MCE441: Intr. Linear Control Systems

Lecture 9: Feedback Performance Characteristics
Steady-State Error and System Type
Dorf Sect. 4.6, 5.6

Feedforward Disturbance Compensation

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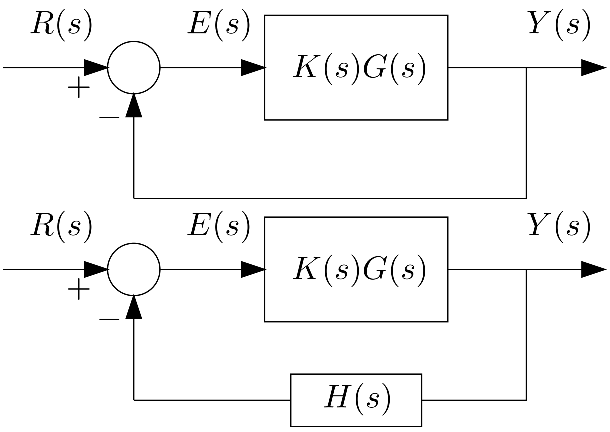
Unity vs. Non-Unity Feedback Loops



$G(s)K(s)H(s)$ is called the open loop transfer function or just loop transfer function. It is the ratio of the feedback signal to the error signal.

Usual Nomenclature

Dorf and several other textbooks use the following nomenclature: $G(s)$ represents the loop transfer function. It includes $G(s)$, $K(s)$ and $H(s)$.
$T(s)$ represent the closed-loop transfer function, that is

$$T(s) = \frac{G(s)}{1 + G(s)}$$

A lot of the techniques in controls involve determining properties of $T(s)$ like stability, steady error and time response from $G(s)$. Of course, $T(s)$ represents the operation of the system under automatic control.

It is very important to distinguish between $T(s)$ and $G(s)$ and to know which techniques are based on each.

Steady-State Error - Unity Feedback System

The error signal is given by

$$E(s) = \frac{1}{1 + G(s)} R(s)$$

Let's apply the final value theorem:

$$e_{\infty} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

The value of the steady-state error (if any) depends on the shape of $r(t)$ and on the system *type*. Next...

System Type

System Type is the number of free integrators in the loop TF

That is, if the loop TF in pole-zero form is

$$G(s) = \frac{(s+z_1)(s+z_2)...(s+z_m)}{s^N(s+p_1)(s+p_2)...(s+p_n)}$$

where $z_i \neq 0$ and $p_i \neq 0$, then the system is of type N.

Steady-State Error - Step Inputs

For a step input, $R(s) = \frac{A}{s}$. Substituting into the expression for e_∞ gives

$$e_\infty = \frac{A}{1+G(0)}$$

- If the system is of type 0 we have $G(0) = K_p$ (a nonzero constant), so

  $$e_\infty = \frac{A}{1+K_p} \neq 0$$

  We have a permanent offset.
- If the system is of type 1 or higher, we have $G(0) = \infty$, so $e_\infty = 0$. The error is rejected.

Steady-State Error - Ramp Inputs

For a ramp input $r(t) = At$, $R(s) = \frac{A}{s^2}$. Substituting into the expression for e_∞ gives

$$e_\infty = \frac{A}{\lim_{s\to 0} sG(s)}$$

- If the system is of type 0, we have $sG(0) = 0$, so $e_\infty = \infty$. The output does not attain the reference slope.

Ramp Inputs ...

- If the system is of type 1, we have $sG(0) = K_v$ (a nonzero constant), so

  $$e_\infty = \frac{A}{K_v} \neq 0$$

  The output attains the commanded slope, but there is an offset of e_∞.
- If the system is of type 2 or higher, we have $sG(0) = \infty$, so $e_\infty = 0$. The output tracks the reference input.

Steady-State Error - Parabolic Inputs

For a quadratic input $r(t) = At^2/2$, $R(s) = \frac{A}{s^3}$. Substituting into the expression for e_∞ gives

$$e_\infty = \frac{A}{\lim_{s\to 0} s^2 G(s)}$$

- If the system is of type 0 or 1, we have $s^2 G(0) = 0$, so $e_\infty = \infty$. The output does not attain the reference acceleration.

Parabolic Inputs ...

