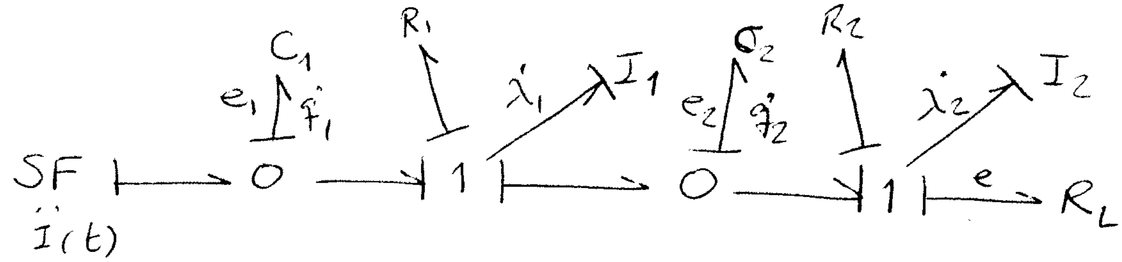
MCE 403/503 HW4 SOLUTION FALL 2010

5-4



$$\dot{q}_1 = I(t) - \frac{\lambda_1}{I_1}$$

$$\dot{\lambda}_1 = e_1 - e_2 - R_1\frac{\lambda_1}{I_1} = \frac{q_1}{C_1} - \frac{q_2}{C_2} - R_1\frac{\lambda_1}{I_1}$$

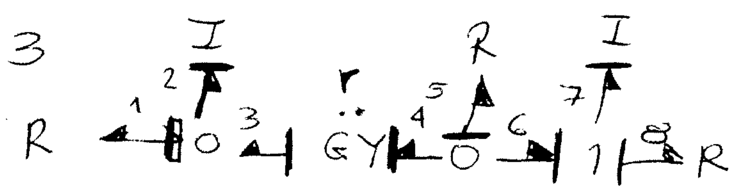
$$\dot{q}_2 = \frac{\lambda_1}{I_1} - \frac{\lambda_2}{I_2}$$

$$\dot{\lambda}_2 = e_2 - R_2\frac{\lambda_2}{I_2} - e = \frac{q_2}{C_2} - R_2\frac{\lambda_2}{I_2} - R_L\frac{\lambda_2}{I_2}$$

Output: $e = R_L\frac{\lambda_2}{I_2}$

5-13

a)



- Algebraic loop on e_1
- state vars. p_2, p_7

$$\dot{p}_2 = e_2 = e_1 \quad \text{(alg. loop var)}$$

$$e_1 = R_1 f_1 = R_1(\cancel{f_3} - f_2) = R_1\left(f_3 - \frac{p_2}{I_2}\right) = R_1\left(\frac{e_4}{r} - \frac{p_2}{I_2}\right)$$

$$e_1 = R_1\left(\frac{R_5 f_5}{r} - \frac{p_2}{I_2}\right) = R_1\left(\frac{R_5}{r}(-f_4 - f_6) - \frac{p_2}{I_2}\right) = R_1\left(\frac{-R_5 \overset{e_1}{\overset{=}{e_3}}}{r^2} - \frac{R_5}{r}\frac{p_7}{I_7} - f_2/I_2\right)$$

C_8 has derivative causality.

State vars: p_1, q_4, q_6, p_{11}

$$\dot{p}_1 = e_1 = e_3 - e_2 = \frac{q_4}{C_4} - R_2 \overset{= f_1}{f_2} = \frac{q_4}{C_4} - R_2 \frac{p_1}{f_1}$$

$$\dot{q}_4 = f_4 = f_5 - \overset{= f_1}{f_3} = f_7 - \frac{p_1}{f_1} = \overset{= f_{11}}{f_9} - f_8 - \frac{p_1}{f_1} = \frac{p_{11}}{I_{11}} - \dot{q}_8 - \frac{p}{f} \quad \text{----(3}$$

$\dot{q}_8$ needs to be linked to other q's (& p's)

$$q_8 = C_8 e_8 = C_8 e_7 = C_8 (e_6 + e_5) = C_8\left(\frac{q_6}{C_6} + \frac{q_4}{C_4}\right)$$

so $$\dot{q}_8 = C_8\left(\frac{\dot{q}_6}{C_6} + \frac{\dot{q}_4}{C_4}\right) \quad \text{-----(1)}$$

$$\dot{q}_6 = f_6 \overset{= f_7}{=} \overset{= f_{11}}{f_9} - f_8 = \frac{p_{11}}{I_{11}} - \dot{q}_8 \quad \text{----(2)}$$

$\star$, (1) and (2) are a system of 3 equations in 3 unknowns: $\dot{q}_4$, $\dot{q}_6$ and $\dot{q}_8$.

