MCE441/541 In-class 2nd Midterm Solution

– Fall 2011 –

1. At $\omega = 20$ rad/s, from the chart: mag $= -10$ dB

$$\rightarrow \underbrace{K\left|\frac{j\omega+1}{2-\omega^2+0.6j\omega}\right|}_{\omega=20} = 10^{-10/20} = 0.3162$$

$$\frac{K\sqrt{1+(20)^2}}{\sqrt{(2-20^2)^2+(0.6\times 20)^2}} = 0.0503\,K = 0.3162$$

a. $K = 6.287$

b. $G(s) = \left[\frac{6.287}{2}\right](s+1)\left[\frac{2}{s^2+0.6s+2}\right]$

$20\log_{10}\left(\frac{6.287}{2}\right) \approx 10\,\text{dB}$ (9.948)

$\tau = 1$

$\omega_n = \sqrt{2}$

$2\zeta\omega_n = 0.6$

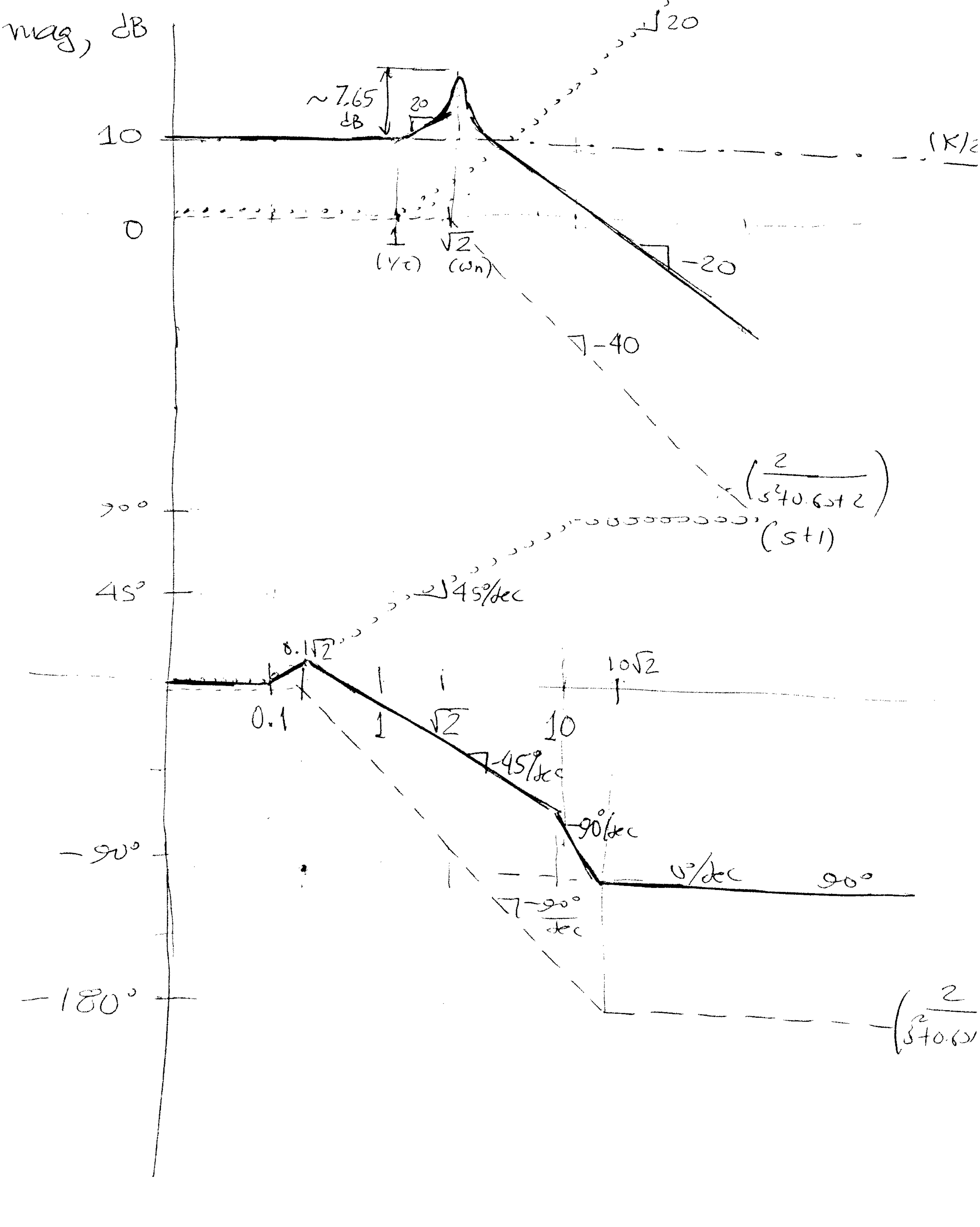
$\rightarrow \zeta = 0.2121$

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 2.412 \quad (7.647\,\text{dB})$$