- If the system is of type 2, we have $s^2 G(0) = K_a$ (a nonzero constant), so

$$e_\infty = \frac{A}{K_a} \neq 0$$

  The output attains the commanded acceleration, but there is an offset of e_∞.
- If the system is of type 3 or higher, we have $s^2 G(0) = \infty$, so $e_\infty = 0$. The output tracks the reference input.
- *See the pattern?*

Summary

	Step Input $R(s) = \frac{A}{s}$	Ramp Input $R(s) = \frac{A}{s^2}$	Parabolic Input $R(s) = \frac{A}{s^3}$
Type 0 System	$\frac{A}{1+K_p}$	∞	∞
Type 1 System	0	$\frac{A}{K_v}$	∞
Type 2 System	0	0	$\frac{A}{K_a}$

Static Error Constants

Position Error Constant:

$$K_p = \lim_{s \to 0} G(s)$$

Velocity Error Constant:

$$K_v = \lim_{s \to 0} sG(s)$$

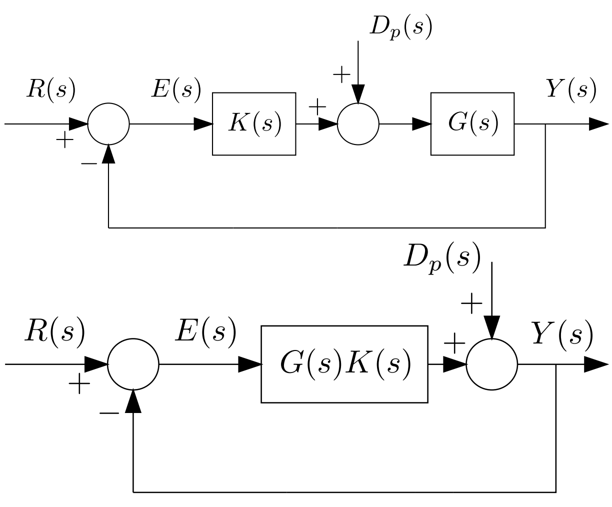
Acceleration Error Constant:

$$K_a = \lim_{s \to 0} s^2 G(s)$$

Examples - Dorf 5.3, 5.4, 5.5

Disturbance Rejection

Distinguish between process disturbance and output disturbance:



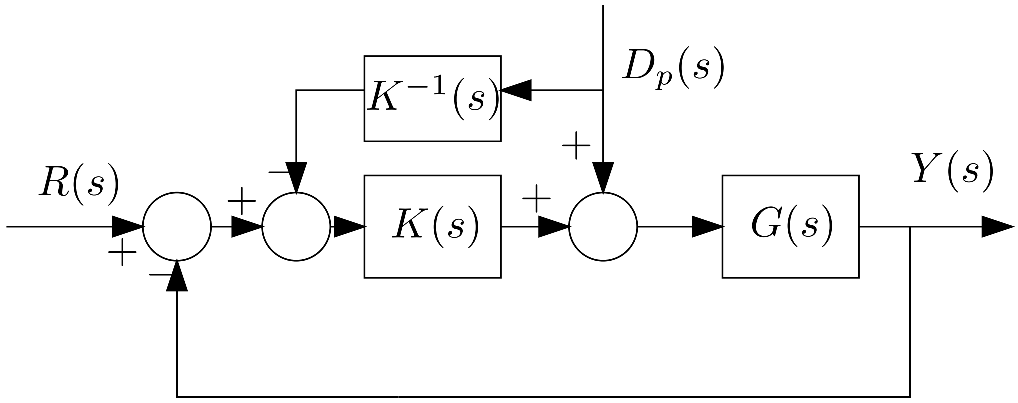
When $G(s)$ and $K(s)$ are known, input and output disturbances can be combined. Studying one case is sufficient.

Example - Dorf 5.10

Feedforward Disturbance Compensation

- Sometimes the disturbance can be measured, or at least estimated.
- Example: Continuous measurement of outside temperature in climate control systems.
- Example: Shock measurements using accelerometers (used to deploy airbags)
- In such cases, we can improve the performance by “undoing” the disturbance.

A Structure for Disturbance Compensatio



Potential Problem: Is $K^{-1}(s)$ always realizable in practice?