

TABLE 7.2(c)
DATA FOR CLAMPED-FREE BEAM

Characteristic function and its derivatives

$$\begin{aligned}\phi_r &= \cosh \lambda_r x - \cos \lambda_r x - \sigma_r (\sinh \lambda_r x - \sin \lambda_r x), \\ \frac{1}{\lambda_r} \frac{d\phi_r}{dx} &= \phi'_r = \sinh \lambda_r x + \sin \lambda_r x - \sigma_r (\cosh \lambda_r x - \cos \lambda_r x), \\ \frac{1}{\lambda_r^2} \frac{d^2\phi_r}{dx^2} &= \phi''_r = \cosh \lambda_r x + \cos \lambda_r x - \sigma_r (\sinh \lambda_r x + \sin \lambda_r x), \\ \frac{1}{\lambda_r^3} \frac{d^3\phi_r}{dx^3} &= \phi'''_r = \sinh \lambda_r x - \sin \lambda_r x - \sigma_r (\cosh \lambda_r x + \cos \lambda_r x).\end{aligned}$$

Boundary values

$$\phi_r(0) = \phi'_r(0) = 0,$$

$$\phi''_r(l) = \phi'''_r(l) = 0.$$

General data

$$\cos \lambda_r l \cosh \lambda_r l + 1 = 0, \quad \sigma_r = \frac{\sinh \lambda_r l - \sin \lambda_r l}{\cosh \lambda_r l + \cos \lambda_r l}$$

$$a_r = \Delta \rho l, \quad \int_0^l \phi_r(x) \phi_s(x) dx = \int_0^l \phi_r''(x) \phi_s''(x) dx = \begin{cases} 0 & (r \neq s), \\ l & (r = s). \end{cases}$$

Values of σ_r and $\lambda_r l$ and various powers

r	$\lambda_r l$	$(\lambda_r l)^2$	$(\lambda_r l)^3$	$(\lambda_r l)^4$
1	1.875 10	3.516 02	6.592 90	12.362 4
2	4.694 09	22.034 5	103.432	485.519
3	7.854 76	61.697 2	484.617	3 806.55
4	10.995 5	120.902	1 329.38	14 617.3
5	14.137 2	199.860	2 825.45	39 943.8

r	σ_r	$\frac{\omega_r}{\omega_1} = \frac{\lambda_r^2}{\lambda_1^2}$	$\frac{\omega_r^2}{\omega_1^2} = \frac{\lambda_r^4}{\lambda_1^4}$
1	0.734 095 5	1.0	1.0
2	1.018 466 4	6.266 89	39.273 9
3	0.999 224 5	17.547 5	307.914
4	1.000 033 6	34.386 1	1 182.40
5	0.999 998 6	56.842 6	3 231.08

For $r > 5$
 $\lambda_r l \doteq (2r-1) \pi/2$
 $\sigma_r \doteq 1.0$

TABLE 7·2 (c) (cont.)

CHARACTERISTIC FUNCTIONS AND DERIVATIVES
CLAMPED-FREE BEAM

First mode

$\frac{x}{l}$	ϕ_1	$\phi_1' = \frac{1}{\lambda_1} \frac{d\phi_1}{dx}$	$\phi_1'' = \frac{1}{\lambda_1^2} \frac{d^2\phi_1}{dx^2}$	$\phi_1''' = \frac{1}{\lambda_1^3} \frac{d^3\phi_1}{dx^3}$
0·00	0·000 00	0·000 00	2·000 00	-1·468 19
0·02	0·001 39	0·073 97	1·944 94	-1·468 17
0·04	0·005 52	0·145 88	1·889 88	-1·468 05
0·06	0·012 31	0·215 72	1·834 83	-1·467 73
0·08	0·021 68	0·283 50	1·779 80	-1·467 10
0·10	0·033 55	0·349 21	1·724 80	-1·466 07
0·12	0·047 84	0·412 86	1·669 85	-1·464 55
0·14	0·064 49	0·474 46	1·614 96	-1·462 45
0·16	0·083 40	0·534 00	1·560 16	-1·459 68
0·18	0·104 52	0·591 48	1·505 49	-1·456 17
0·20	0·127 74	0·646 92	1·450 96	-1·451 82
0·22	0·153 01	0·700 31	1·396 60	-1·446 56
0·24	0·180 24	0·751 67	1·342 47	-1·440 32
0·26	0·209 36	0·801 00	1·288 59	-1·433 02
0·28	0·240 30	0·848 32	1·235 00	-1·424 59
0·30	0·272 97	0·893 64	1·181 75	-1·414 97
0·32	0·307 30	0·936 96	1·128 89	-1·404 10
0·34	0·343 22	0·978 31	1·076 46	-1·391 91
0·36	0·380 65	1·017 71	1·024 51	-1·378 34
0·38	0·419 52	1·055 16	0·973 09	-1·363 34
0·40	0·459 77	1·090 70	0·922 27	-1·346 85
0·42	0·501 31	1·124 35	0·872 09	-1·328 84
0·44	0·544 08	1·156 12	0·822 62	-1·309 24
0·46	0·588 00	1·186 06	0·773 92	-1·288 01
0·48	0·633 01	1·214 18	0·726 03	-1·265 12
0·50	0·679 05	1·240 52	0·679 05	-1·240 52
0·52	0·726 03	1·265 12	0·633 01	-1·214 18
0·54	0·773 92	1·288 01	0·588 00	-1·186 06
0·56	0·822 62	1·309 24	0·544 08	-1·156 12
0·58	0·872 09	1·328 84	0·501 31	-1·124 35
0·60	0·922 27	1·346 85	0·459 77	-1·090 70
0·62	0·973 09	1·363 34	0·419 52	-1·055 16
0·64	1·024 51	1·378 34	0·380 65	-1·017 71
0·66	1·076 46	1·391 91	0·343 22	-0·978 31
0·68	1·128 89	1·404 10	0·307 30	-0·936 96
0·70	1·181 75	1·414 97	0·272 97	-0·893 64
0·72	1·235 00	1·424 59	0·240 30	-0·848 32
0·74	1·288 59	1·433 02	0·209 36	-0·801 00
0·76	1·342 47	1·440 32	0·180 24	-0·751 67
0·78	1·396 60	1·446 56	0·153 01	-0·700 31
0·80	1·450 96	1·451 82	0·127 74	-0·646 92
0·82	1·505 49	1·456 17	0·104 52	-0·591 48
0·84	1·560 16	1·459 68	0·083 40	-0·534 00
0·86	1·614 96	1·462 45	0·064 49	-0·474 46
0·88	1·669 85	1·464 55	0·047 84	-0·412 86
0·90	1·724 80	1·466 07	0·033 55	-0·349 21
0·92	1·779 80	1·467 10	0·021 68	-0·283 50
0·94	1·834 83	1·467 73	0·012 31	-0·215 72
0·96	1·889 88	1·468 05	0·005 52	-0·145 88
0·98	1·944 94	1·468 17	0·001 39	-0·073 97
1·00	2·000 00	1·468 19	0·000 00	0·000 00