Solving:
$$\begin{cases} \dot{q}_4 + \dot{q}_8 = \dfrac{p_{11}}{I_{11}} - \dfrac{p_1}{f_1} \\[2ex] \dfrac{C_8}{C_4}\dot{q}_4 + \dfrac{C_8}{C_6}\dot{q}_6 - \dot{q}_8 = 0 \\[2ex] \dot{q}_6 + \dot{q}_8 = \dfrac{p_{11}}{I_{11}} \end{cases}$$

gives the s-t eq's for $\dot{q}_4$, $\dot{q}_8$, $\dot{q}_6$.

so $$e_1 = R_1\left(\frac{-R_5 e_1}{r^2} - \frac{R_5}{r}\frac{p_7}{I_7} - \frac{p_2}{I_2}\right)$$

$$\left[1+\frac{R_1R_5}{r^2}\right]e_1 = -R_1\left(\frac{R_5}{rI_7}p_7 + \frac{p_2}{I_2}\right)$$

$$e_1 = \frac{-R_1}{\left(1+\frac{R_1R_5}{r^2}\right)}\left(\frac{R_5}{rI_7}p_7+\frac{p_2}{I_2}\right) = \dot{p}_2$$

$$\dot{p}_7 = e_7 = e_6 - e_8 = e_4 - R_8 \overset{=f_7}{f_8} = rf_3 - R_8\frac{p_7}{I_7}$$

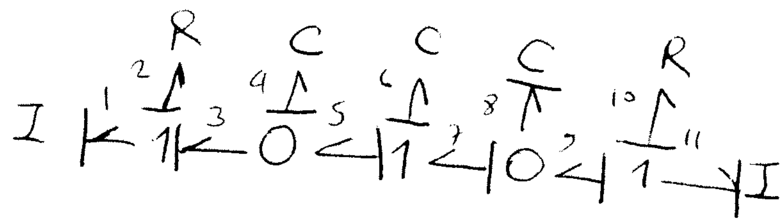
$$\dot{p}_7 = r(f_1+f_2) - R_8\frac{p_7}{I_7} = r\left(\frac{e_1}{R_1}+\frac{p_2}{I_2}\right) - R_8\frac{p_7}{I_7}$$

$$\dot{p}_7 = \frac{-r}{\left(1+\frac{R_1R_5}{r^2}\right)}\left(\frac{R_5}{rI_7}p_7+\frac{p_2}{I_2}\right)+\frac{rp_2}{I_2} - R_8\frac{p_7}{I_7}$$

$$\dot{p}_7 = \frac{r}{I_2}\left(1-\frac{1}{\left(1+\frac{R_1R_5}{r^2}\right)}\right)p_2 - \frac{R_8}{I_7}\left(1+\frac{R_5}{\left(1+\frac{R_1R_5}{r^2}\right)}\right)p_7$$

b) inconsistent B.G. No causality assignment possible.

c)



(by inspection)

Finally $\dot{p}_{11} = e_{11} = -e_{10} - \underbrace{e_9}_{=e_8} = -R_{10} f_{10} - \underbrace{\dot{q}_8}_{=f_{11}}$

$$\dot{p}_{11} = -R_{10} \frac{p_{11}}{I_{11}} - \dot{q}_8$$

available once syst. of eq's is solved

5-17

State variables: $p_5,\ p_{10},\ q_{12}$

$$\dot{p}_5 = e_5 = e_4 - e_6 = \tau f_3 - R\, \underbrace{e_7}_{=e_9} = \tau\left(\frac{e_2}{R_w}\right) - R(\underbrace{e_{11}}_{=e_{12}} - e_{10})$$

$$\dot{p}_5 = \frac{\tau}{R_w}(e_1 - e_3) - R\left(k_w q_{12} - \dot{p}_{10}\right) = \frac{\tau}{R_w}(e - \tau f_4) - R(k_w q_{12} - \dot{p}_{10})$$

$$\dot{p}_5 = \frac{\tau}{R_w}\left(e - \tau \frac{p_5}{I_m}\right) - R\left(k_w q_{12} - \dot{p}_{10}\right)$$

$$\dot{p}_{10} = e_{10} = e_{11} - e_9 = k_w q_{12} - \dot{p}_8$$

$\dot{p}_8$ can be found:

$$f_8 = f_9 + f_7 = \frac{p_{10}}{M_f} + R f_6 = \frac{p_{10}}{M_f} + R \frac{p_5}{I_m}$$

$$f_8 = \frac{p_8}{M_m} \rightarrow p_8 = \frac{M_m}{M_f} p_{10} + \frac{M_m R}{I_m} p_5$$

$$\dot{p}_8 = \frac{M_m}{M_f} \dot{p}_{10} + \frac{M_m R}{I_m} \dot{p}_5$$

so $\dot{p}_{10} = k_w q_{12} - \frac{M_m}{M_f}\dot{p}_{10} - \frac{M_m R}{I_m}\dot{p}_5$

