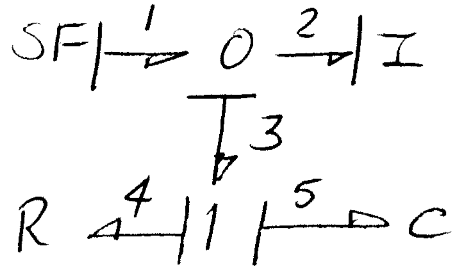
5-1 b)



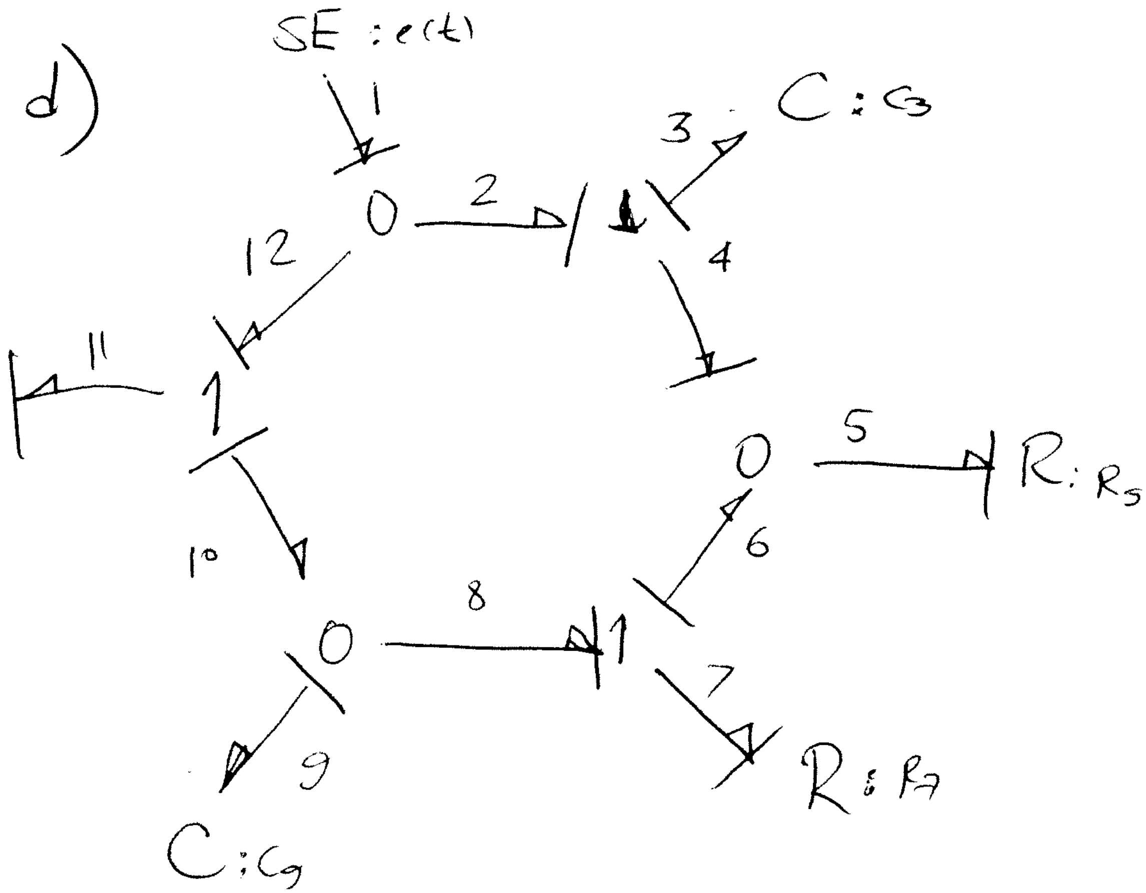
$$\dot{p}_2 = e_2 = e_3 = e_4 + e_5 = R f_4 + \frac{q_5}{C} = R f_3 + \frac{q_5}{C}$$

$$= R(f_1 - f_2) + \frac{q_5}{C} = R\left(f_1(t) - \frac{p_2}{I}\right) + \frac{q_5}{C}$$

$$\dot{q}_5 = f_5 = f_3 = f_1 - f_2 = f_1(t) - \frac{p_2}{I}$$

— x —

d)



