Department of Mechanical Engineering
Cleveland State University

MCE 441: Introduction to Linear Control Systems
MCE 541: Linear Control Systems
Homework 3 - Fall 2009

OUT: 09-28-09. DUE: 10-07-09

Figure 1 shows a rack-and-pinion mechanism driven by a DC motor. The rack and pinion can be considered massless and friction- free. The DC motor dynamics from voltage to shaft speed $w = \dot{\theta}$ may be described by a second-order differential equation or by a simplified, first-order differential equation obtained by neglecting inductive effects. The second-order model is given by Eq. 1:

$$\ddot{w} + \left(\frac{R_A}{L_A} + \frac{b}{J}\right)\dot{w} + \left(\frac{R_A b + \alpha^2}{JL_A}\right) w = \frac{\alpha}{JL_A} V_i - \frac{1}{J}\dot{T}_L - \frac{R_A}{JL_A} T_L \tag{1}$$

while the simplified, first-order model is given by Eq. 2

$$\dot{w} + \left(\frac{b}{J} + \frac{\alpha^2}{JR_A}\right) w = \frac{\alpha}{JR_A} V_i - \frac{1}{J} T_L \tag{2}$$

Refer to the numerical values of Table 1 and do the following

1. Use the second-order motor model to find the transfer function from V_i to x. Call it $G(s)$
2. Find the poles and zeros of $G(s)$ using Matlab and print a pole-zero map. Then apply the pole dominance criterion to neglect any fast poles and obtain a reduced transfer function. Call this transfer function $H(s)$.
3. Use $H(s)$ to obtain a detailed hand-sketch of the response of x to a unit voltage step input. Find the settling time, final value and percent overshoot using the methods and formulas shown in class.
4. Use the first-order motor model to find the transfer function from V_i to x. Call it $K(s)$.
5. Use $K(s)$ to obtain a detailed hand-sketch of the response of x to a unit voltage step input. Find the settling time, final value and percent overshoot using the methods and formulas shown in class.
6. Finally, obtain three superimposed step responses using Matlab's `step` command (use `hold on` and `legend`). Make a conclusion of how good are $H(s)$ and $K(s)$ as approximations to $G(s)$.

Note: All Matlab commands and their output must be included

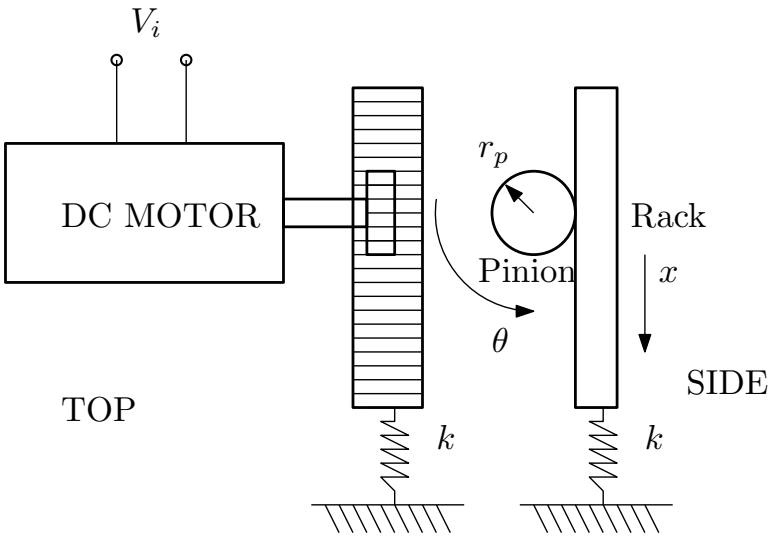

Figure 1: Rack-and-Pinion Mechanism

Parameter	Value	Units
L_A	1.5×10^{-3}	H
R_A	3.5	Ω
J	1×10^{-4}	kg-m^2
b	0.001	N-m-s
α	0.075	N-m-A^{-1}
k	350	N-m^{-1}
r_p	0.02	m

Table 1: System parameters