Minimum vs. Nonminimum Phase System

Among all systems having the same magnitude plot, those with the least phase shift range are called minimum phase. The others are called nonminimum phase. The TF of a MP system can be determined from the magnitude alone.

Fact: Systems having rhp zeros are nonminimum phase. But they're not the only ones.

Fact: Every rational transfer function (ratio of polynomials) has a high-frequency magnitude asymptote with slope \(-20(n - m)\) dB/dec, where \(n\) is the number of poles and \(m\) is the number of zeros.

Fact: Every rational, minimum phase transfer function has high-frequency phase asymptote at \(-90(n - m)\).

Use these facts to detect nonminimum phase systems from their Bode plot (phase-magnitude consistency). Can be done from an experimental Bode plot (no model).
Example

Compare the Bode plots of \( \frac{1 + Ts}{1 + T_1 s} \) and \( \frac{1 - Ts}{1 + T_1 s} \) when \( 0 < T < T_1 \).

As we see, the nonminimum phase system introduces extra phase lag for the same attenuation.

NMP Step Response Example

Compare the step responses of \( \frac{1 + s}{s^2 + s + 1} \) and \( \frac{-s}{s^2 + s + 1} \).

When hitting the system with a step, the NMP system actually responds in a direction opposite to the intended one! (out-of-phase at the high frequencies involved in the jump).
**Abstract**—This paper addresses the path control problem for a ship steering in restricted waters using sliding mode techniques. The ship's dynamic equations and numerical computation of the bank disturbance forces are briefly described. The ship's path control system is presented which leads to a nonminimum phase system. Therefore, the traditional input-output linearization strategy in geometric control theory is modified to convert the controller design of a fourth-order nonlinear system to that of a second-order linear system. A continuous sliding mode controller is designed to ensure the system's robustness and better performance. Simulation and experimental studies validate the controller design. The simulation results demonstrate the effectiveness of the proposed controller and its practicality in the steering control and navigation of the ship.

**Index Terms**—Control systems, differential geometry, dynamics, marine vehicle control, navigation, simulation, stability, variable structure systems.

Other control techniques have also been introduced to improve the safety of a ship steering in restricted waters, such as linear quadratic Gaussian (LQG) [9], stochastic control [13], and adaptive control [12]. It is worth mentioning that Kallstrom and Astrom developed a controller that makes the ship move as desired and rejects the disturbances adaptively [8]. The adaptive controller, although mostly designed based on the energy cost, in fact has no advantage on fuel consumption to most of the ships [15], and presents less robustness against speed and environment changes [20]. In the literature, disturbance forces acting on a ship moving in restricted waters are usually assumed as stochastic noises. However, the difference between the stochastic noises, including the white or colored noises, and the disturbance forces are significant from the point of view of hydrodynamics [24]. Thus the assumption may lead to the unreliability in the control simulation results. This paper will present...
Delay in the Laplace Domain

Let $x(t)$ be the original signal and $y(t) = x(t - T)$ be the delayed signal. By using the definition of Laplace transform we can show that $Y(s) = e^{-sT}$, so

$$\frac{Y(s)}{X(s)} = e^{-sT}$$

A system with delay introduces a TF which is not a ratio of polynomials. Sometimes we can use a series expansion: the Padé approximation.

$$e^{-Ts} \approx \frac{1}{1 + \frac{T}{2}s + \frac{(Ts)^2}{2} + \frac{(Ts)^3}{3} + ...}$$

First order approx: $e^{-Ts} \approx 1 - Ts$

Second order approx: $e^{-Ts} \approx \frac{1}{Ts+1}$

Frequency Response of a Pure Delay

$$|G(jw)| = |e^{-Twj}| = |\cos wT - j \sin wT| = 1$$

$$\angle G(jw) = \angle e^{-Twj} = -wT$$

A time delay introduces a lot of phase lag!!

There is no phase-magnitude consistency, therefore a system with delay is also NMP. Systems with delay need to be compensated with predictive action.
Example

Sketch a Bode plot for

\[ G(s) = \frac{e^{-s}}{1 + s} \]

Solution
TF from Bode Plots: Example

Find the mystery TF from the following Bode plot:

Check with Matlab: `bode(num, den)`

Polar and Nyquist Plots

A polar plot is a magnitude-angle chart in polar coordinates, for \( w = 0 \) to \( \infty \). A Nyquist plot also includes the \( w = 0 \) to \( -\infty \) portion, which is symmetrical to the positive frequency part.
Example

Draw an approximate polar plot from the mystery TF considered earlier.