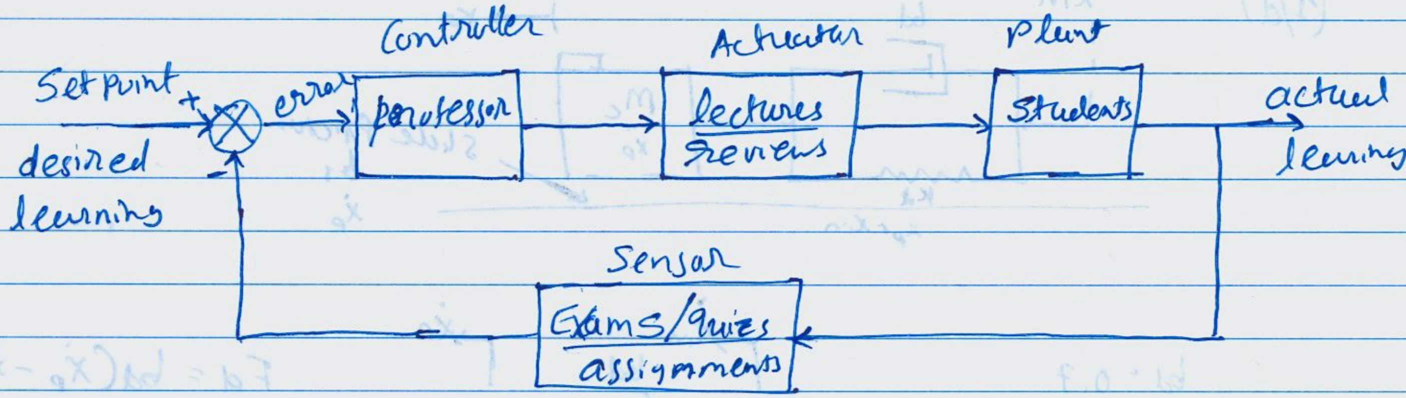
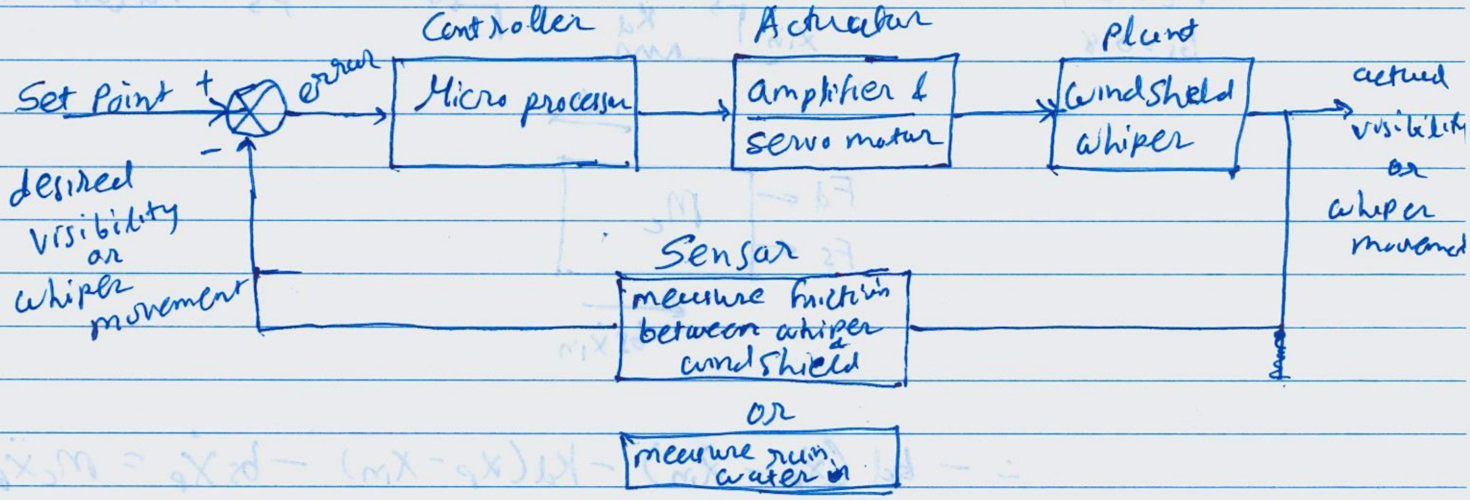
H.W. 1

