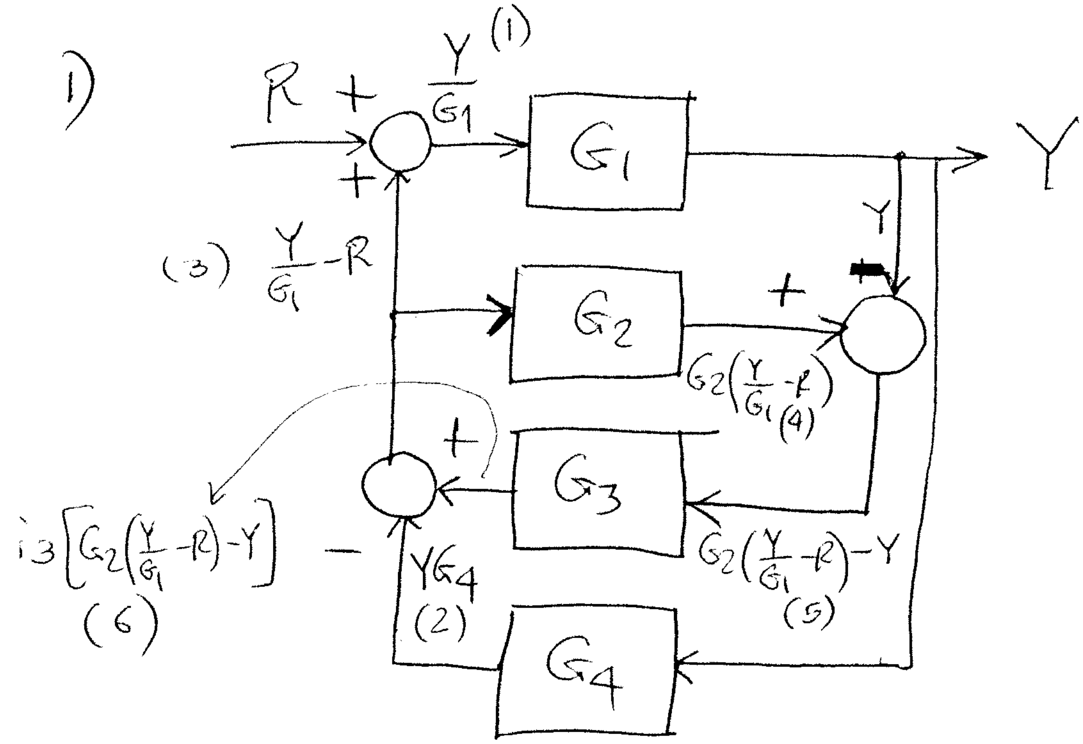
MCE 441/541 1st MIDTERM SOLUTION - FALL '11

1)



numbers in parenthesis are solution steps

At the lower summing node: $G_3\left[G_2\left(\frac{Y}{G_1} - R\right) - Y\right] - YG_4 = \frac{Y}{G_1} - R$

simplify:

$$\frac{G_3 G_2}{G_1} Y - G_3 G_2 R - G_3 Y - Y G_4 = \frac{Y}{G_1} - R$$

$$Y\left[\frac{G_3 G_2}{G_1} - G_3 - G_4 - \frac{1}{G_1}\right] = \left(G_3 G_2 - 1\right) R$$

$$\frac{Y}{R} = \frac{G_3 G_2 - 1}{\left[\frac{G_3 G_2}{G_1} - G_3 - G_4 - \frac{1}{G_1}\right]}$$

2 a. Since the gears are ideal and there is no torque on the output shaft:

$$\frac{n T_L r_1}{r_2} = 0 \quad (\text{torque @ output shaft})$$

$$\rightarrow T_L = 0$$

Also $\omega_o = \frac{r_1}{n r_2}\omega \rightarrow \omega = \frac{n r_2}{r_1}\omega_o$

Substituting:

