

Department of Mechanical Engineering
Cleveland State University

MCE 441: Introduction to Linear Control Systems
MCE 541: Linear Control Systems
Homework 4 - Fall 2009

OUT: 10-26-09. DUE: 11-4-09 in class. No homework will be accepted past the due date and time.

1.(MCE441: 60pts; MCE541:40pts) In this exercise you will be tuning P, PI and PID controllers for the following plant:

$$G(s) = \frac{s + 1}{(s^2 + s + 1)(s + 2)(s + 7)}$$

A tuning method introduced in 1942 by J.G. Ziegler and N.B. Nichols is applicable when complete information about the plant is unavailable, provided that experimental step responses can be recorded. The method is as follows:

1. A PID control loop is established around the plant, with the integral and derivative gains set to zero. The proportional gain K_p is set to a small number.
2. The proportional gain is increased and the step response observed. For the plants for which the method applies, an increasingly oscillatory response is observed as the gain is increased. Eventually, for a critical value of the gain, a sustained oscillation of constant amplitude develops (onset of instability). Let this gain be K_u and the period of the oscillations T_u .
3. The gains are tuned according to Table 1.

You are asked to:

1. Use the Simulink model `hw4f09.mdl` which has been uploaded to the course website. This model facilitates testing different gains. In the Simulation Parameters dialog box, set the relative tolerance to 1e-5 and the refine factor to 4. This will allow you to accurately find K_u and T_u from a step response plot.
 2. Find K_u by trial-and-error simulations. Write down the corresponding period T_u . You may find the `ginput(n)` command handy for measuring n points from the plot by using the cursor and clicking.
 3. Tune the gains for a P, PI and PID controller using the Ziegler-Nichols method. For each case, set the tuned gains in the PID block and simulate and print the unit step response. For each, find the settling time and percent overshoot for the plots. Which control configuration would you pick? Use fine tuning to find a set of gains giving a better overshoot and response time.
- 2.(MCE441: 40pts; MCE541:20pts) Manipulate the characteristic equation corresponding to the plant under PID control so that K_d appears as the root locus parameter. Assign values to the ratios K_p/K_d and K_i/K_d corresponding to the best tuning found in part 1).
1. Hand-sketch a detailed root locus using K_d as the free parameter. Find angles of asymptotes and their center as well as any break-in or break-away points.
 2. Determine from the root locus whether a positive value of K_d exists that destabilizes the closed loop.

3.(MCE441: 0pts; MCE541:20pts) Obtain the first column of the Routh-Hurwitz table as a function of K_p and K_i , assigning a numerical value to K_d corresponding to the best tuning found in part 1). Then write a Matlab routine that will plot a 2D shaded region representing the allowable values of K_p and K_i for stability.

	P	PI	PID
K_p	$0.5K_u$	$0.45K_u$	$0.6K_u$
K_i	0	$\frac{0.54K_u}{T_u}$	$\frac{1.2K_u}{T_u}$
K_d	0	0	$0.075K_uT_u$

Table 1: Ziegler-Nichols PID Tuning