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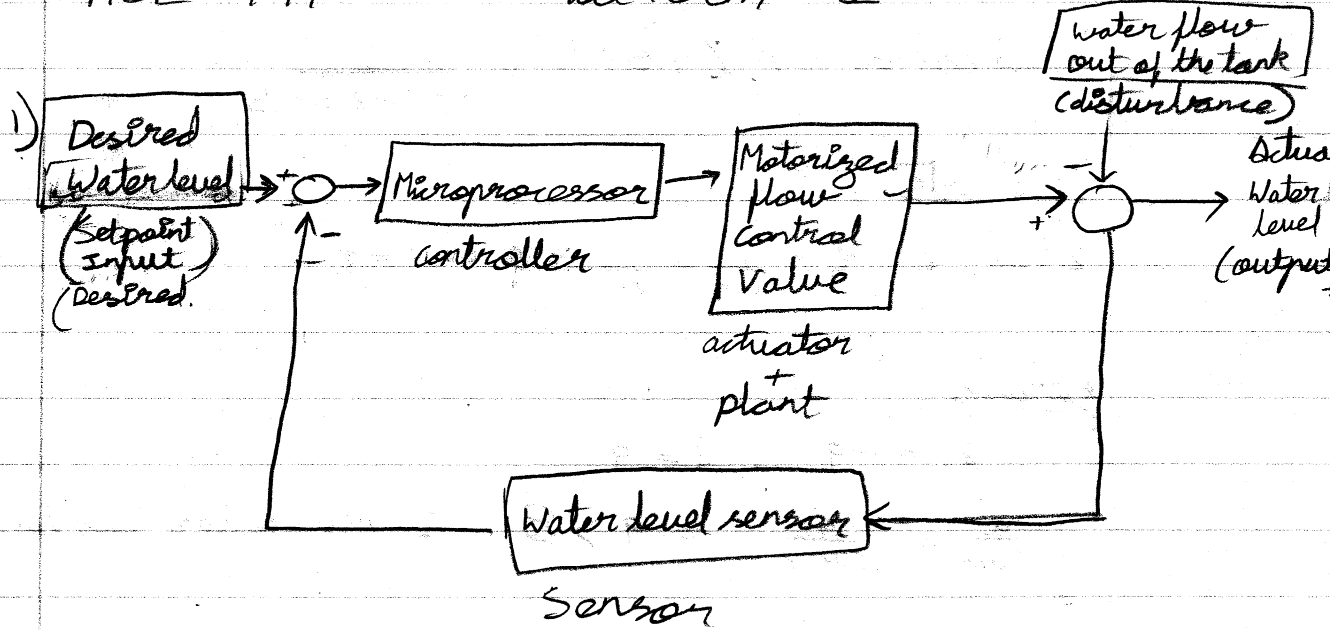
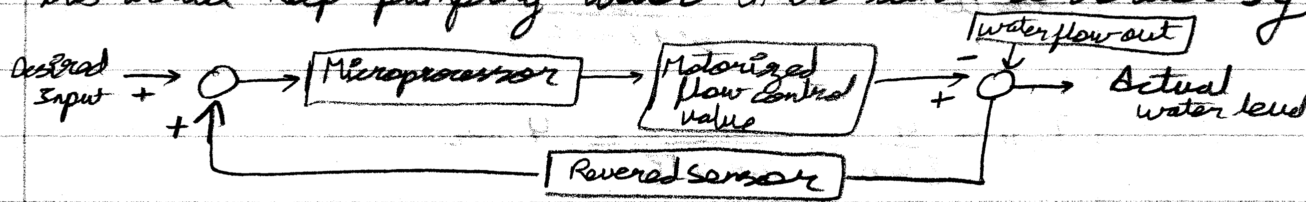
MCE 441 MidTerm – I

1)



If the terminals are reversed on the sensor then the output signal get multiplied by a −1 that combined with the negative feedback loop would result in a <u>positive feedback loop</u> this would keep pumping water in the tank continuously