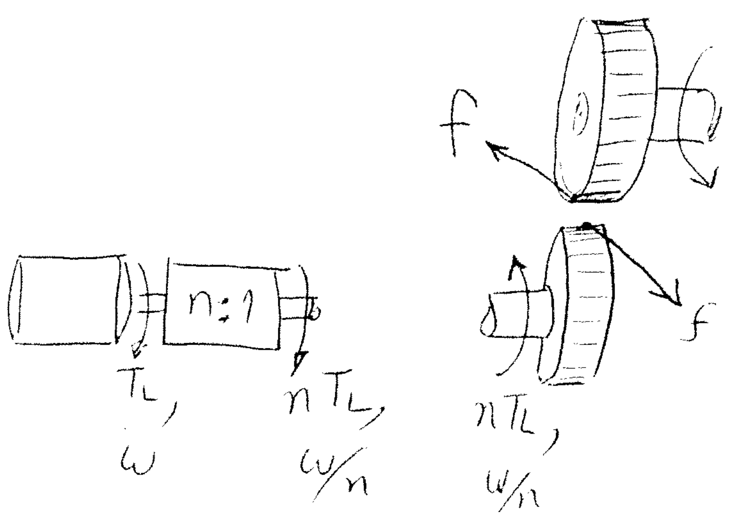
$$\frac{n r_2}{r_1}\dot{\omega}_o + \frac{n r_2}{\tau r_1}\omega_o = a V_i$$

Taking Laplace:

$$\left[\frac{n r_2}{r_1}s + \frac{n r_2}{\tau r_1}\right] W_o = a V_i$$

$$\frac{W_o(s)}{V_i(s)} = \frac{a}{\left[\frac{n r_2}{r_1}s + \frac{n r_2}{\tau r_1}\right]}$$

2b)



$$\left.\begin{aligned} f r_2 &= J_2 \dot{\omega}_o \\ -f r_1 + n T_L &= J_1 \frac{\dot{\omega}}{n} \\ \omega &= \frac{n r_2}{r_1}\omega_o \end{aligned}\right\}$$

Then $T_L = \frac{1}{n}\left[J_1 \frac{\dot{\omega}}{n} + f r_1 \right]$, $f = \frac{J_2 \dot{\omega}_o}{r_2}$

$$T_L = \frac{1}{n}\left[\frac{J_1}{n}\left(\frac{n r_2}{r_1} \dot{\omega}_o \right) + \frac{r_1}{r_2} J_2 \dot{\omega}_o \right]$$

Substituting:

$$\frac{n r_2}{r_1} \dot{\omega}_o + \frac{n r_2}{\tau r_1} \omega_o = a V_i - \frac{1}{nJ}\left[\cdot \right]$$

Simplifying:

$$\frac{n r_2}{r_1} \dot{\omega}_o + \frac{n r_2}{\tau r_1} \omega_o = a V_i - \frac{J_1}{J} \frac{n r_2}{r_1} \dot{\omega}_o - \frac{r_1 J_2}{n r_2 J} \dot{\omega}_o$$

$$\left[\frac{n r_2}{r_1} + \frac{J_1}{J} \frac{n r_2}{r_1} + \frac{r_1 J_2}{n r_2 J} \right] \dot{\omega}_o + \frac{n r_2}{\tau r_1} \omega_o = a V_i$$

$\downarrow$ Laplace:

$$\frac{W_o(s)}{V_i(s)} = \frac{a}{\left[\left(\frac{n r_2}{r_1} + \frac{J_1 n r_2}{J r_1} + \frac{r_1 J_2}{n r_2 J} \right) s + \frac{n r_2}{\tau r_1} \right]}$$

$//$

3. Poles: $s = -13 \rightarrow \tau = 1/13 = 0.0769$ fast

$s = \underbrace{-1.5}_{\xi\omega_n} \pm (\ )i \longrightarrow \tau = \frac{1}{1.5} = 0.66 \downarrow$ slow

The real pole has a τ >8 times faster than the cplx. poles.

$\rightarrow$ ignore the real pole.

$$G_{red}(s) = \frac{\frac{1}{13}[s+6]}{s^2+3s+6} \quad ; \quad G_{red}(0) = 1/13 = G(0)$$

(Include the $1/13$ to maintain $G(0)$).

Zero: $s = -6$, $a = 6$. $\frac{a}{\xi\omega_n} = \frac{6}{1.5} = 4$

Since $\frac{a}{\xi\omega_n} < 8$, we use the correction chart to find P.O: for $\frac{a}{\xi\omega_n} = 4$ and $\xi = 0.61$

