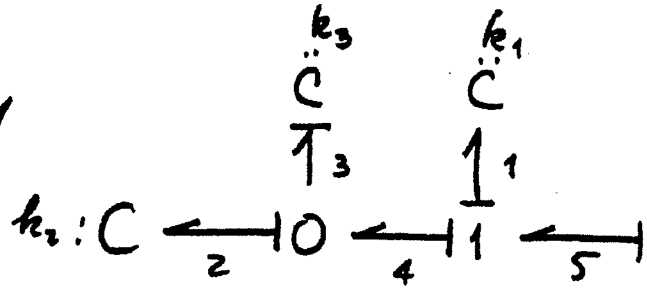
7-1



State Equs: $\dot{q}_1 = f_5 \rightarrow \underline{q_1 = q_5} + \cancel{const}^{\,0}$

$\dot{q}_2 = -f_3 + f_4 = -\dot{q}_3 + f_5 \rightarrow q_2 = -q_3 + q_5 + \cancel{const.}^{\,0}$

Deriv. Caus.

$$q_3 = \frac{e_3}{k_3} = \frac{e_2}{k_2} = \frac{k_2 q_2}{k_3}$$

so $$q_2 = -\frac{k_2}{k_3} q_2 + q_5$$

or $$\underline{q_2 = \frac{k_3}{k_2 + k_3} q_5}$$

Output Equ: $e_5 = e_1 + e_4 = k_1 q_1 + k_2 q_2$

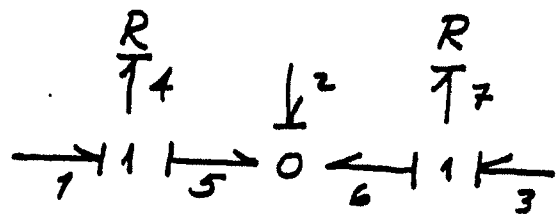
$$e_5 = k_1 q_5 + \frac{k_2 k_3}{k_2 + k_3} q_5$$

$$e_5 = \underbrace{\left[k_1 + \frac{k_2 k_3}{k_2 + k_3} \right]}_{k_{eq}} q_5$$

7-13

$$K = \begin{bmatrix} R_4 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & R_7 \end{bmatrix}$$

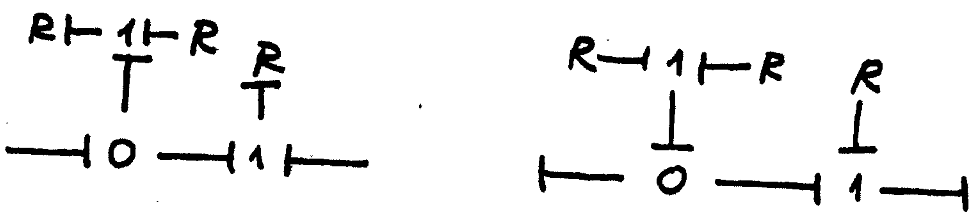
Casimir form



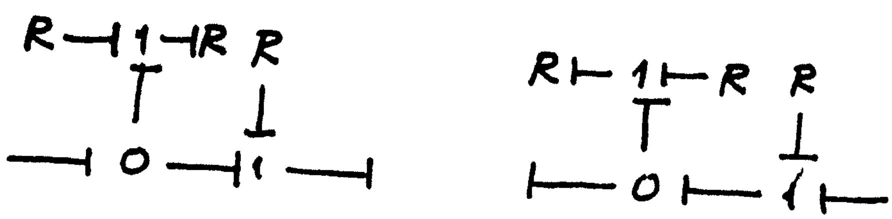
$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1/R_4 & -1/R_4 & 0 \\ -1/R_4 & 1/R_4 + 1/R_7 & -1/R_7 \\ 0 & -1/R_7 & 1/R_7 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Onsager form