2 C's w/ integral causality: 2 states

(q_3, q_9)

$$\dot q_3 = f_3 = f_4 = f_5 - \overset{= f_7}{f_6} = \frac{\overset{= e_4}{e_5}}{R_5} - \frac{\overset{e_8 - e_6}{e_7}}{R_7} = \frac{e_2 - e_3}{R_5} - \overset{= e_4}{\frac{e_8 - e_6}{R_7}}$$

$$\dot q_3 = \frac{e_1(t)}{R_5} - \frac{q_3}{C_3 R_5} - \frac{q_9}{C_9 R_7} + \frac{(e_2 - e_3)}{R_7}$$

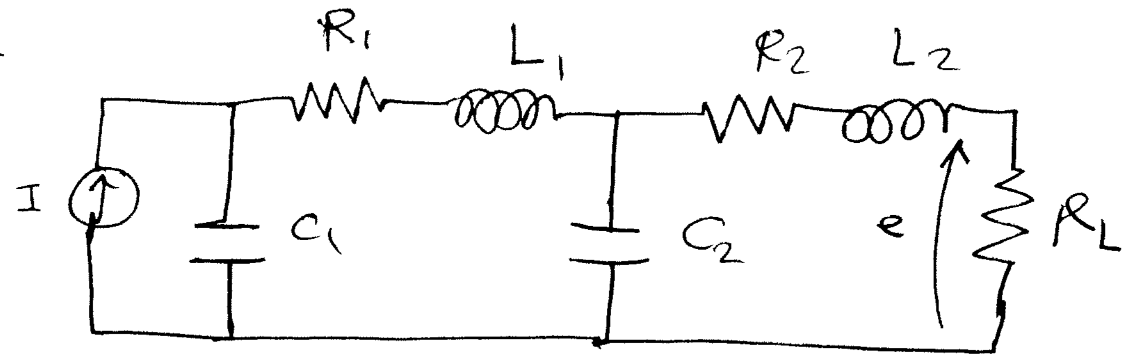
$$= \frac{e_1(t)}{R_5} - \frac{q_3}{C_3 R_5} - \frac{q_9}{C_9 R_7} + \frac{e_1(t)}{R_7} - \frac{q_3}{C_3 R_7}$$

same

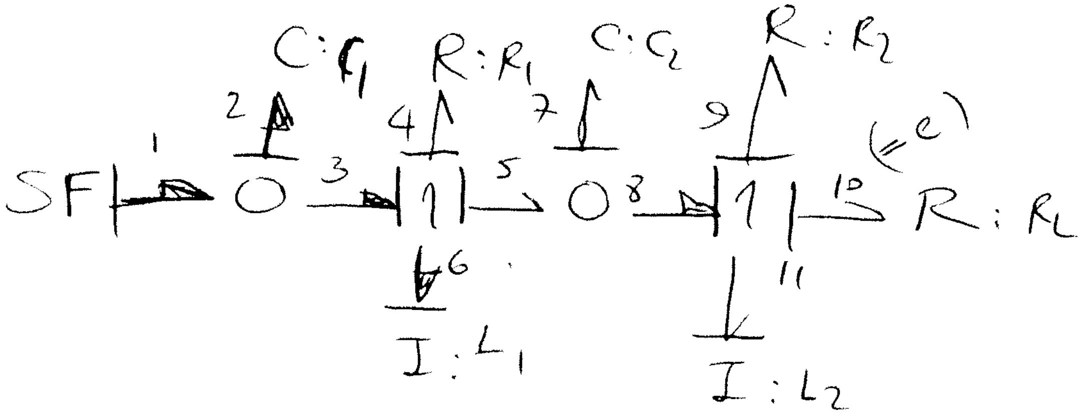
$$\dot q_9 = f_9 = \overset{= f_{11}}{f_{10}} - \overset{= f_7}{f_8} = \frac{e_{11}}{R_{11}} - \frac{e_7}{R_7} = \frac{\overset{= e_1}{e_{12}} - \overset{= e_9}{e_{10}}}{R_{11}} - \frac{e_8 - e_6}{R_7}$$

$$\dot q_9 = \frac{e_1(t)}{R_{11}} - \frac{q_9}{C_9 R_{11}} - \frac{q_9}{C_9 R_7} + \frac{e_1(t)}{R_7} - \frac{q_3}{C_3 R_7}$$

