# Engraving Machine Example

MCE441 - Fall 08
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## 1 Basic Design

The X-axis of the engraving machine has the transfer function

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

In this basic example, we use a proportional gain to achieve the following specifications:

1. Zero steady-state error to step inputs
2. Overshoot: 37%
3. Settling Time: 16 sec.

### 1.1 Translating the specifications into the frequency domain

The steady-state error specification is already satisfied, since the plant has one free integrator (type I). Assuming second order dominant poles, we find the required damping ratio for 37% overshoot. From the chart (Dorf, Fig. 5.8), we take $\zeta = 0.3$. This implies a phase margin of approximately $100\zeta$=30 degrees. Continuing with the assumption of dominant poles, we use the settling time formula to determine the required natural frequency:

$$T_{set} = \frac{4}{\zeta w_n} < 16$$

from where we obtain $w_n > 0.88$ rad/sec. Since $\zeta$ is in the range 0.3 to 0.8, we use the approximate formula to determine the closed-loop bandwidth:

$$w_b \approx (-1.19\zeta + 1.85)w_n = 1.322$$

Finally, we take $w_c = w_b/1.6 = 0.8263$ rad/sec. The specifications in the frequency domain become:

1. Magnitude slope at low frequencies of -20 dB/dec (already satisfied)
2. Phase margin of 30 degrees
3. Magnitude crossover frequency of 0.826 rad/sec.

### 1.2 Gain selection

Since we are using proportional control, the gain can be used to adjust either the crossover frequency **or** the phase margin to the desired values, but the two cannot be changed simultaneously. We select the gain to give the desired crossover frequency and check the value of the phase margin.

$$K \frac{1}{|jw_c(jw_c+1)(jw_c+2)|} = 1$$

from where we get $K = 2.32$. The angle of $KG(jw_c)$ is calculated to be -145 degrees, from where PM=-152+180=28 degrees. We expect the overshoot to be somewhat higher than the target of 37%. Simulating the response with $K = 2.32$ gives and actual overshoot of 44% and a settling time of 15 seconds. The gain can be reduced to trade off overshoot and settling time. Taking $K = 1.86$ gives an actual settling time of 15.8 seconds and overshoot of 36.3%.

# 2 Advanced Example - Using SISO Tool

Now we change the specifications to:

1. Zero steady-state error to ramp inputs
2. Overshoot: 5%
3. Settling Time: 1 sec.

## 2.1 Translating the specifications into the frequency domain

The steady-state error specification dictates two free integrators in $GK$. Therefore the extra integrator needed has to be incorporated by the controller. This implies a magnitude slope of -40dB/dec at low frequencies. Assuming second order dominant poles, we find the required damping ratio for 5% overshoot. From the chart (Dorf, Fig. 5.8), we take $\zeta = 0.65/$. This implies a phase margin of approximately $100\zeta$=65 degrees. Continuing with the assumption of dominant poles, we use the settling time formula to determine the required natural frequency:

$$T_{set} = \frac{4}{\zeta w_n} < 1$$

from where we obtain $w_n > 6.154$ rad/sec. Since $\zeta$ is in the range 0.3 to 0.8, we use the approximate formula to determine the closed-loop bandwidth:

$$w_b \approx (-1.19\zeta + 1.85)w_n = 6.625$$

Finally, we take $w_c = w_b/1.6 = 4.14$ rad/sec. The specifications in the frequency domain become:

1. Magnitude slope at low frequencies of -40 dB/dec
2. Phase margin of 65 degrees
3. Magnitude crossover frequency of 4.14 rad/sec.

## 2.2 Initial assessment

We import $G(s)$ and an initial $K(s) = 1$ into SISO tool. As seen in the root locus, the speed of response cannot be increased to meet the specifications, since the complex branches are to the right of -0.4, implying a large time constant. Alternatively, look at the $GK$ magnitude and phase. If the magnitude plot is shifted up (by increasing the gain) to give a crossover frequency of 4.14, the phase margin would be negative (unstable closed-loop). Figures 1 and 2 show the initial loop and closed-loop step response, respectively.

## 2.3 Adding an Integrator

We need to include an extra integrator to achieve the steady-state error requirement. Figure 3 shows how the loop is destabilized by this operation. The phase at $w_c = 4.14$ rad/sec has worsened.