7-6



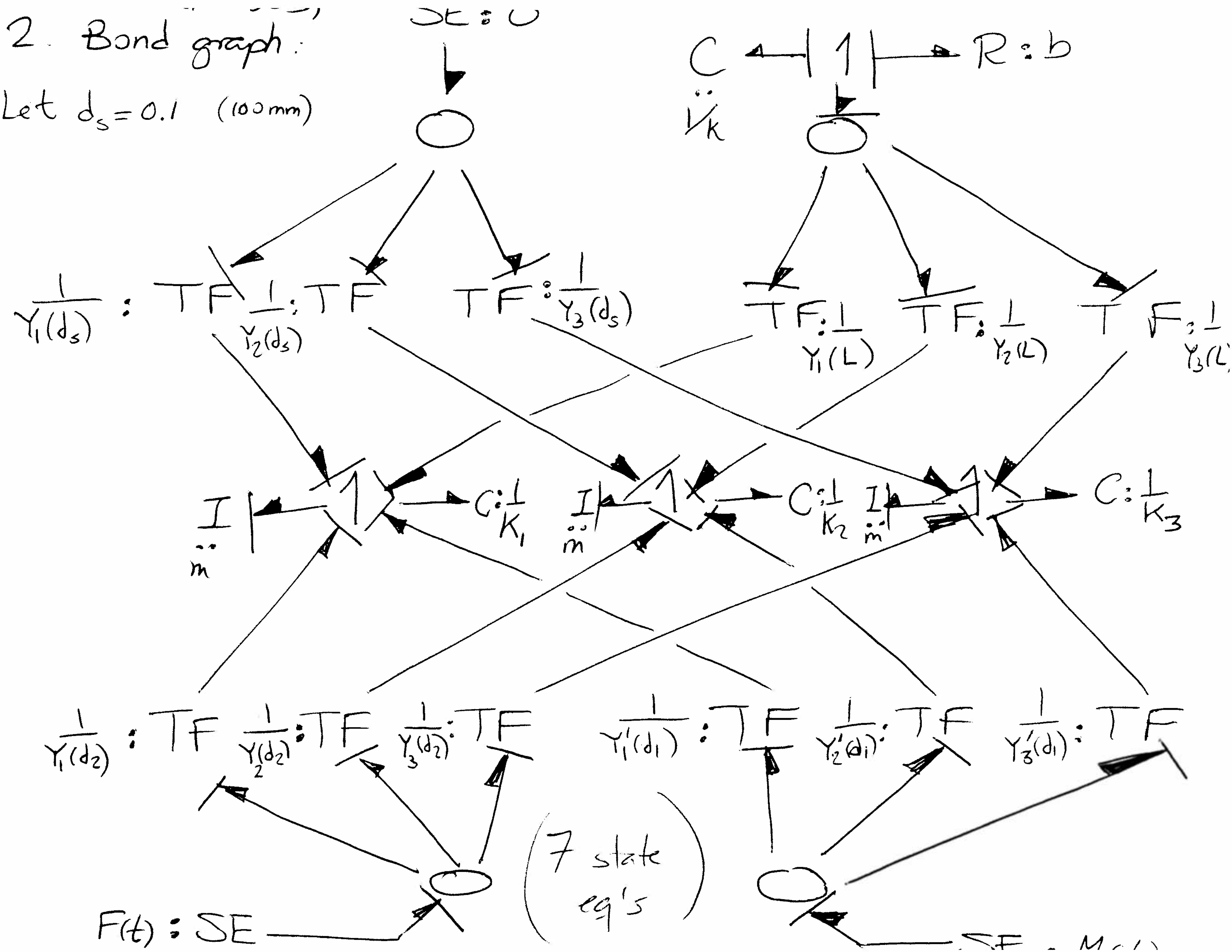
Onsager Forms



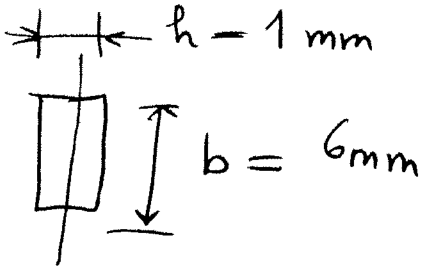
Casimir Forms

2. Bond graph:

Let $d_s = 0.1$ (100 mm)



Physical parameters :



$$A = (6\times10^{-3})(1\times10^{-3})\ m^2$$

$$A = 6\times10^{-6}\ m^2$$

$$I = \frac{1}{12} bh^3 = \frac{1}{12}(6\times10^{-3})(1\times10^{-3})^3$$

$$I = 0.5\times10^{-12}\ m^4 = 5\times10^{-13}\ m^4$$

$E = 210\times10^9\ N/m^2$, $\rho = 7850\ kg/m^3$

$L = 0.3\ m$

Frequencies : $$\omega_n^2 = \frac{EI}{\rho A}\frac{(k_n L)^4}{L^4}$$

$$\omega_n^2 = 275.222\,(k_n L)^4$$

For $n = 1$: $k_n L = 1.8751$ (table data for clamped-free)

$\rightarrow$ $\omega_1 = 58.33$ rad/s (9.28 Hz)

For $n = 2$, $k_n L = 4.69409$

$\rightarrow$ $\omega_2 = 365.55$ rad/s (58.18 Hz)

For $n = 3$, $k_n L = 7.85476$

$\rightarrow$ $\omega_3 = 1023.5$ rad/s (162.9 Hz)

Modal mass

$$m_1 = m_2 = m_3 = \rho A L = 0.0141 \text{ kg}$$

Modal stiffnesses:

$$K_1 = m_1 \omega_1^2 = 48.075 \text{ N/m} \rightarrow C_1 = \frac{1}{K_1} = 0.0208 \frac{m}{N}$$

$$K_2 = m_2 \omega_2^2 = 1888.1 \text{ N/m} \rightarrow C_2 = \frac{1}{K_2} = 5.29\times10^{-4} \text{ m/N}$$

$$K_3 = m_3 \omega_3^2 = 1.4803\times10^{4} \text{ N/m} \rightarrow C_3 = \frac{1}{K_3} = 6.75\times10^{-[illegible]} \text{ m/N}$$

Mode shape function evaluations:

distance x	x/L	$Y_1(x)$	$Y_2(x)$	$Y_3(x)$	$Y_1'(x)$	$Y_2'(x)$	$Y_3'(x)$
d_1	0.666	—	—	—	~~6.7191~~ ~~8.5241~~ 8.7262	−19.73	−7.82
d_2	0.5	0.67905	1.42733	0.0394	—	—	—
$l_s = 100$ mm	0.333	0.3312	1.1785	1.4438	—	—	—
L	1	2	−2	2	—	—	—

(Y_1–Y_2: From table; Y_3: From formula)

The transformer moduli are the reciprocal of the Y′s