Problem 2 | 4 page

$$G(s) = \frac{300\left(\frac{1}{9}s+1\right)}{(s^2+2s+5)(s^2+22s+120)}$$

$$p_{1,2} = -\frac{2}{2\cdot 1} \pm \frac{1}{2\cdot 1}\sqrt{2^2-4\cdot 5\cdot 1} = -1 \pm \frac{1}{2}\sqrt{-16} = -1 \pm \frac{1}{2}\sqrt{16}\,i = -1 \pm 2i$$

$$p_{1,2} = -1 \pm 2i$$

$$p_{3,4} = -\frac{22}{2\cdot 1} \pm \frac{1}{2\cdot 1}\sqrt{22^2-4\cdot 120\cdot 1} = -11 \pm \frac{1}{2}\sqrt{4} = -11 \pm 1$$

$$p_3 = -10$$

$$p_4 = -12$$

$G(0) = 0.5$

$$\tau_{1,2} = \frac{1}{1} = 1$$

$$\tau_3 = \frac{1}{10} = 0.1$$

$$\tau_4 = \frac{1}{12} = 0.083$$

$\Rightarrow$ poles $p_3 = -10$, $p_4 = -12$ with time constants $\tau_3 = 0.1$ and $\tau_4 \doteq 0.083$ eliminated (fast time constants)

$$G_{red}(s) = \frac{\frac{300}{120}\left(\frac{1}{9}s+1\right)}{s^2+2s+5} = \frac{2.5\left(\frac{1}{9}s+1\right)}{s^2+2s+5} =$$

$$= \frac{\frac{2.5}{9}s+2.5}{s^2+2s+5} = \frac{\frac{2.5}{9}(s+9)}{s^2+2s+5}$$

zero: $\omega = 9$

$$\frac{\omega}{\zeta\omega_n} = \frac{9}{1} = 9 > 8 \; - \text{zero eliminated}$$

$$G_{red}(s) = \frac{2.5}{s^2+2s+5} = \frac{2.5}{5} \cdot \frac{5}{s^2+2s+5} =$$

scaling factor

$$= \frac{1}{2} \; \frac{5}{s^2+2s+5}$$

Final value:

Final value $= y_\infty = G(0) = \frac{1}{2}$

$\boxed{F.V. = 0.5}$

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Setting time:

$$T_S = \frac{4}{\zeta \omega_n} = \frac{4}{1} = 4 \text{ (sec)}$$

$\boxed{T_S = 4 \text{(sec)}}$

Percent overshoot:

$\zeta\omega_n = 1$

$\omega_n^2 = 5$

$$\Rightarrow \zeta = \frac{1}{\sqrt{5}} \Rightarrow \boxed{\zeta = 0.447}$$

using chart M_p vs. ζ

for $\zeta = 0.447$ $\boxed{P.O. \approx 20\%}$

Peak value:

$$y_{peak} = 0.5 \cdot P.V. = 1.2 \cdot 0.5 = 0.6$$

$\boxed{y_{peak} = 0.6}$

$$T_{peak} = \frac{\pi}{\omega_d} = \frac{3.14}{\sqrt{5}\cdot\sqrt{1-0.447^2}} = 1.57 \text{ (sec)}$$

Rise time:

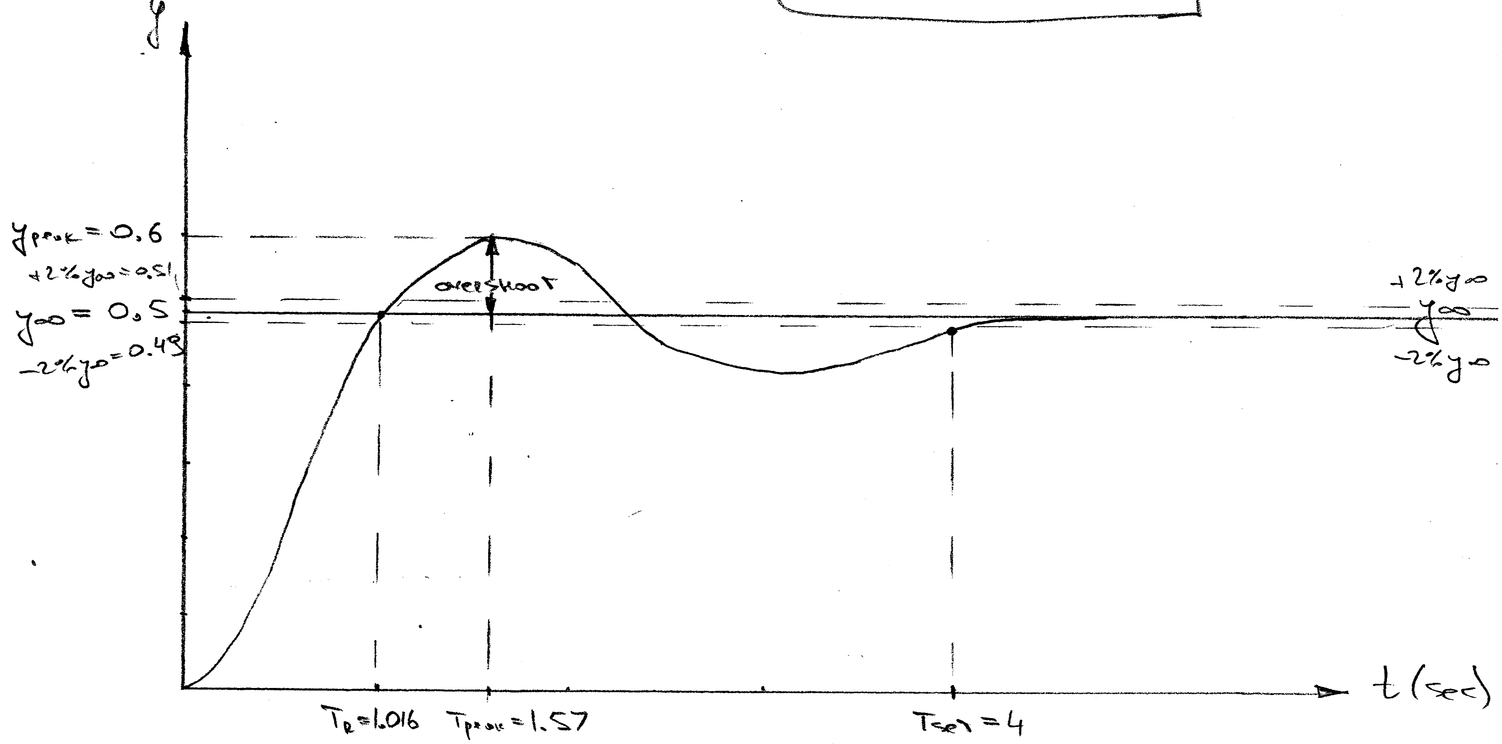
$$T_R = \frac{\pi - \beta}{\omega_d}$$

$P_{1,2} = -1 \pm 2i$

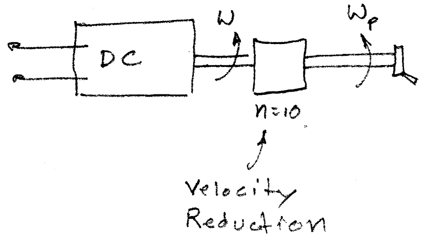
$$\beta = \tan^{-1}\left(\frac{2}{1}\right) = 1.107$$

$$T_{rise} = \frac{3.14 - 1.107}{\sqrt{5}\cdot\sqrt{1-0.447^2}} = 1.016 \text{ (sec)}$$

$\boxed{T_{rise} = 1.016 \text{ (sec)}}$



3.) GIVEN:



DC MOTOR:

$$\dot{\omega} + \frac{1}{\tau}\omega = aV - \frac{T_L}{J}$$

WHERE:

$\tau = 0.1$ sec.

$a = 1200 \text{ Volt}^{-1} \sec^{-2}$

T_L (N·m)

$J = 0.1 \text{ kg-m}^2$

CUTTING TORQUE:

$$T_c = k\omega_p$$

WHERE: $k = 2$ N·m·sec

* Spindle and workpiece have negligible inertia

$\therefore T_L \propto T_c$ (proportional based on gearing)

a.) FIND THE T.F. V to ω_p

$$\omega_p = \frac{1}{10}\omega \longrightarrow \omega = 10\,\omega_p, \quad \dot{\omega} = 10\,\dot{\omega}_p$$

$$T_c = 10\,T_L \longrightarrow T_L = \frac{1}{10}T_c$$

$$T_L = \frac{k}{10}\omega_p$$

$$10\,\dot{\omega}_p + \frac{10}{\tau}\omega_p + \frac{k\omega_p}{10\,J} = aV$$

$$10\,\dot{\omega}_p + \omega_p\left(\frac{10}{\tau} + \frac{k}{10J}\right) = aV$$

F = ma

N = kg · m/s^2

$$10\,\dot{\omega}_p + \omega_p\left[\frac{10}{(0.1\sec)} + \frac{(2\,\cancel{N}\cdot\cancel{m}\cdot\sec)}{10\,(0.1\,\cancel{kg}\cdot\cancel{m^2})\sec}\right] = \left(\frac{1200}{\text{Volt}\cdot\sec^2}\right)V$$

$$10\,\dot{\omega}_p + \left(\frac{102}{\sec}\right)\omega_p = \left(\frac{1200}{\text{Volt}\cdot\sec^2}\right)V$$

$\downarrow \mathcal{L}$

$$10\,s\,W_p(s) + 102\,W_p(s) = 1200\,V(s)$$

$$W_p(s)\left[10s + 102\right] = 1200\,V \ldots$$

$$\boxed{G(s) = \frac{W_p(s)}{V(s)} = \frac{1200}{(10s + 102)}}$$

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3.) (CONT.)

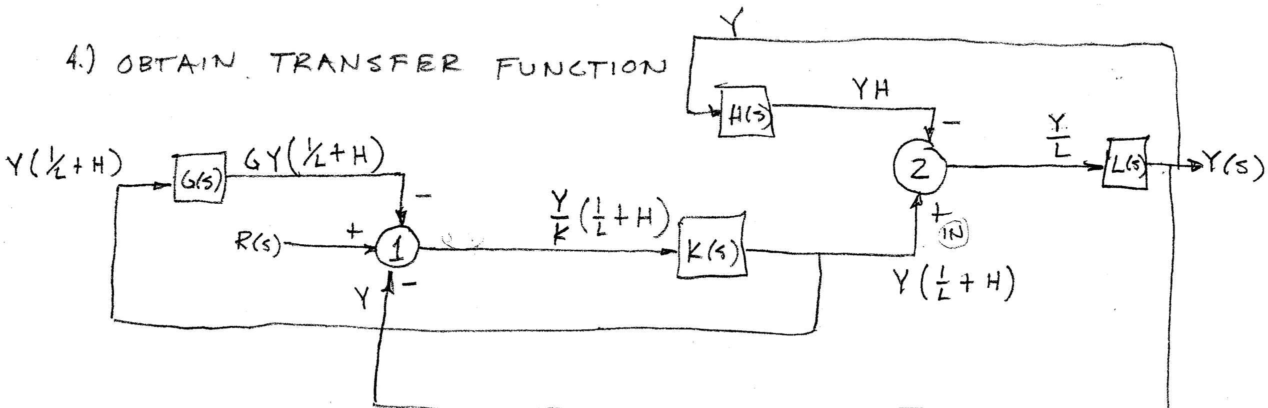
b.) FIND STEADY-STATE WORKPIECE VELOCITY (RPM) WHEN THE INPUT IS A 400-VOLT STEP.