5-4



By inspection:



4 states

q_2 q_7

p_6 p_{11}

$$\dot q_2 = f_2 = f_1 - \overset{= f_6}{f_3} = I(t) - \frac{p_6}{L_1}$$

$$\dot q_7 = f_7 = \overset{= f_6}{f_5} - \overset{= f_{11}}{f_8} = \frac{p_6}{L_1} - \frac{p_{11}}{L_2}$$

$$\dot p_6 = e_6 = e_3 - e_4 - e_5 = e_2 - R_1 \overset{= f_6}{f_4} - e_7 = \frac{q_2}{C_1} - R_1 \frac{p_6}{L_1} - \frac{q_7}{C_2}$$

$$\dot p_{11} = e_{11} = \overset{= e_7}{e_8} - e_9 - e_{10} = \frac{q_7}{C_2} - R_2 \overset{= f_{11}}{f_9} - \overset{= f_{11}}{\cancel{f_{10}}} R_L$$

$$= \frac{q_7}{C_2} - \frac{R_2 p_{11}}{L_2} - R_L \frac{p_{11}}{L_2}$$

Output eq: $e = e_{10} = R_L f_{10} = R_L f_{11} = R_L \frac{p_{11}}{L_2}$ //

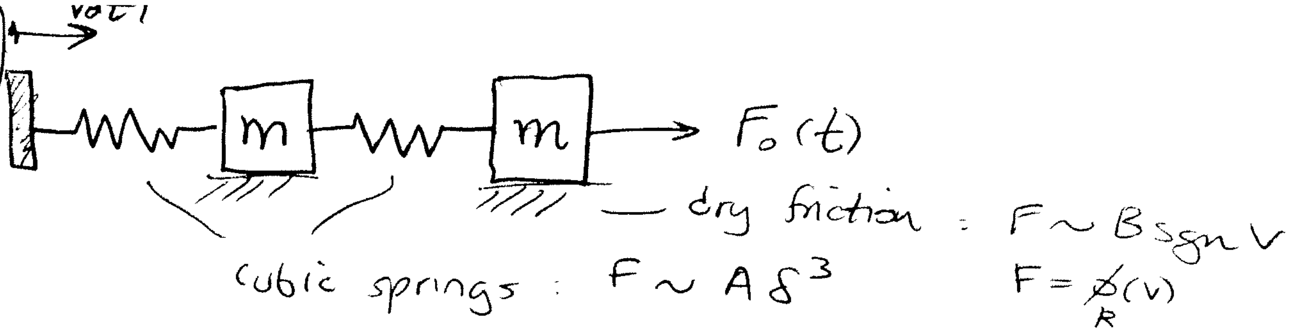
— x —

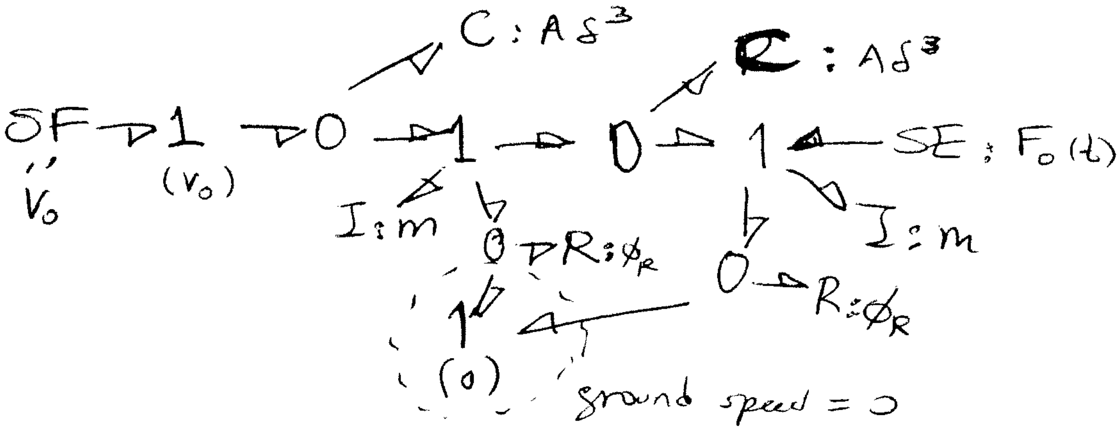
<u>Matrix form</u>

$$\begin{bmatrix} \dot q_2 \\ \dot q_7 \\ \dot p_6 \\ \dot p_{11} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_1} & -\frac{1}{L_2} \\ \frac{1}{C_1} & -\frac{1}{C_2} & -R_1/L_1 & 0 \\ 0 & 1/C_2 & 0 & \frac{R_2 - R_L}{L_2} \end{bmatrix} \begin{bmatrix} q_2 \\ q_7 \\ p_6 \\ p_{11} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} I(t)$$

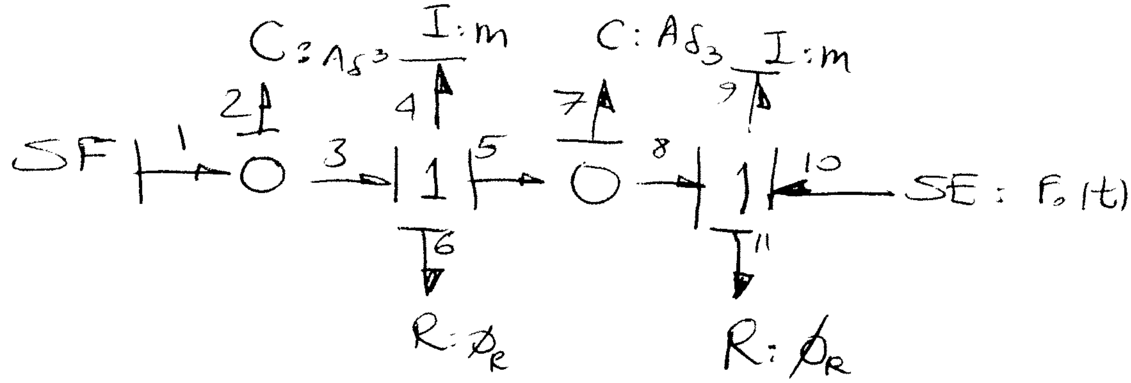
$$e = \begin{bmatrix} 0 & 0 & 0 & \frac{R_L}{L_2} \end{bmatrix} \begin{bmatrix} q_2 \\ q_7 \\ p_6 \\ p_{11} \end{bmatrix}$$

5-5





Simplify & assign causality :



State variables : $q_2,\ p_4,\ q_7,\ p_9$

Inputs : $f_1 = V_0(t)$

$e_{10} = F_0(t)$

<u>Derivation</u> : $\dot{q}_2 = f_2 = f_1 - f_3 = V_0(t) - \frac{p_4}{m}$, $f_3 = f_4 = \frac{p_4}{m}$

$$\dot{p}_4 = e_4 = e_3 - e_5 - e_6 = e_2 - e_7 - B\,\mathrm{sgn}(f_6), \quad f_6 = f_4$$

$$= A q_2^{\,3} - A q_7^{\,3} - B\,\mathrm{sgn}\left(\frac{p_4}{m}\right)$$

$$\dot{q}_7 = f_7 = f_5 - f_8 = f_4 - f_9 = \frac{p_4}{m} - \frac{p_9}{m}$$

$$\dot{p}_9 = e_9 = e_{10} + \overset{= e_7 = A q_7^3}{e_8} - e_{11} = F_0(t) + A q_7^3 - B\,\mathrm{sgn}(\underset{\overset{\|}{f_9 = \frac{p_9}{m}}}{f_{11}})$$

$$\begin{cases} \dot{q}_2 = V_0(t) - \dfrac{p_4}{m} \\ \dot{p}_4 = A q_2^3 - A q_7^3 - B\,\mathrm{sgn}\left(\dfrac{p_4}{m}\right) \\ \dot{q}_7 = \dfrac{p_4}{m} - \dfrac{p_9}{m} \\ \dot{p}_9 = F_0(t) + A q_7^3 - B\,\mathrm{sgn}\left(\dfrac{p_9}{m}\right) \end{cases}$$