TABLE 7.2 (c) (cont.)

Second mode

$\frac{x}{l}$	ϕ_2	$\phi_2' = \frac{1}{\lambda_2} \frac{d\phi_2}{dx}$	$\phi_2'' = \frac{1}{\lambda_2^2} \frac{d^2\phi_2}{dx^2}$	$\phi_2''' = \frac{1}{\lambda_2^3} \frac{d^3\phi_2}{dx^3}$
0.00	0.000 00	0.000 00	2.000 00	-2.036 93
0.02	0.008 53	0.178 79	1.808 77	-2.036 66
0.04	0.033 01	0.339 62	1.617 64	-2.034 83
0.06	0.071 74	0.482 53	1.426 80	-2.030 02
0.08	0.123 05	0.607 54	1.236 60	-2.020 97
0.10	0.185 26	0.714 75	1.047 50	-2.006 58
0.12	0.256 70	0.804 28	0.860 04	-1.985 90
0.14	0.335 73	0.876 31	0.674 84	-1.958 14
0.16	0.420 70	0.931 08	0.492 61	-1.922 67
0.18	0.510 02	0.968 92	0.314 09	-1.879 01
0.20	0.602 11	0.990 20	0.140 07	-1.826 82
0.22	0.695 44	0.995 39	-0.028 65	-1.765 92
0.24	0.788 52	0.985 02	-0.191 23	-1.696 25
0.26	0.879 92	0.959 70	-0.346 87	-1.617 91
0.28	0.968 27	0.920 13	-0.494 75	-1.531 13
0.30	1.052 27	0.867 07	-0.634 10	-1.436 24
0.32	1.130 68	0.801 36	-0.764 19	-1.333 73
0.34	1.202 36	0.723 89	-0.884 31	-1.224 16
0.36	1.266 26	0.635 65	-0.993 84	-1.108 21
0.38	1.321 41	0.537 64	-1.092 22	-0.986 67
0.40	1.366 94	0.430 94	-1.178 95	-0.860 40
0.42	1.402 09	0.316 65	-1.253 65	-0.730 34
0.44	1.426 19	0.195 93	-1.316 00	-0.597 48
0.46	1.438 71	0.069 95	-1.365 78	-0.462 91
0.48	1.439 20	-0.060 12	-1.402 89	-0.327 72
0.50	1.427 33	-0.193 07	-1.427 33	-0.193 07
0.52	1.402 89	-0.327 72	-1.439 20	-0.060 12
0.54	1.365 78	-0.462 91	-1.438 71	0.069 95
0.56	1.316 00	-0.597 48	-1.426 19	0.195 93
0.58	1.253 65	-0.730 34	-1.402 09	0.316 65
0.60	1.178 95	-0.860 40	-1.366 94	0.430 94
0.62	1.092 22	-0.986 67	-1.321 41	0.537 64
0.64	0.993 84	-1.108 21	-1.266 26	0.635 65
0.66	0.884 31	-1.224 16	-1.202 36	0.723 89
0.68	0.764 19	-1.333 73	-1.130 68	0.801 36
0.70	0.634 10	-1.436 24	-1.052 27	0.867 07
0.72	0.494 75	-1.531 13	-0.968 27	0.920 13
0.74	0.346 87	-1.617 91	-0.879 92	0.959 70
0.76	0.191 23	-1.696 25	-0.788 52	0.985 02
0.78	0.028 65	-1.765 92	-0.695 44	0.995 39
0.80	-0.140 07	-1.826 82	-0.602 11	0.990 20
0.82	-0.314 09	-1.879 01	-0.510 02	0.968 92
0.84	-0.492 61	-1.922 67	-0.420 70	0.931 08
0.86	-0.674 84	-1.958 14	-0.335 73	0.876 31
0.88	-0.860 04	-1.985 90	-0.256 70	0.804 28
0.90	-1.047 50	-2.006 58	-0.185 26	0.714 75
0.92	-1.236 60	-2.020 97	-0.123 05	0.607 54
0.94	-1.426 80	-2.030 02	-0.071 74	0.482 53
0.96	-1.617 64	-2.034 83	-0.033 01	0.339 62
0.98	-1.808 77	-2.036 66	-0.008 53	0.178 79
1.00	-2.000 00	-2.036 93	0.000 00	0.000 00