t_{80}$, $\Delta x = 0.8856$

$\rightarrow \Delta f = 20.28$

Answer: d)

5)

x	$\lvert x-\bar{x}\rvert$	$\frac{\lvert x-\bar{x}\rvert}{\sigma}$
90.13	1.1	0.54
89.13	0.1	0.05
88.13	0.9	0.45
91.13	2.1	1.04
93.13	4.1	2.02
86.13	2.9	1.43
87.13	1.9	0.94

According to Table ~~3.5~~, the maximum acceptable deviation is 1.96 (for 10 points)

$\longrightarrow$ 93.13 is eliminated (1 point)

Answer = c)

6) $\bar{x} - 87.5816 = \frac{t\sigma}{\sqrt{n}} \longrightarrow t = 2.262$

Using Table 3.7 we see that $p = 95\%$

and $\bar{x} + \frac{t\sigma}{\sqrt{n}} = 90.4784$

Answer: d)

7) a) is false. Precision (repeatability) is prefered
b) is true. Calibration fixes inaccuracy

c) is false. Calibration cannot remove randomness

d) Is true.

Answer: e)

8)

Distance	Voltage	Sensitivity $\left(\frac{\Delta V}{\Delta D}\right)$
−2	−12	—
−1	−7.5	4.5
0	−3	4.5
1	1.5	4.5
2	7	5.5
3	13	6
4	19	6
5	28	9

a) is false, since the sensitivity changes.

b) true

c) false

d) true

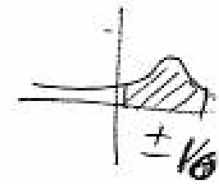
e) true

Answer: e)

9) Using table 3.3:

Chances (proportion) of data falling within K std's:

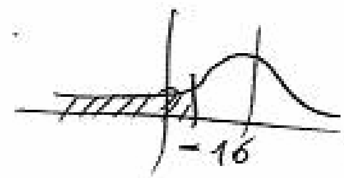
	p
K = 1	68.27%



a) is true

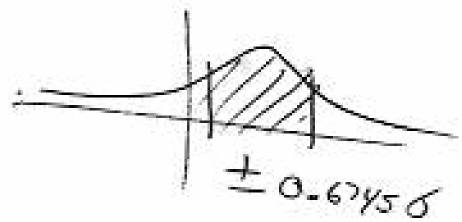
The data <u>outside</u> $\pm 1\sigma$ is $100 - 68.27\% = 31.73$.

The low range is half of this $\approx 16\%$



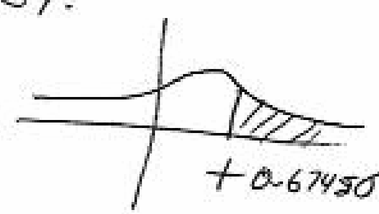
b) is true

For 0.6745σ, $p = \dfrac{1}{1+1} = 0.5$



The data <u>outside</u> $\pm 0.6745\sigma$ is $100 - 50\% = 50\%$.

The high range is half of this $\approx 25\%$



c) is true.

Answer: d)

10) $T = \frac{PV}{mR}$

The worst case is obtained using $P \pm \Delta P$, $V + \overset{0}{\Delta V}$, $m \mp \Delta m$

$$T_{(high)} = \frac{(2\times10^6 + 10\times10^3) \times 1\times10^{-4}}{(0.002 - 1\times10^{-5}) \times 287.06} = 351.8603$$

$$T_{(low)} = \frac{(2\times10^6 - 10\times10^3) \times 1\times10^{-4}}{(0.002 + 1\times10^{-5}) \times 287.06} = 344.893$$

The nominal T is $T_0 = 348.3592$

High dev $= -T_0 + 351.8603 = 3.5011$

Low dev $= T_0 - 344.893 = 3.4662$

Answer: b)

11) $$\Delta T = \sqrt{\left(\frac{\partial T}{\partial P}\Big|_0 \Delta P\right)^2 + \left(\frac{\partial T}{\partial V}\Big|_0 \Delta V\right)^2}$$

$$\frac{\partial T}{\partial P} = \frac{V}{mR}\Big|_0 = 1.7418\times10^{-4}$$

$\rightarrow \Delta T = 3.8948\ °K$

$$\frac{\partial T}{\partial V} = \frac{P}{mR}\Big|_0 = 3.4836\times10^{6}$$

Interval: [344.4644, 352.2540] (a)