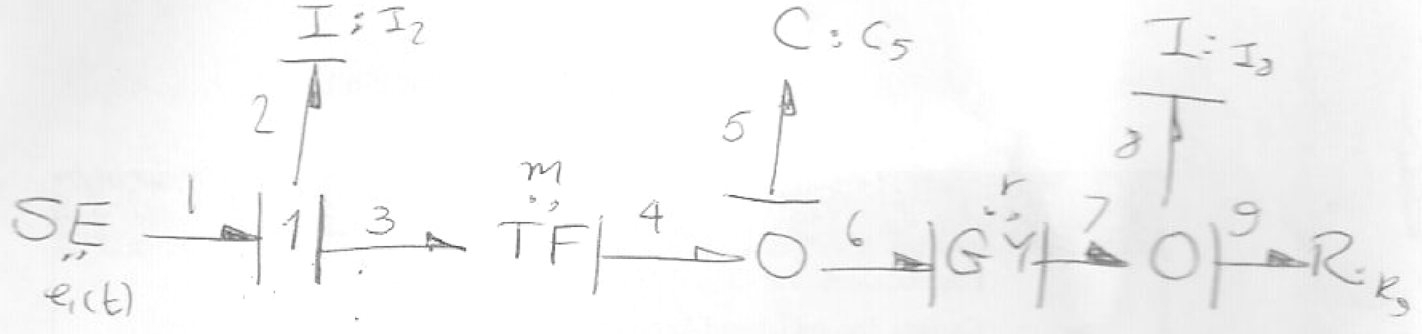
MCE403/503 2nd MIDTERM SOLUTION: FALL 2010

1)



$$e_1 = e_1(t) \qquad e_3 = m e_4 \qquad f_6 = \frac{e_7}{r} \qquad f_8 = \frac{p_8}{I_8}$$

$$f_1 = f_2 = f_3 \qquad f_4 = m f_3 \qquad f_7 = \frac{e_6}{r} \qquad e_9 = R_9 f_9$$

$$e_2 = e_1 - e_3 \qquad e_4 = e_5 = e_6 \qquad e_7 = e_8 = e_9$$

$$f_2 = \frac{p_2}{I_2} \qquad f_5 = f_4 - f_6 \qquad f_9 = f_7 - f_8$$

$$e_5 = \frac{q_5}{C_5}$$

3 states: p_2, q_5, p_8

$$\dot{p}_2 = e_2 = e_1 - e_3 = e_1(t) - m e_4 = e_1(t) - m e_5 = \boxed{e_1(t) - m\frac{q_5}{C_5}}$$

$$\dot{q}_5 = f_5 = f_4 - f_6 = m f_3 - \frac{e_7}{r} = m f_2 - \frac{e_9}{r} = \frac{m p_2}{I_2} - \frac{R_9}{r} f_9$$

$$\dot{q}_5 = \frac{m p_2}{I_2} - \frac{R_9}{r}(f_7 - f_8) = \frac{m p_2}{I_2} - \frac{R_9}{r}\left(\frac{\overset{=e_5}{e_6}}{r} - \frac{p_8}{I_8}\right)$$

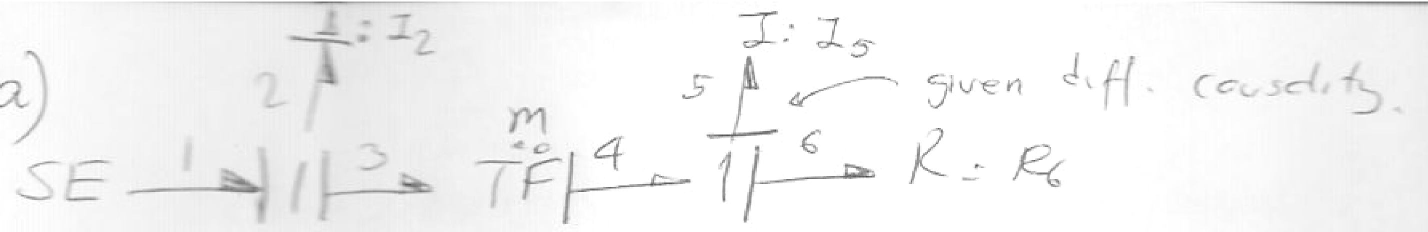
$$\boxed{\dot{q}_5 = \frac{m p_2}{I_2} - \frac{R_9}{r}\left(\frac{q_5}{r C_5} - \frac{p_8}{I_8}\right)}$$

$$\dot{p}_8 = e_8 = e_9 = R_9 f_9 = R_9(f_7 - f_8) = \boxed{R_9\left(\frac{q_5}{r C_5} - \frac{p_8}{I_8}\right)}$$

$$f_1 = f_2 = \frac{p_2}{I_2}$$

$$e_9 = R_9 f_9 = R_9\left(\frac{q_5}{r C_5} - \frac{p_8}{I_8}\right)$$

2a)