$n=2$, $m=1$ } Final mag. slope $= -20(n-m)\frac{\text{dB}}{\text{dec}} = -20\frac{\text{dB}}{\text{dec}}$

Final phase $= -90(n-m)^\circ = -90^\circ$

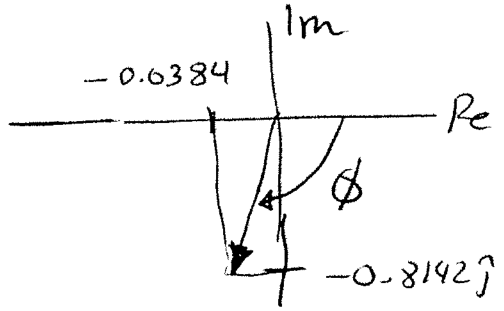
sketch:



mag, dB
(s+1)
20
~7.65 dB
20
10
(K/2
0
(1/τ)
√2
(ωn)
-20
-40
(2 / (s²+0.6s+2))
90°
(s+1)
45°
45°/dec
0.1√2
10√2
0.1
1
√2
10
-45°/dec
-90°/dec
-90°
0°/dec
90°
-90°/dec
-180°
(2 / (s²+0.6s

c. When $u(t) = 2\sin(8t)$, $\omega = 8$ rad/s

$$G(j\omega) = \frac{6.287(1+8j)}{(2-64)+0.6(8)j} = -0.0384 - 0.8142j$$



$$\phi = -92.698^\circ$$

For $\omega = 8$ r/s $\quad T = \frac{2\pi}{\omega} = 0.7854$ sec

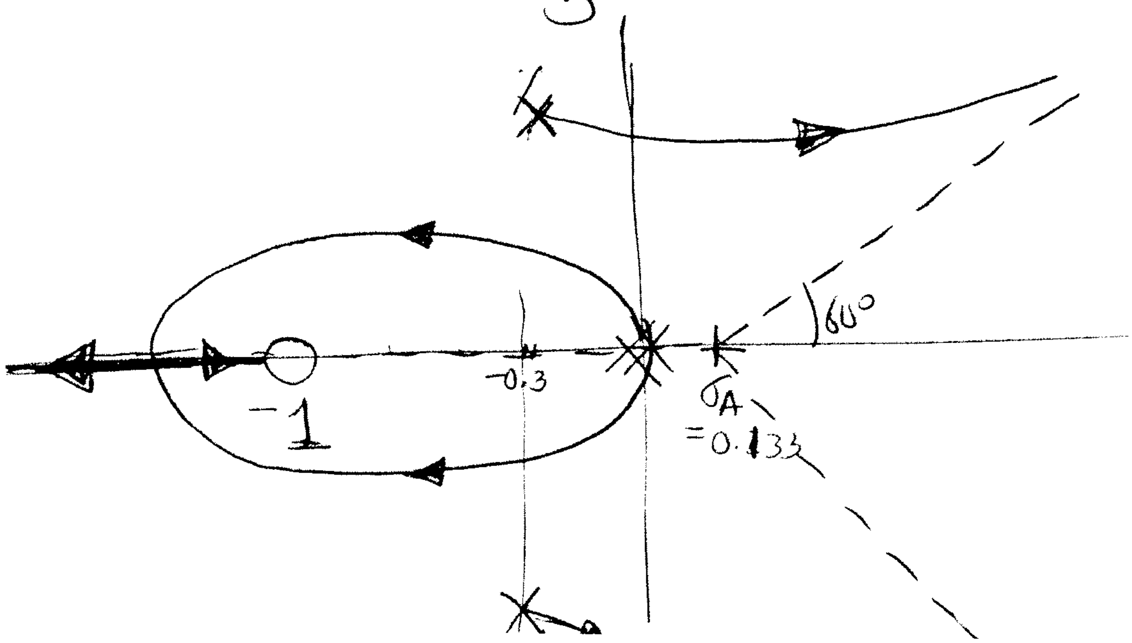
$$\frac{\Delta t}{T} = \frac{\phi}{360^\circ} \rightarrow \Delta t = -0.2022 \text{ sec}$$