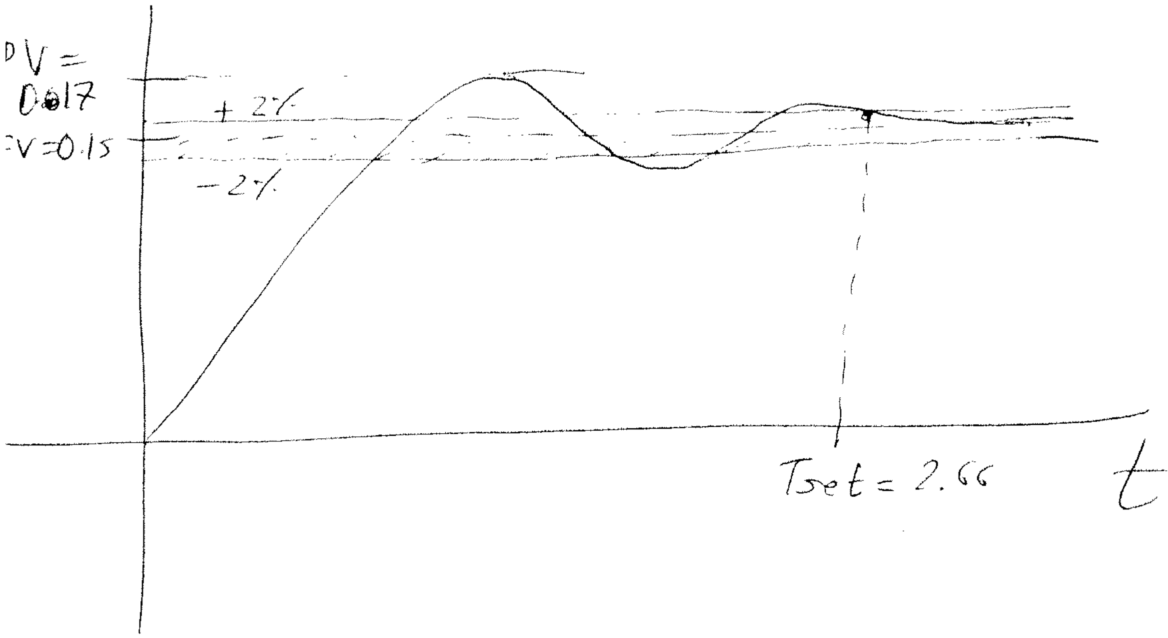
0.61 $\qquad \sqrt{6}$

PO $\approx$ 11 %

Settling time: $T_{set} = \frac{4}{\xi\omega_n} = 2.66$

$$F.V. \cong \underset{\text{step size}}{2} \times G(0) = 2/13 = 0.154$$

$$P.V = 0.154 \times (1.11) = 0.17$$



4. Closed-loop TF: $T(s) = \dfrac{GK}{1+GK}$

$$= \frac{numGK}{numGK + denGK} = \frac{s+40}{s^2 + \underbrace{8}_{2\xi\omega_n} s + \underbrace{40}_{\omega_n^2}}$$

Poles: $s = -4 \pm (\ \ )i$

$\omega_n = \sqrt{40}$, $\xi = 0.632$

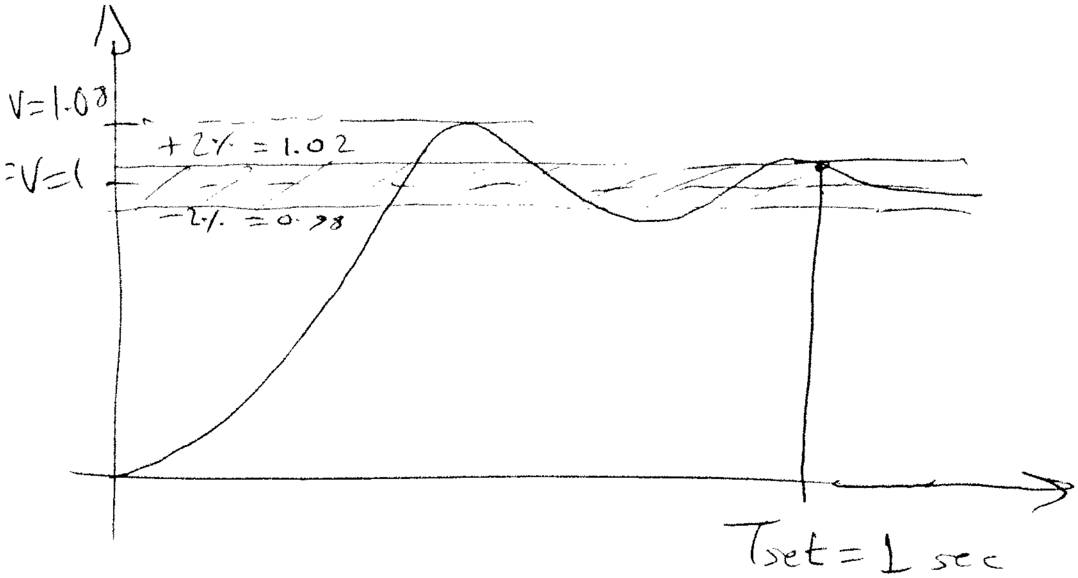
$a = 40$, $a/\xi\omega_n = 40/4 = 10 > 8 \rightarrow$ ignore zero.

P.O. (chart) with $\xi = 0.632$: PO $\approx 8\%$

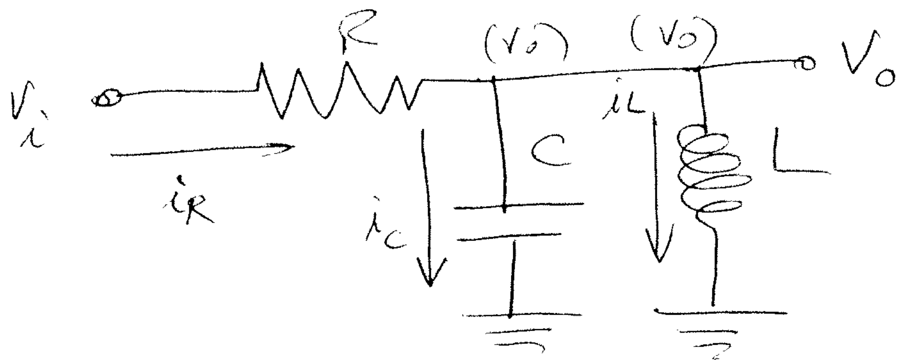
$FV = 1 \times T(0) = 1$

$PV = 1.08$

$T_{set} = \dfrac{4}{\xi\omega_n} = \dfrac{4}{4} = 1$



5a.



$$\boxed{\begin{aligned} V_i - i_R R &= V_o \\ C\dot{V}_o &= i_C \\ V_o &= L\frac{di_L}{dt} \\ i_R &= i_C + i_L \end{aligned}}$$

$\longrightarrow \quad i_R = \dfrac{V_i - V_o}{R}$

$\longrightarrow \quad i_L = \dfrac{1}{L}\int V_o\,dt$

subs:

$$\longrightarrow \frac{V_i - V_o}{R} = C\dot{V}_o + \frac{1}{L}\int V_o\,dt$$

Laplace: $$\frac{V_i - V_o}{R} = CsV_o + \frac{1}{Ls}V_o$$

$$V_i - V_o = RCsV_o + \frac{R}{Ls}V_o$$

$$V_o\left[RCs+\frac{R}{Ls}+1\right]=V_i$$

$$\frac{V_o(s)}{V_i(s)}=\frac{1}{RCs+\frac{R}{Ls}+1}=\frac{s}{RCs^2+s+\frac{R}{L}}$$

5b)

$$\boxed{\begin{aligned} V_i - i_R R &= V_o \\ C\dot{V}_o &= i_c \\ V_o &= L\frac{di_L}{dt} \\ i_R &= i_c + i_L \end{aligned}} \xrightarrow{\text{Laplace}} \begin{aligned} V_i - RI_R &= V_o \quad -(1) \\ CsV_o &= I_C \quad -(2) \\ V_o &= LsI_L \quad -(3) \\ I_R &= I_C + I_L \quad -(4) \end{aligned}$$

From (1) & (2): $V_i - RI_R = \frac{I_c}{Cs}$ ($\star$)

From (3) & (4) & (2): $I_R = I_c + \frac{V_o}{Ls} = I_c + \frac{I_c}{LCs^2}$ ($\star\star$)

Substitute ($\star\star$) into ($\star$):

$$V_i - R\left(I_c + \frac{I_c}{LCs^2}\right) = \frac{I_c}{Cs}$$

$$V_i - \left(R+\frac{1}{LCs^2}\right)I_c = \frac{I_c}{Cs}, \quad V_i = \left(\frac{1}{Cs}+R+\frac{1}{LCs^2}\right)I_c$$

$$\rightarrow \frac{I_c(s)}{V_i(s)} = \frac{1}{1/Cs + R + 1/LCs^2} = \frac{LCs^2}{LRCs^2+Ls+1}$$