STEADY-STATE $\longrightarrow$ FINAL VALUE WITH 400 VOLT INPUT

$$\text{Velocity} = 400 \times \frac{1200}{102} = 4705.9\ s^{-1}$$

$$= 4705.9 \frac{1}{s}\left(\frac{1\ \text{rev}}{2\pi}\right)\left(\frac{60\ \text{sec}}{1\ \text{min}}\right) = \boxed{44938\ \text{RPM}}$$

4.) OBTAIN TRANSFER FUNCTION



$$G(s) = \frac{Y(s)}{R(s)}$$

AT NODE 2: $IN - YH = \frac{Y}{L}$

$$IN = \frac{Y}{L} + YH = Y\left(\frac{1}{L} + H\right)$$

AT NODE 1: $R - GY\left(\frac{1}{L} + H\right) - Y = \frac{Y}{K}\left(\frac{1}{L} + H\right)$

$$R = \frac{Y}{K}\left(\frac{1}{L} + H\right) + Y + GY\left(\frac{1}{L} + H\right)$$

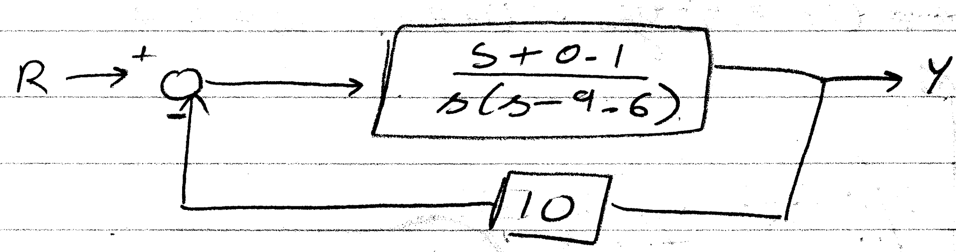
LET $Z = \frac{1}{L} + H$

$$R = \frac{YZ}{K} + Y + GYZ$$

$$Y\left(\frac{Z}{K} + GZ + 1\right) = R$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{1}{\left[\frac{Z}{K(s)} + G(s)Z + 1\right]}}$$

5) step input = 2



$$\frac{Y}{R} = \frac{\dfrac{s+0.1}{s(s-9.6)}}{1+\dfrac{10(s+0.1)}{s(s-9.6}} = \frac{(s+0.1)}{s(s-9.6)+10(s+0.1)}$$

$$= \frac{s+0.1}{s^2-9.6s+10s+1} =$$

$$\therefore \frac{Y(s)}{R(s)} = \frac{s+0.1}{s^2+0.4s+1} = G(s)$$

poles = $s_{1,2} = -0.2 \pm 0.980i$ $\therefore$

$G_{red}(s) = SF(s+0.1) \times \frac{1}{s^2+0.4s+1}$

$\therefore \omega_n = 1;\ 2\zeta\omega_n = 0.4 \ \therefore \zeta = \frac{0.4}{2\omega_n} = \frac{0.4}{2(1)} = 0.2$

Zeroes: $z_1 = -0.1$ $\quad \frac{a}{\zeta\omega_n} = \frac{0.1}{0.2(1)} = 0.5 \ngtr 8$

$\therefore$ zeroes have to be accounted in the numerator

$G_{red}(s) = SF \frac{(s+0.1)}{s^2+0.4s+1}$ $\quad (S.F = 1)$

$\therefore y(\infty) = G(0) \times step = 0.1 \times 2 = 0.2$

$P.O = 750\% = 7.5$ (from the other Table)

$$T_{sett} = \frac{4}{\xi \omega_n} = \frac{4}{0.2(1)} = 20$$

$$T_{rise} = \frac{\pi - \cos^{-1}(\xi)}{\omega_d} = \frac{\pi - \cos^{-1}(0.2)}{0.980} = 1.808$$

$$Peak = (1 + 7.5)\, y(\infty) = (1+7.5)(0.2) = 1.7$$