1 state: p_2.

$$\dot{p}_2 = e_2 = e_1(t) - e_3 = e_1(t) - m e_4 = e_1(t) - m(e_5 + e_6)$$

$$\dot{p}_2 = e_1(t) - m(\dot{p}_5 + R_6 f_6) = e_1(t) - m(\dot{p}_5 + R_6 \underbrace{f_6}_{= f_4})$$

$$\dot{p}_2 = e_1(t) - m(\dot{p}_5 + R_6 m \underbrace{f_3}_{= f_2}) = e_1(t) - m\left(\dot{p}_5 + R_6 m \frac{p_2}{I_2}\right)$$

link to $\dot{p}_2$

$$f_5 = f_4 = m f_3 = m f_2$$

Then $$p_5 = I_5 f_5 = I_5 m f_2 = I_5 m \frac{p_2}{I_2}$$

$$\dot{p}_5 = \frac{m I_5}{I_2} \dot{p}_2$$

Resuming main derivation:

$$\dot{p}_2 = e_1(t) - m\left(\frac{m I_5}{I_2}\dot{p}_2 + \frac{m R_6}{I_2} p_2\right)$$

$$\left[1 + \frac{m^2 I_5}{I_2}\right]\dot{p}_2 = e_1(t) - \frac{m^2 R_6}{I_2} p_2$$

$$\dot{p}_2 = \frac{1}{\left[1 + \frac{m^2 I_5}{I_2}\right]}\left\{e_1(t) - \frac{m^2 R_6}{I_2} p_2\right\}$$

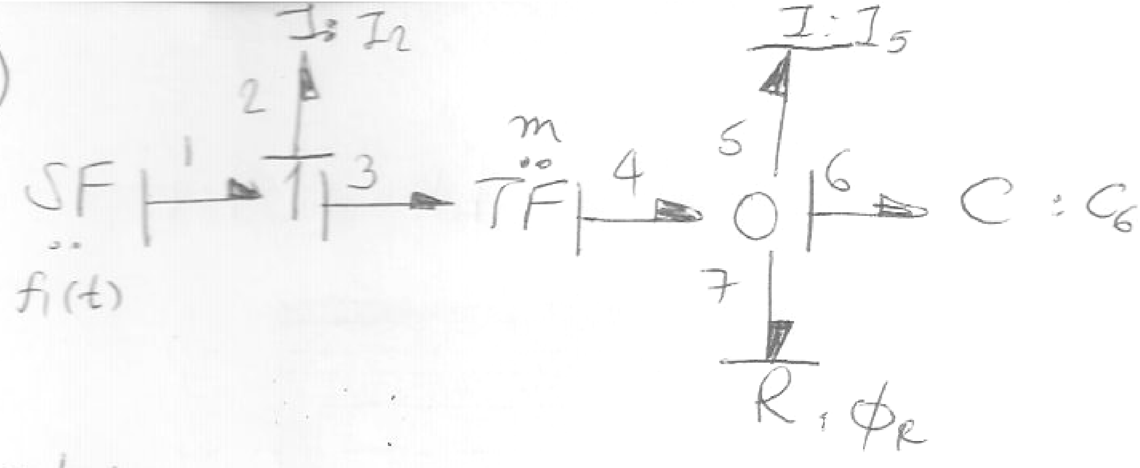
In matrix form:

$$\begin{bmatrix} \dot{p}_2 \\ \dot{q}_5 \\ \dot{p}_8 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{m}{C_5} & 0 \\ \frac{m}{I_2} & -\frac{R_9}{r^2 C_5} & \frac{R_9}{r I_8} \\ 0 & \frac{R_9}{r C_5} & -\frac{R_9}{I_8} \end{bmatrix}}_{A} \begin{bmatrix} p_2 \\ q_5 \\ p_8 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{B} e_1(t)$$

$$\begin{bmatrix} f_1 \\ e_9 \end{bmatrix} = \underbrace{\begin{bmatrix} 1/I_2 & 0 & 0 \\ 0 & \frac{R_9}{r C_5} & -\frac{R_9}{I_8} \end{bmatrix}}_{C} \begin{bmatrix} p_2 \\ q_5 \\ p_8 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} e_1(t)$$

X

2b)



States: q_6, p_5

$$\dot{p}_5 = e_5 = e_6 = \frac{q_6}{C_6}$$

$$\dot{q}_6 = f_6 = f_4 - f_5 - f_7 = m f_3 - \frac{p_5}{I_5} - \phi_R^{-1}(\underset{e_6}{\underset{\parallel}{e_7}})$$

$$\dot{q}_6 = m f_1(t) - \frac{p_5}{I_5} - \phi_R^{-1}\left(\frac{q_6}{C_6}\right)$$

$\phi_R(f)$ is known:

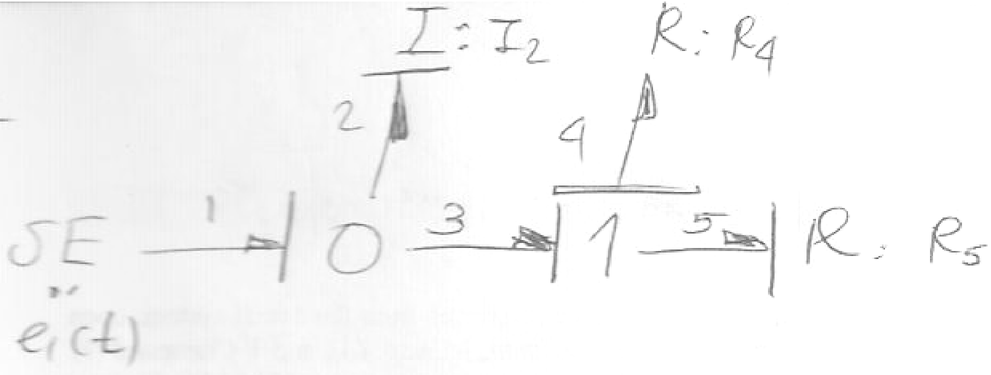
$$e = \phi_R(f) = 2f + 3$$

$$\rightarrow \quad f = \phi_R^{-1}(e) = \frac{e-3}{2}$$

So

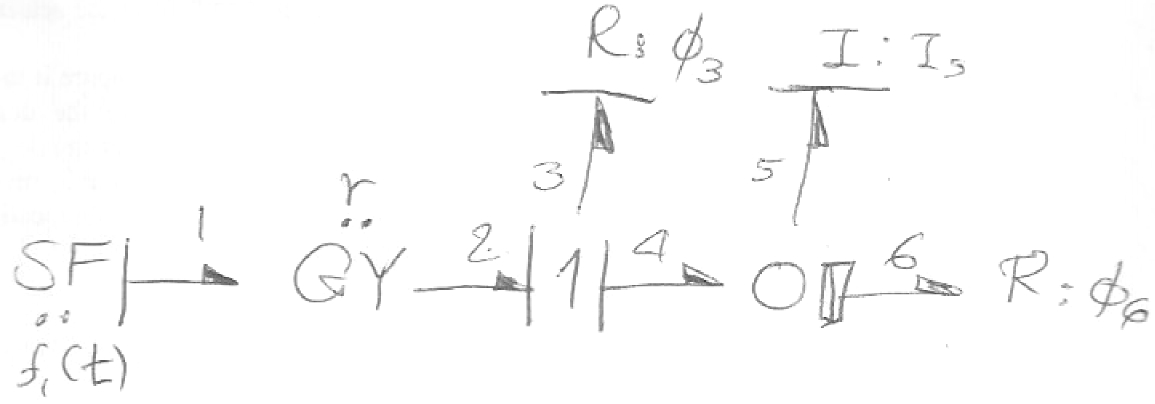
$$\dot{q}_6 = m f_1(t) - \frac{p_5}{I_5} - \left[\frac{\frac{q_6}{C_6} - 3}{2}\right]$$

3a.



$$\dot{p}_2 = e_2 = e_1(t)$$

3b)



$\dot{p}_5 = e_5 = e_6$ $\Big\}$ e_6 is involved in an algebraic loop:

$$e_6 = e_4 = e_2 - e_3 = r f_1 - \phi_3(f_3) = r f_1(t) - \phi_3(f_4)$$

$$e_6 = r f_1(t) - \phi_3(f_5 + f_6) = r f_1(t) - \phi_3\left(\frac{p_5}{I_5} + \phi_6^{-1}(e_6)\right)$$

DAE system:

$$\begin{cases} \dot{p}_5 = e_6 \\ e_6 = r f_1(t) - \phi_3\left(\frac{p_5}{I_5} + \phi_6^{-1}(e_6)\right) \end{cases}$$