MCE 441/541 Linear control System

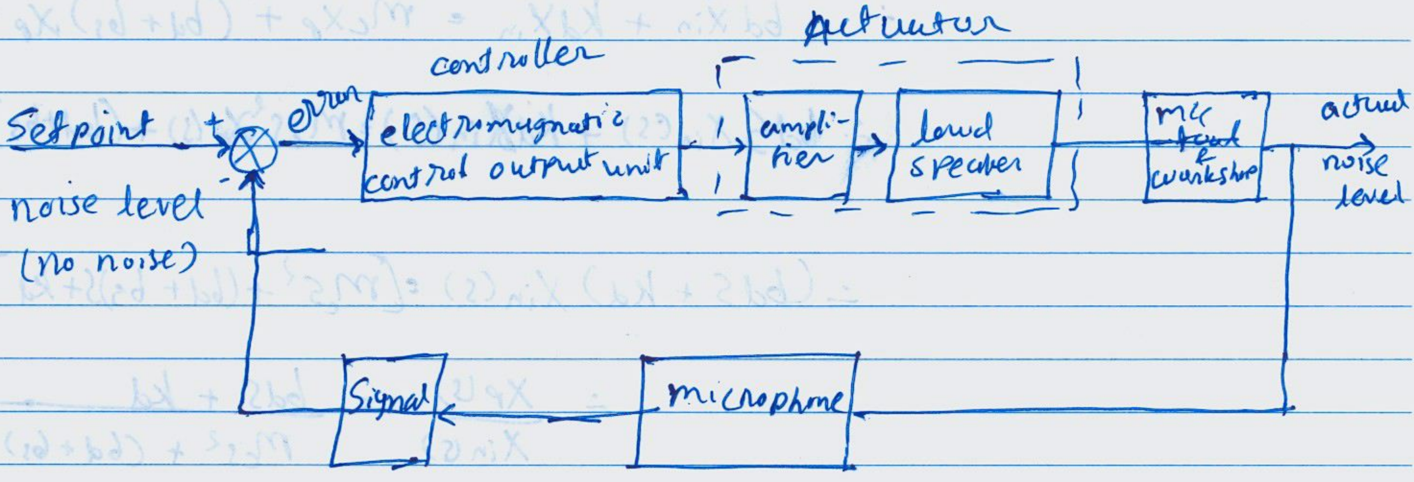
(1/a)



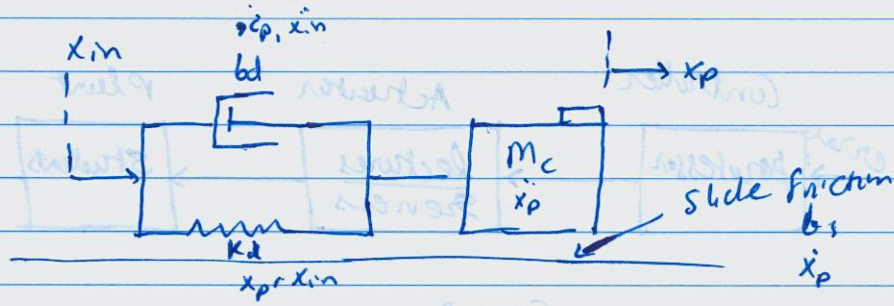
(1/b)



(1/c)



(1/d)

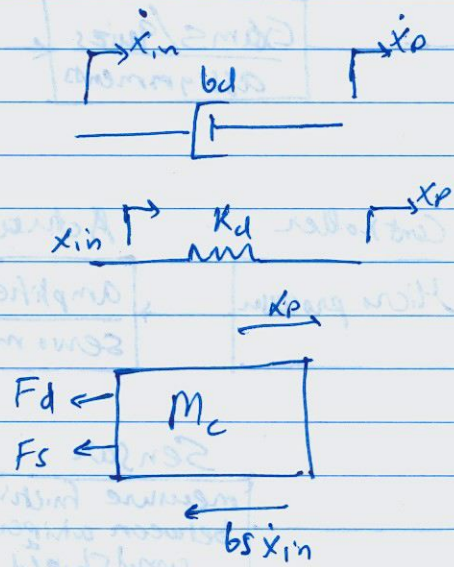


$b_d = 0.7$

$K_d = 2$

$M_c = 1$

$b_s = 0.8$



$F_d = b_d(\dot{x}_p - \dot{x}_{in})$

$F_s = K_d(x_p - x_{in})$

$$\therefore -b_d(\dot{x}_p - \dot{x}_{in}) - K_d(x_p - x_{in}) - b_s\dot{x}_p = M_c\ddot{x}_p$$

$$\therefore b_d\dot{x}_{in} + K_d x_{in} = M_c\ddot{x}_p + (b_d + b_s)\dot{x}_p + K_d x_p$$