(lagging)

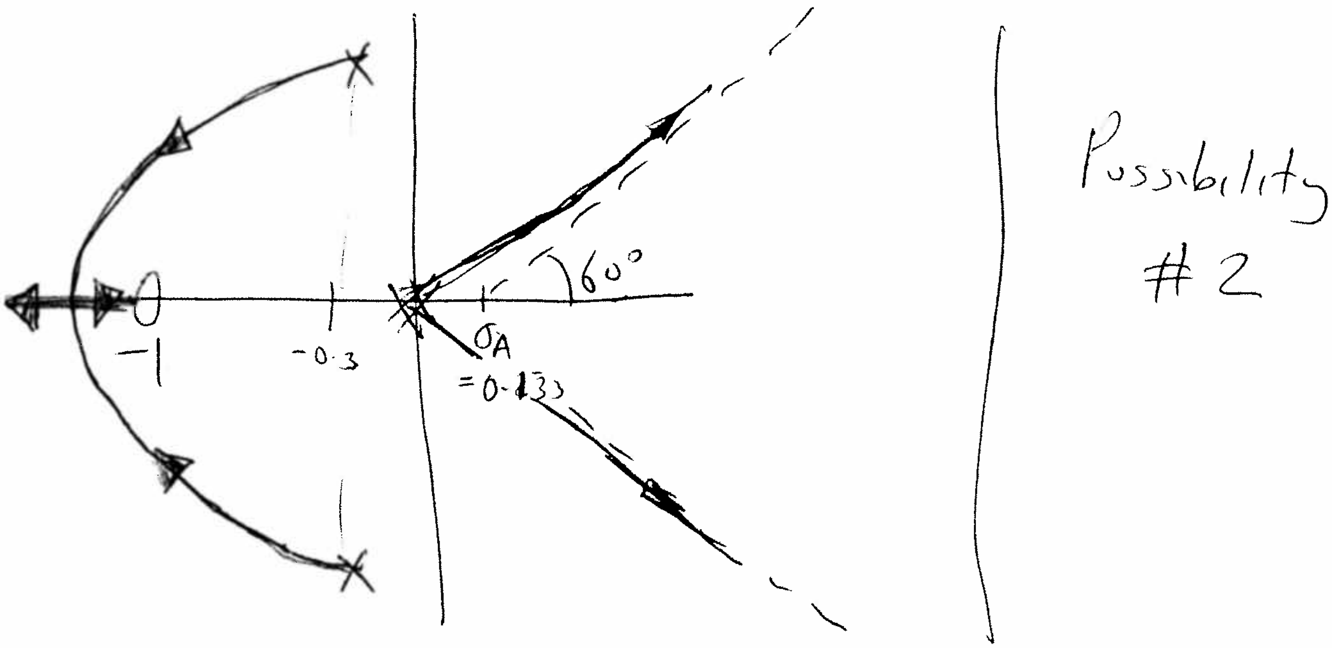
2a. $n = 4$, $m = 1 \rightarrow n - m = 3$, $\phi_A = \begin{cases} 60^\circ \\ 180^\circ \\ 300^\circ \end{cases}$