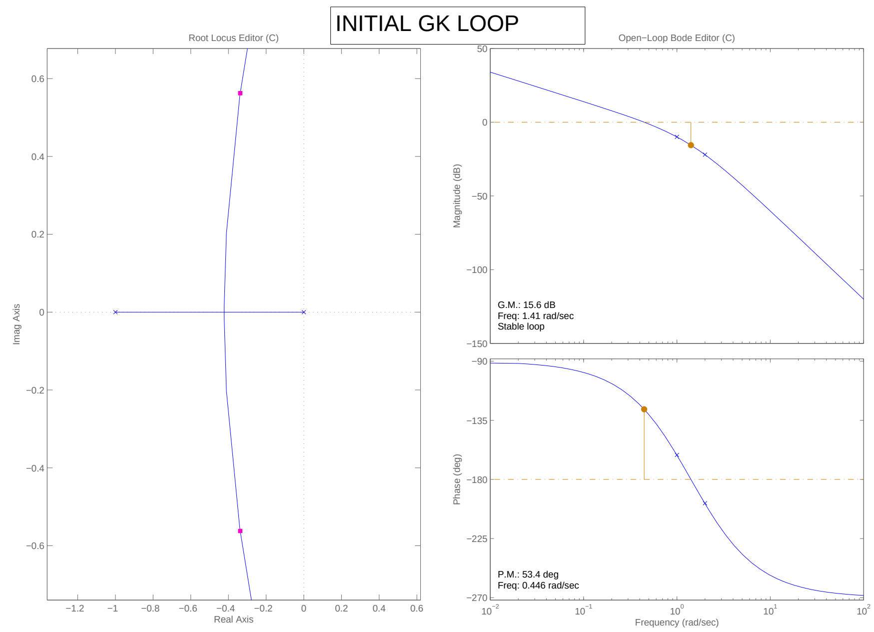


Figure 1: Initial Loop

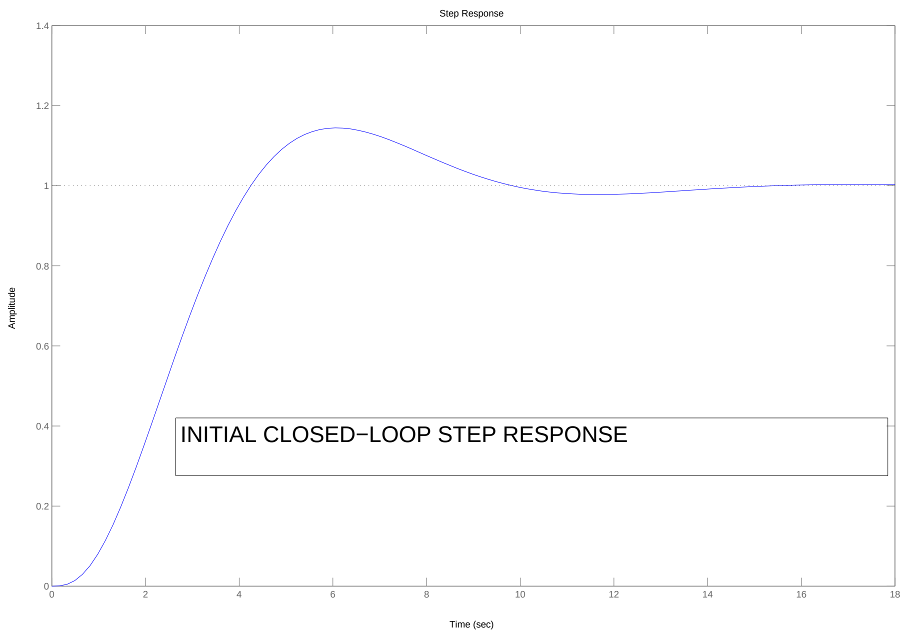


Figure 2: Initial Step Response

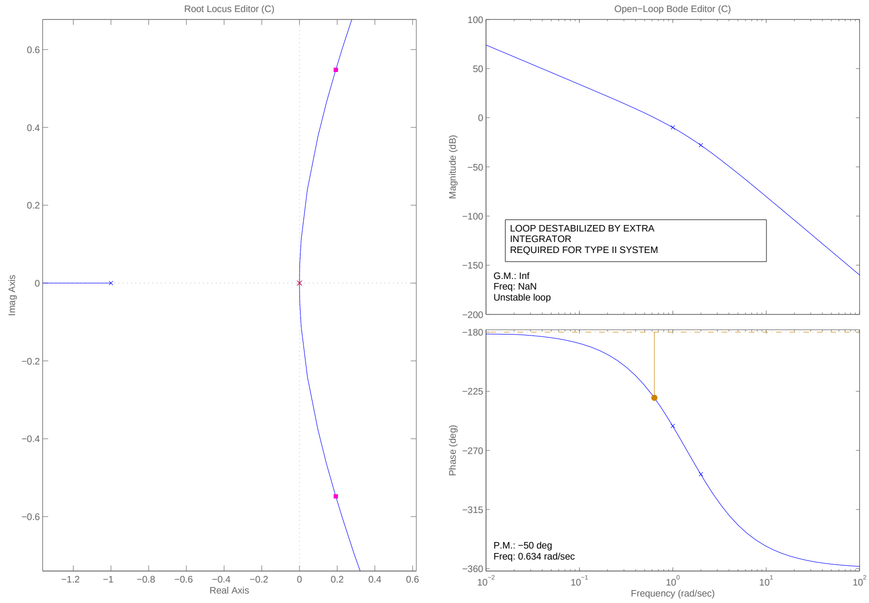


Figure 3: Integrator Added

## 2.4 Adding Complex Zeros

Our controller is currently a single integrator. We must keep the controller transfer function as simple as possible. Adding a real zero would improve the phase at crossover. However, since the total phase swing of a real zero is 90 degrees, we see that a single real zero is far insufficient. We add a complex zero pair, which can improve the phase by 180 degrees at crossover. We place the zeros interactively in SISO tool to give the maximum phase at crossover. We get a positive PM of 33.8 degrees. This operation can be observed in Figure 4

## 2.5 Adding a Real Zero

The PM is 33.8 degrees after adding the complex zeros. We could improve it by lowering the gain, however, the crossover frequency would also be lowered to an off-specification value. We add a real zero to further improve the phase at crossover. Since we foresee the addition of poles to make the controller causal, we exceed the required PM and crossover frequency. This operation can be observed in Figure 5 Now the PM is close to 90 degrees, with $w_c = 7.5$ rad/sec.

## 2.6 Adding a Complex Pole Pair

Our controller meets the specifications in the frequency domain, however it has more zeros than poles. We must have at least as many poles as zeros. To fix this problem, we introduce a complex pole pair. The poles are introduced as far to the left as possible in the root locus. In the frequency domain, the poles must not significantly change the shape of the magnitude and phase near the crossover region (where the action takes place). We add poles with time constants an order of magnitude smaller than the projected dominant poles. This operation can be observed in Figure 6 Now the PM is 65 degrees, with $w_c = 7.25$ rad/sec. The step response indicates that some minor tweaking is required, as shown in Fig. 7.

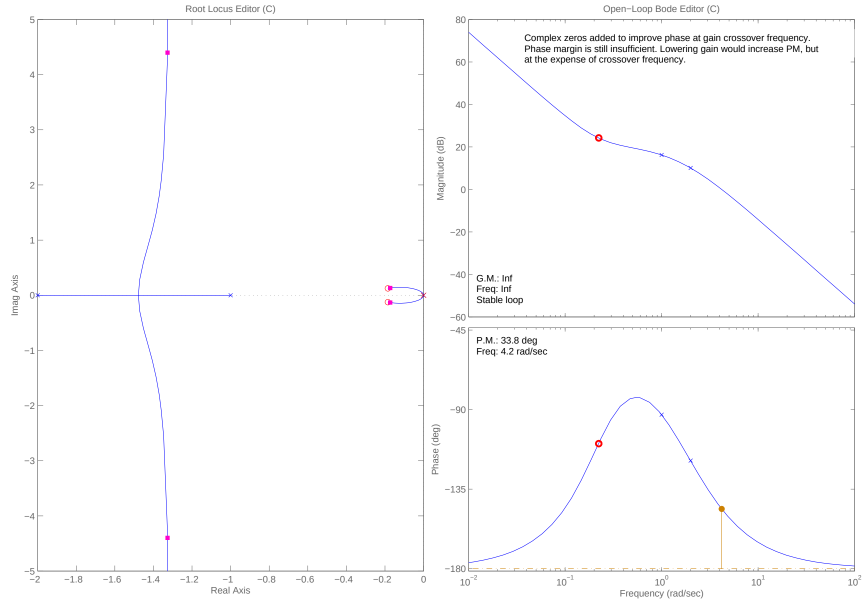


Figure 4: Added complex zero pair

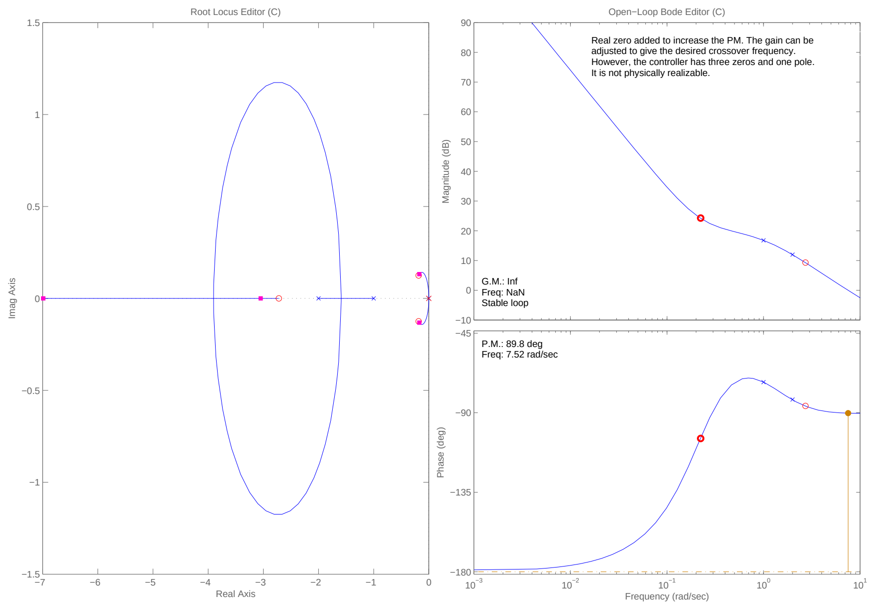


Figure 5: Added real zero

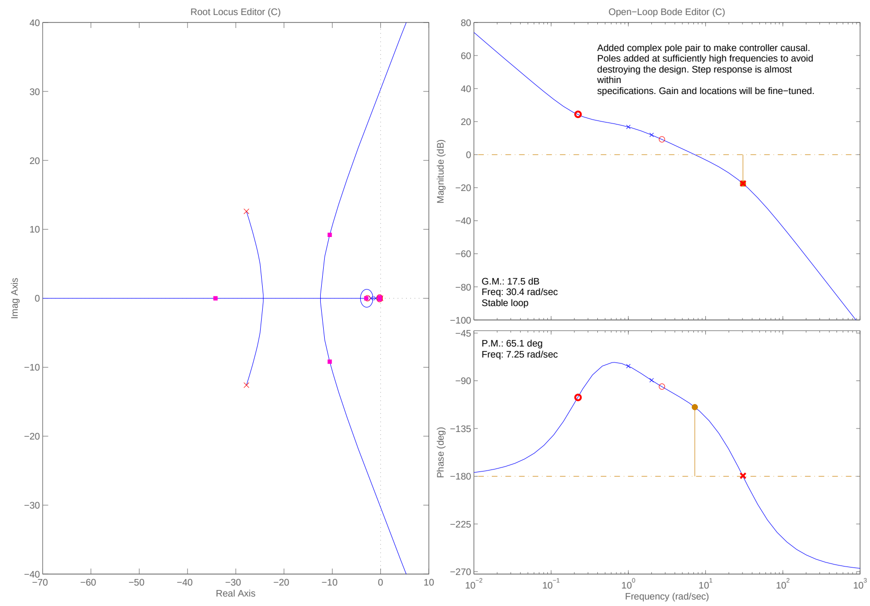


Figure 6: Added complex poles

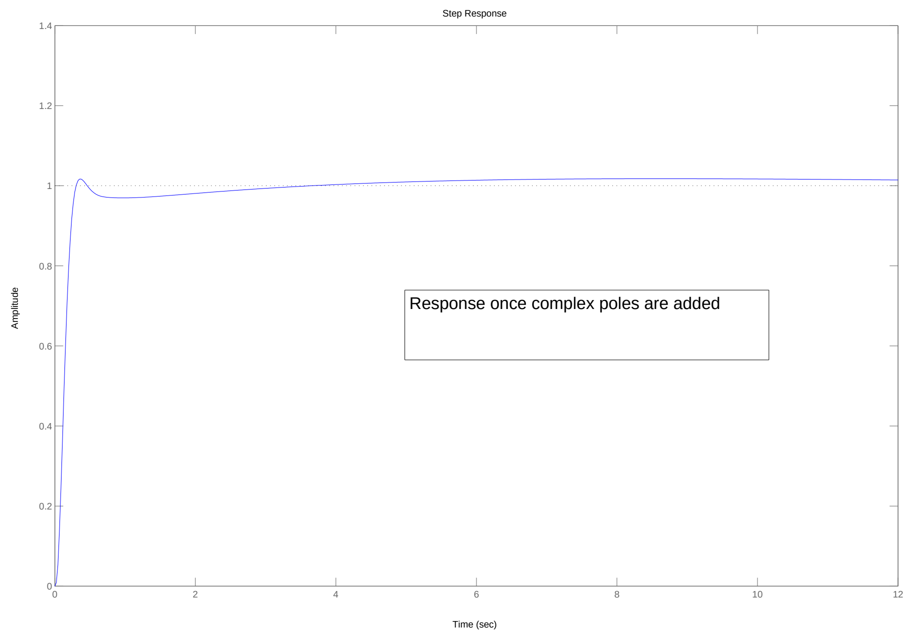


Figure 7: Step response after adding complex poles

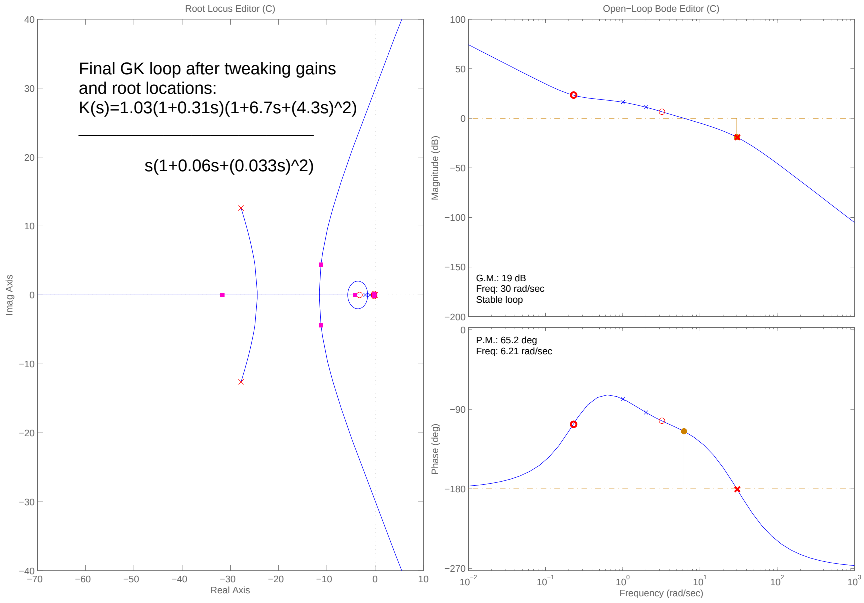


Figure 8: Loop after final adjustments

## 2.7 Fine-Tuning

Now we fine-tune aided by SISO tool, watching the step response in real time as we update pole/zero locations and loop gain. The resulting controller is

$$K(s) = 1.03\frac{(1 + 0.31s)(1 + 6.7s + (4.3s)^2)}{s(1 + 0.06s + (0.033s)^2}$$

The final loop and step response can be seen in Figs. 8 and 9, respectively.

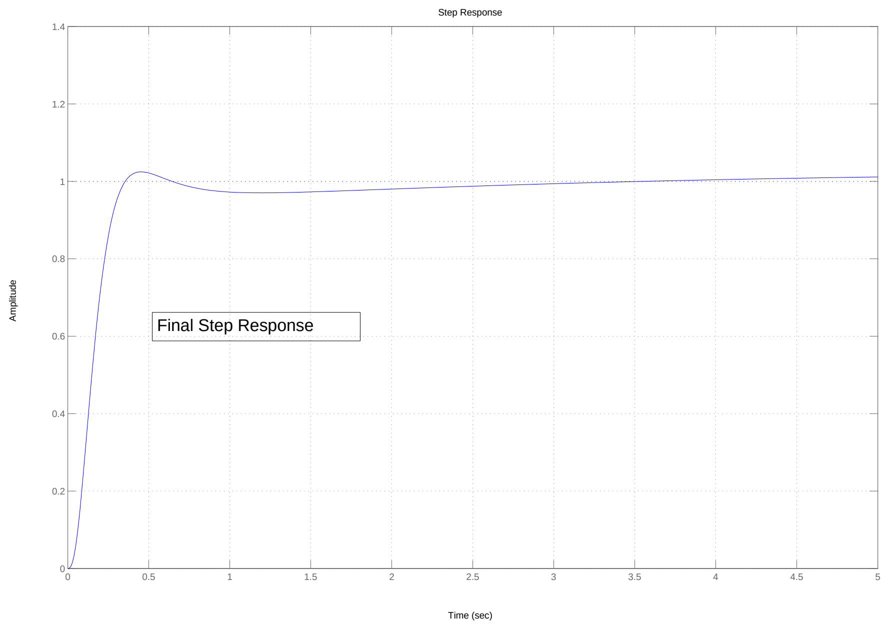


Figure 9: Final step response