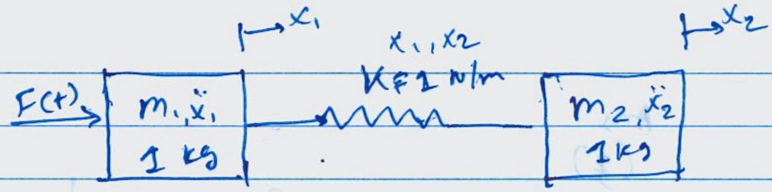
$$\therefore b_d S X_{in}(s) + K_d X_{in}(s) = M_c S^2 X_p(s) + (b_d + b_s) S X_p(s) + K_d X_p(s)$$

$$\therefore (b_d S + K_d) X_{in}(s) = [M_c S^2 + (b_d + b_s)S + K_d] X_p(s)$$

$$\therefore \frac{X_p(s)}{X_{in}(s)} = \frac{b_d S + K_d}{M_c S^2 + (b_d + b_s) S + K_d}$$

$$\frac{X_p(s)}{X_{in}(s)} = \frac{0.7S + 2}{S^2 + 1.5S + 2}$$

(1/e) (e)



$m_1 = 1k, \quad m_2 = 1$ kg, $k = 1$ N/m

$$m_1\ddot{x}_1 + k(x_1 - x_2) = F(t) \quad --- \quad (I)$$
$$m_2\ddot{x}_2 + k(x_2 - x_1) = 0 \quad --- \quad (II)$$

$$\therefore \ m_2 s^2 X_2(s) + kX_2(s) - kX_1(s)$$

$$\therefore \ X_1(s) = \left(\frac{m_2 s^2 + k}{k}\right) X_2(s)$$

now from eq (1)

$$m_1 s^2 X_1(s) + kX_1(s) - kX_2(s) = F(s)$$

$$\therefore \ m_1 s^2 \left(\frac{m_2 s^2 + k}{k}\right) X_2(s) + k\left(\frac{m_2 s^2 + k}{k}\right) X_2(s) - kX_2(s) = F(s)$$

$$\therefore \ m_1 s^2 (m_2 s^2 + k) X_2(s) + k(m_2 s^2 + k) X_2(s) - k^2 X_2(s) = F(s).k$$

$$\therefore \ [(m_1 s^2 + k)(m_2 s^2 + k) - k^2] X_2(s) = F(s)\ k$$

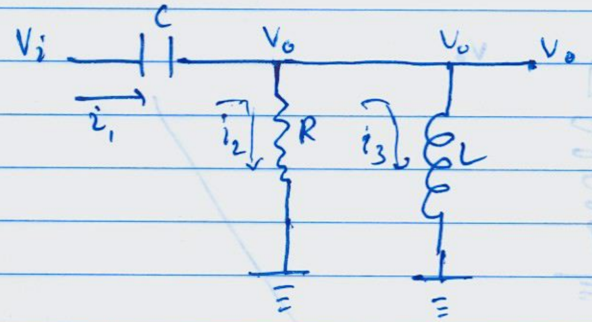
$$\therefore \ [(s^2 + 1)(s^2 + 1) - 1] X_2(s) = F(s)$$

$$\therefore \ (s^4 + s^2 + s^2 + 1 - 1) X_2(s) = F(s)$$

$$\therefore \ \frac{X_2}{F(s)} = \frac{1}{s^4 + 2s^2}$$

$$\boxed{\frac{X_2}{F(s)} = \frac{1}{s^2(s^2+2)}}$$

Ans
(2)



$i_1 = i_2 + i_3$

$i_1 = C(\dot{V}_i - \dot{V}_o)$

$V_o = i_2 R$

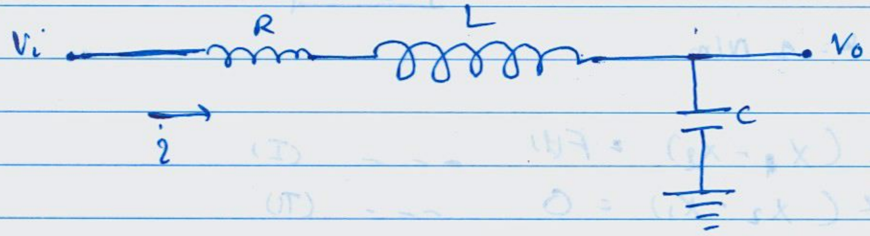
$V_o = L \dfrac{di_3}{dt}$

$$\therefore \; C(\dot{V}_i - \dot{V}_o) = \frac{V_o}{R} + \int \frac{V_o}{L}\, dt$$

$$C\ddot{V}_i - C\ddot{V}_o = \frac{\dot{V}_o}{R} + \frac{V_o}{L}$$

$$C\ddot{V}_i = C\ddot{V}_o + \frac{\dot{V}_o}{R} + \frac{V_o}{L}$$

Ans
(3)



$v_C = V_o$

$i = C\dot{V_o}$

$V_R = iR$

$V_L = L\frac{di}{dt}$

$$V_i - V_R - V_L - V_c = 0$$

$$V_i = V_R + V_L + V_c$$

$$V_i = V_R + L\frac{di}{dt} + V_o$$

$$V_i = iR + L\frac{di}{dt} + V_o$$

$$\boxed{V_i = \frac{C\,dV_o R}{dt} + LC\frac{d^2V_o}{dt^2} + V_o}$$