$\rightarrow \left(1 + \frac{M_m}{M_f}\right)\dot{p}_{10} = k_w q_{12} - \frac{M_m}{I_m} R \dot{p}_5$

$\dot{q}_{12} = f_{12} = f_{13} - f_{11} = V_i - \frac{p_{10}}{M_f}$ //

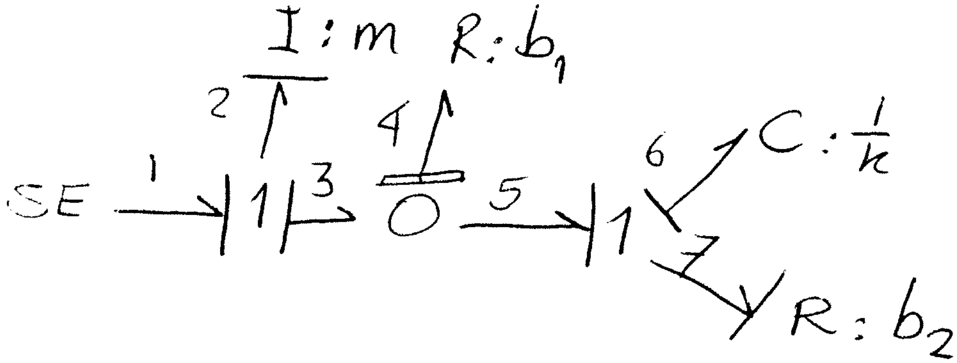
The following must be solved:

$$\begin{cases} \dot{p}_5 = \frac{\tau}{R_w}\left(e - \tau\frac{p_5}{I_m}\right) - R\left(k_w q_{12} - \dot{p}_{10}\right) \\ \left(1 + \frac{M_m}{M_f}\right)\dot{p}_{10} = k_w q_{12} - \frac{M_m}{I_m} R \dot{p}_5 \end{cases}$$

— x —

5-19

a)



Algebraic loop involving e_4

state vars : p_2, q_6

b) $\dot{p}_2 = e_2 = e_1 - e_3 = F - e_4$ (loop var.)

$$\dot{q}_6 = f_6 = f_7 = \frac{e_7}{b_2} = \frac{e_5 - e_6}{b_2} = \frac{e_4 - k q_6}{b_2}$$

e_4 needs to be figured out:

$$e_4 = b_1 f_4 = b_1 (f_3 - \overset{= f_7}{f_5}) = b_1 \left(\frac{p_2}{m} - \frac{e_7}{b_2} \right)$$

$$e_4 = b_1 \left(\frac{p_2}{m} - \frac{(\overset{= e_4}{e_5 - e_6})}{b_2} \right) = b_1 \left(\frac{p_2}{m} - \frac{e_4}{b_2} + k \frac{q_6}{b_2} \right)$$

$$\left(1 + \frac{b_1}{b_2}\right) e_4 = \frac{b_1 p_2}{m} + \frac{b_1 k}{b_2} q_6$$

$$e_4 = \frac{1}{\left(1 + \frac{b_1}{b_2}\right)} \left[\frac{b_1 p_2}{m} + \frac{b_1 k}{b_2} q_6 \right]$$

Substituting:

$$\dot{p}_2 = F - \left(\frac{b_1 b_2}{b_1 + b_2} \right) \left(\frac{p_2}{m} + \frac{k q_6}{b_2} \right)$$

$$\dot{q}_6 = \frac{b_1}{(b_1 + b_2)} \left(\frac{p_2}{m} + k \frac{q_6}{b_2} \right) - \frac{k q_6}{b_2}$$

$$= \frac{b_1}{m(b_1 + b_2)} p_2 + \frac{K}{b_2} \left(\frac{b_1}{b_1 + b_2} - 1 \right) q_6$$

$$= \frac{b_1}{m(b_1 + b_2)} p_2 - \frac{k}{b_1 + b_2} q_6$$

c) One possibility is to consider a concentrated mass at the spring/damper attachment point: