Department of Mechanical Engineering  
Cleveland State University  

MCE 541: Linear Control Systems  
Final Project - Fall 2009  
Speed and Position Control of a DC Servomotor  

OUT: 11-02-09. Designed controller and Simulink file due: 11-30-09 in class. Report due: 12-04-09 by 5PM. Late submissions will not be accepted.

1 Plant Model

The transfer function of a DC motor was studied in class:

\[
\ddot{w} + \left( \frac{R_A}{L_A} + \frac{b}{J} \right) \dot{w} + \left( \frac{R_A b + \alpha^2}{J L_A} \right) w = \frac{\alpha}{J L_A} V_i - \frac{1}{J} \dot{T}_L - \frac{R_A}{J L_A} T_L
\]

In real DC motors, the viscous friction (coefficient \( b \)) is very small compared to the dry (Coulomb) friction. It can be safely assumed that \( b = 0 \). You must account for the dry friction by considering it to be a load torque. This torque has the form:

\[ T_L = \tau_f \text{ sign}(w) \]

That is, the torque always opposes the direction of motion. As it can be seen in Fig. 1, the signum function is defined by

\[
\text{sgn} \left( x \right) = \begin{cases} 
1, & x > 0 \\
-1, & x < 0 \\
0, & x = 0 
\end{cases}
\]

This function has a sharp discontinuity at zero, which creates difficulties with numerical solvers. Instead, use the saturation function shown in Fig. 1, defined as

\[
\text{sat} \left( \frac{x}{\delta} \right) = \begin{cases} 
1, & x > \delta \\
-1, & x < -\delta \\
\frac{x}{\delta}, & |x| \leq \delta 
\end{cases}
\]

![Figure 1: Finite-slope approximation to signum function](image)

The value of \( \delta \) is chosen to be small enough so that accuracy is maintained and the simulation is fast.
2 Task 1: Simulink Model

Build a Simulink model that allows to simulate the position and speed responses given a voltage input. You can ignore the $\dot{T}_L$ term. Use the motor parameters from Table 1. **It is a requirement that you create a Matlab script (m-file) to assign values to all parameters. The Simulink blocks must contain only symbolic variables, not numerical quantities.** Use a 40V step input and plot the resulting velocity (RPM) and shaft position (degrees) against times. **Check with the instructor to see if the response plots are correct before continuing with the rest of the project.**

3 Task 2: Speed Controller Design

You will use $T_L = 0$ to obtain a transfer function for design and regard $T_L$ as an external (step) disturbance input for controller design purposes. The specifications are as follows:

- Settling time to a step velocity command: 0.5 seconds or less.
- Overshoot of 1 percent or less.
- Zero steady-state error under no disturbances.
- Steady-state error to a step friction torque disturbance less than 2 rpm.
- Roll-off rate of -40 dB/dec to minimize the impact of sensor noise and unmodeled dynamics.
- The control voltage cannot exceed 30 V for a speed change command of 2500 rpm
- Except for a possible PID solution, there cannot be more zeros than poles in the controller transfer function.

You must accomplish the following design tasks:

1. Obtain the transfer function from voltage to speed
2. Display a root locus to decide on a preliminary controller structure (how many poles and zeros, need for integrators, approximate location of poles and zeros)
3. Translate the design specifications into the frequency domain and sketch the design boundaries in a Bode plot.
4. Use SISO tool to obtain a controller that accomplishes all specifications except for torque disturbance.
5. Find an expression for the steady state error due to the torque disturbance input as a function of controller gains (or pole/zero locations). Use the expression to re-tune the controller to meet the torque disturbance specification.
6. Implement the controller in Simulink, including the $T_L$ term in the simulation. Check for the control voltage limitation of 30V. Fine-tune or re-tune if necessary.
4 Task 3: Position Controller Design

You will use $T_L = 0$ to obtain a transfer function for design and regard $T_L$ as an external (step) disturbance input for controller design purposes. The specifications are as follows:

- Settling time to a step position command: 0.1 seconds or less.
- Overshoot of 5 percent or less.
- Zero steady-state error under no disturbances.
- Steady-state error to a step friction torque disturbance less than 2 degrees.
- Roll-off rate of -40 dB/dec to minimize the impact of sensor noise and unmodeled dynamics.
- The control voltage cannot exceed 30 V for a position change of 180 degrees.
- Except for a possible PID solution, there cannot be more zeros than poles in the controller transfer function.

You must accomplish the following design tasks:

1. Obtain the transfer function from voltage to shaft position.
2. Display a root locus to decide on a preliminary controller structure (how many poles and zeros, need for integrators, approximate location of poles and zeros)
3. Translate the design specifications into the frequency domain and sketch the design boundaries in a Bode plot.
4. Use SISO tool to obtain a controller that accomplishes all specifications except for torque disturbance.
5. Find an expression for the steady state error due to the torque disturbance input as a function of controller gains (or pole/zero locations). Use the expression to re-tune the controller to meet the torque disturbance specification.
6. Implement the controller in Simulink, including the $T_L$ term in the simulation. Check for the control voltage limitation of 30V. Fine-tune or re-tune if necessary.

Check with the instructor to determine the validity of the controller and simulations.

5 Controller Deployment

Your controller must be designed and turned in as Matlab/Simulink files by class time on November 30th, no exceptions. The instructor will then perform additional tests for digital implementability and controller deployment will be conducted during class on December 2th, using a remote laboratory setup with networked data transmission capabilities. If you can’t find a controller that meets all specifications for either position or velocity, relax one or more specifications and clearly indicate which ones are being met.
<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armature Resistance (Ω)</td>
<td>$R_A$</td>
<td>4.85</td>
</tr>
<tr>
<td>Inductance, (mH)</td>
<td>$L_A$</td>
<td>1.4</td>
</tr>
<tr>
<td>Dry Friction Torque (N-mm)</td>
<td>$\tau_f$</td>
<td>28.5</td>
</tr>
<tr>
<td>Rotary Inertia, (kg-m$^2$)</td>
<td>$J$</td>
<td>$5.8213 \times 10^{-5}$</td>
</tr>
<tr>
<td>Back-emf Constant, (N-m-A$^{-1}$)</td>
<td>$\alpha$</td>
<td>0.075</td>
</tr>
</tbody>
</table>

Table 1: Experimental Setup Parameters

6 Report

Organize your work in a report (not homework) format. Some pointers:

- Include everything that is necessary for an outsider (who knows controls) to follow your reasoning and to understand the outcome.
- Material presentation will have an influence on the grade.
- Put a title and label the axes of each plot, with units.
- If you were unable to meet the specifications due to a fundamental power limitation, relax the specifications a bit and explain your tradeoff.
- You are encouraged to consult with the instructor at every stage of the design process.