MCE503: Modeling and Simulation of Mechatronic System

Lecture 10: Distributed Parameter Systems

Reading: KMR Chapter 10

Cleveland State University

Mechanical Engineering
Hanz Richter, PhD

Preliminaries

- Physical system parameters (mass, resistance, thermal conductivity...) can be considered to be lumped (concentrated) at specific locations or spatially *distributed* .
- Examples: The mass in a pendulum is pretty much concetrated in the bob. The mass of the thin membrane used in a musical drum is distributed.
- These are extreme cases, where the decision to use lumped or distrbuted parameters is obvious.
- Vibrating beams are fundamentally distributed parameter systems, however lumped models are also used.
- Some problems can hardly be explained without using the distributed parameter model: thermal diffusivity effects.

Distributed Parameters and PDEs

- Lumped parameter modeling leads to ordinary differential equations (ODE) (what we've done so far)
- Distributed parameter modeling leads to partial differential equations (PDE).
- Example: Heat equation:

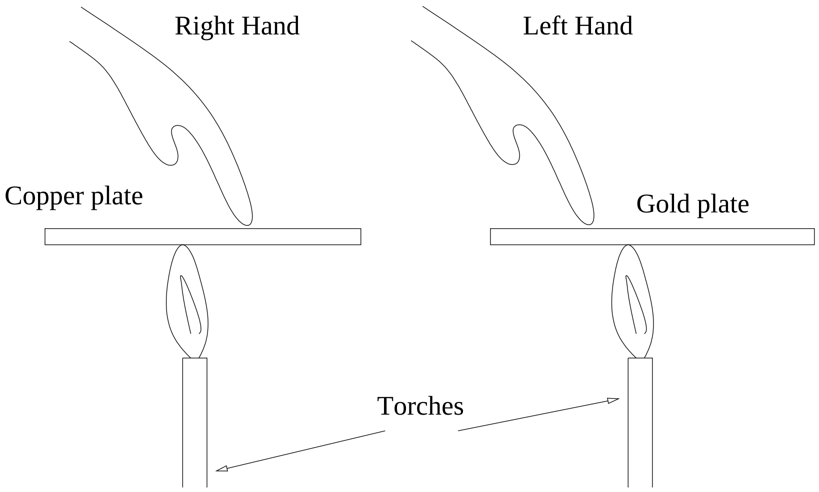
$$\frac{\partial T}{\partial t} = \frac{H}{C_p} + \frac{1}{\rho Cp}\nabla.(k\nabla T)$$

  where k is the thermal conductivity, C_p is the mass heat capacity and ρ is the density.
- If the thermal conductivity is independent of spatial location we define $\kappa = \frac{k}{\rho C_p}$ as the thermal diffusivity and the equation becomes

$$\frac{\partial T}{\partial t} = \frac{H}{C_p} + \kappa\nabla^2 T$$

A Diffusivity Effect

- Conductivity of Copper = 401 W/m/°K. Conductivity of Gold = 317 W/m/°K (at 300 °K) Which hand will you remove first?



- Diffusivity of Copper = 113 mm^2/s . Diffusivity of Gold = 127 mm^2/s

More examples

- The wave equation (used for longitudinal vibrations of a beam)

$$E\frac{\partial^2\psi}{\partial x^2} = \rho\frac{\partial^2\psi}{\partial t^2}$$

- The Navier-Stokes equations for fluids.
- In our case, we'll limit attention to transversal and longitudinal vibrations of elastic beams.
- Elastic beams are very relevant in Mechatronics applications, in particular for recent small-scale devices (MEMS and NEMS).
- Atomic Force Microscopy (AFM) relies on controlling the vibrations of a cantilever beam to a high degree of accuracy.
- Several designs of micro-manipulators and data storage devices (IBM's millipede) are also based on arrays of cantilevers.

The AFM