$$\sigma_A = \frac{\Sigma(\text{real part of poles}) - \Sigma(\text{real part of zeroes})}{n-m}$$

$$\sigma_A = \frac{(0+0-2\overbrace{(0.3)}^{\zeta\omega_n}) - (-1)}{3} = 0.133$$



Possibility #1



You can figure out which one is right by finding the break-in point (not required in this test):

Ch. eq: $$1 + \frac{K(s+1)}{s^2(s^2+0.6s+2)} = 0$$

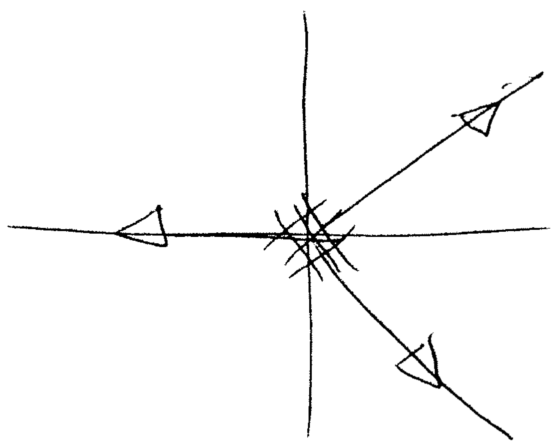
solve for K: $$K = \frac{-s^2(s^2+0.6s+2)}{(s+1)} = p(s)$$

Find $\frac{dp}{ds} = 0$ $\Big\}$ $s = \begin{cases} 0 \\ -1.485 \leftarrow \text{valid solution} \\ \{2 \text{ complex solutions}\} \end{cases}$

Still can't tell. You would have to find the "angle of departure" from the complex poles

(see textbook)

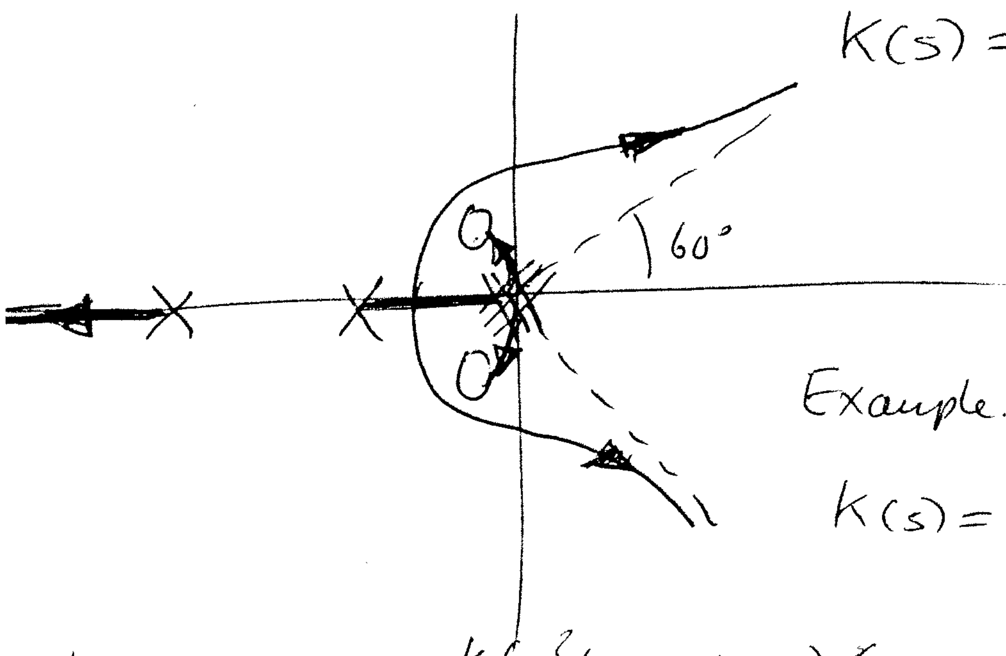
2b) The uncompensated TF has root locus:



It can be stabilized by bringing the 2 unstable branches to the left by using 2 zeroes.

Two poles will be required to have a proper controller

$$K(s) = \frac{k(s^2 + 2\zeta\omega s + \omega^2)}{(s+p_1)(s+p_2)}$$



Example: Take

$$K(s) = \frac{k(s^2 + 0.1s + 0.1)}{(s+1)(s+2)}$$

Ch. eq: $1 + \dfrac{k(s^2 + 0.1s + 0.1)\,G}{s^3(s+1)(s+2)} = 0$

$$s^3(s+1)(s+2) + k(s^2 + 0.1s + 0.1) = 0$$

$$s^5 + 3s^4 + 2s^3 + Ks^2 + 0.1Ks + 0.1K = 0$$

See if there's a value of K for stability using the R-H array:

s^5	1	2	$0.1K$
s^4	3	K	$0.1K$
s^3	$6-\frac{k}{3}$	$0.3K-0.1K = 0.2k/3$	—
s^2	$\frac{(6-k)k-0.6K}{6-k}$ $((6-k)k-0.6k)$	$0.1K$ $(0.1k(6-k))$	—
s^1	$\frac{[(6-k)k-0.6k]\times 0.2k-(6-k)^2 0.1k}{(6-k)k-0.6k}$	—	
s^0	$0.1k(6-k)$		

$(\times 3)$

$(\times (6-k))$
(must be >0 based on s^3 line, 1st col

we need:

$$6-k>0$$

$$(6-k)k-0.6k>0 \Rightarrow (6-k)\cancel{k} > 0.6\cancel{k} \quad (k>$$

$$\underbrace{((6-k)k-0.6k)0.2k-(6-k)^2 0.1k}>0$$

$$k>0$$

$$[(6-k)k-0.6k]0.2\cancel{k} > (6-k)^2 \times 0.1\cancel{K} \quad (k>0)$$

$$\rightarrow \begin{cases} k>0 & \\ 6-k>0 & (k<6) \\ 6-k>0.6 & (k<5.4) \\ \underbrace{0.2(6-k)k-0.12k-0.1(6-k)^2}>0 & \end{cases}$$

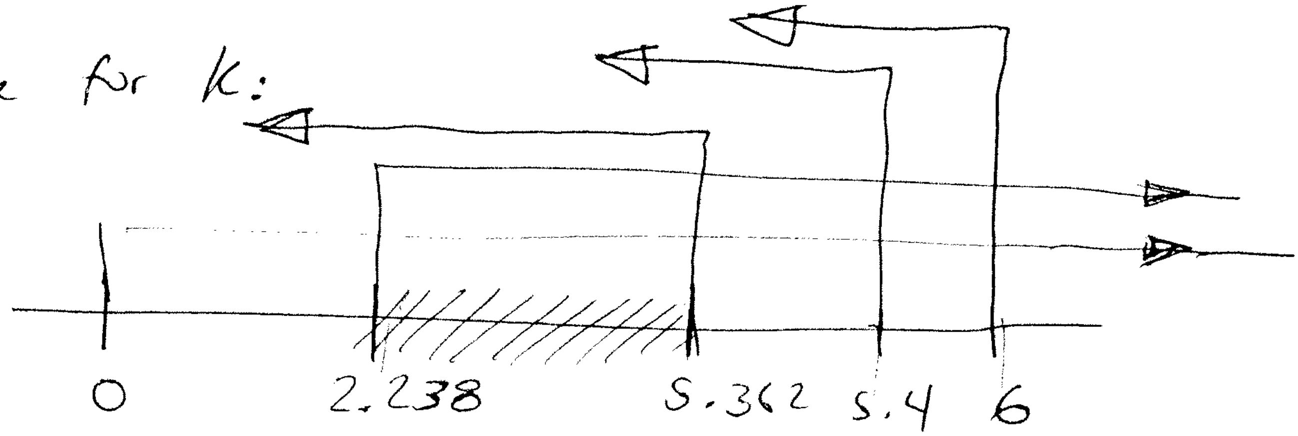
$$-0.3k^2+2.28k-3.6>0$$

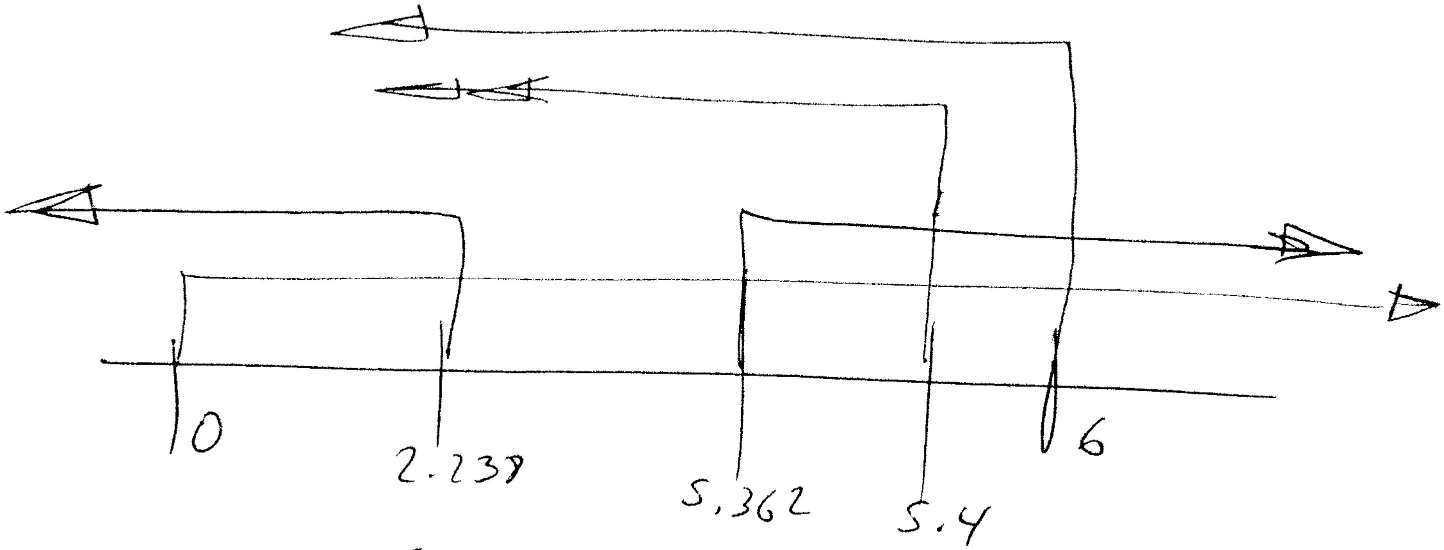
$$0.3k^2-2.28k+3.6<0$$

$$:\ \underbrace{(K-5.362)(k-2.238)}<0$$

either $K<5.362$ and $k>2.238$ or
$k>5.362$ and $k<2.238$

Range for K:





(Impossible)

The range for K is $(2.238, 5.362)$

Example: Pick $K = 4$.

→ The closed-loop poles are:

$$\begin{cases} -2.786 \\ -0.0731 \pm 1.1295i \\ -0.0291 \pm 0.334i \\ \underline{\text{OK}} \end{cases}$$