

Lecture 2: Introduction to Bond Graph Modeling

Reading: KMR Chapter 1 and handout.

Cleveland State University

Mechanical Engineering
Hanz Richter, PhD

MCE503 – p.1/14

Motivating discussion

1. Formal analogies across domains: capacitor, tank, spring
2. Repetitive algebraic manipulations when deriving equations

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Motivating discussion

3. Need for systematic approach

4. Possibility of mechanizing the derivation of equations

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Automated (mechanized) modeling

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- Bond graphs allow *computers* to generate system equations in symbolic form (ultimate reduction to manipulation of the symbols 0 and 1)

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- Model analysis can be performed with the bond graph, without deriving equations.

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- Bond graphs are also used in non-energetic domains: economics, social systems.

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- Please refer to KMR Tables 2.1 thru 2.5. Can you think of a pair of effort and flow variables not belonging to any of those 4 domains?

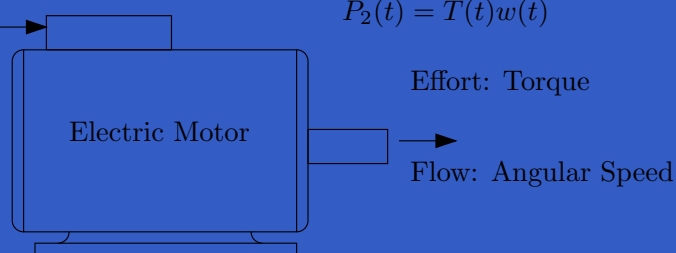
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Example of a multiport

Effort: Voltage

Flow: Current

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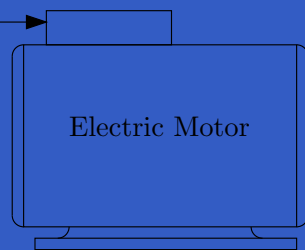
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Effort: Torque

Flow: Angular Speed

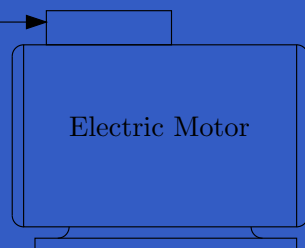
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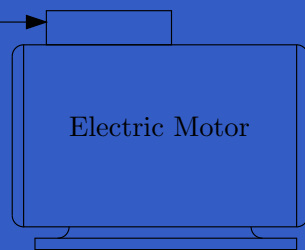
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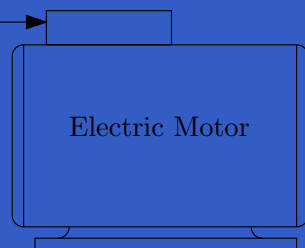
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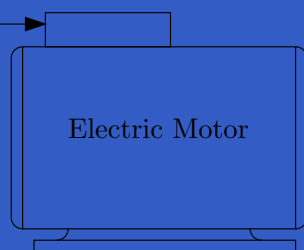
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- We also have to decide about signs for the effort and flow variables.

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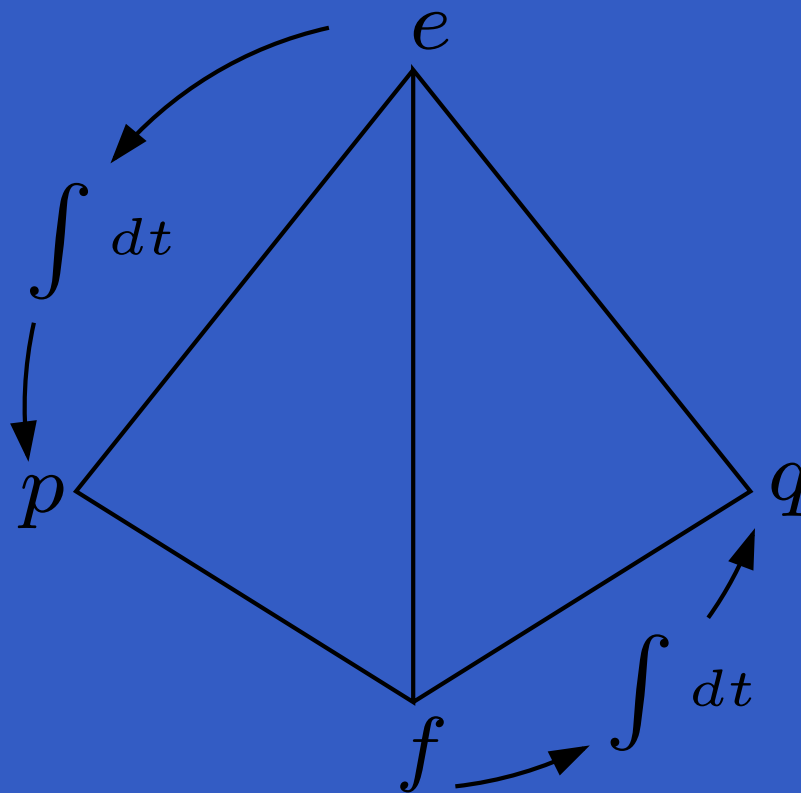
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- Remember the definitions by taking the case of mechanics: flow is velocity, effort is force. Displacement better be the integral of velocity (a flow). Remember also that force is the derivative of momentum. So displacement is the integral of flow and momentum is the integral of force.

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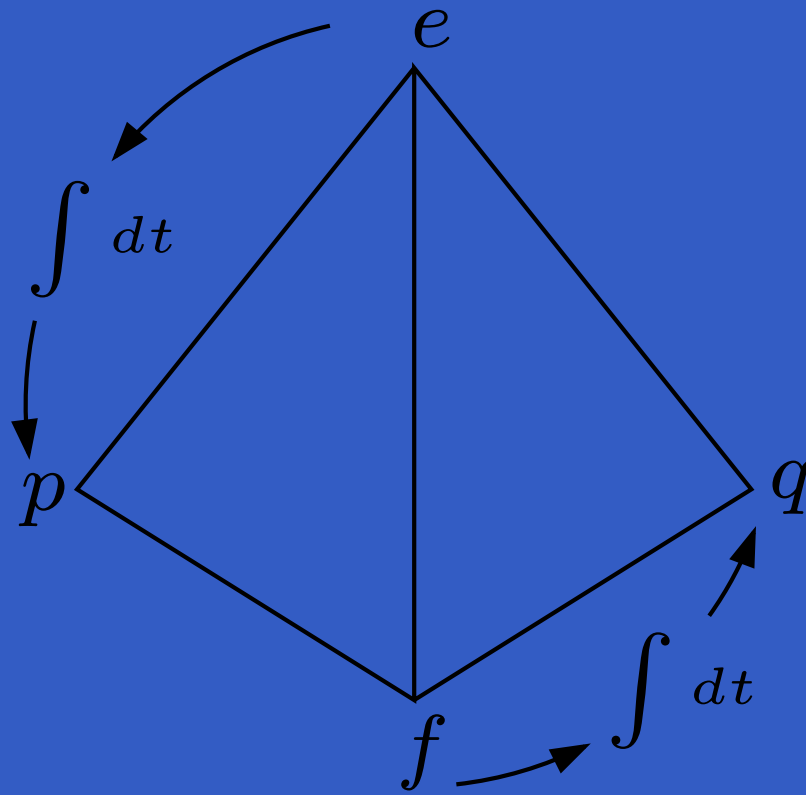
Tetrahedron of state



- Eat Pizza, Feel Queasy

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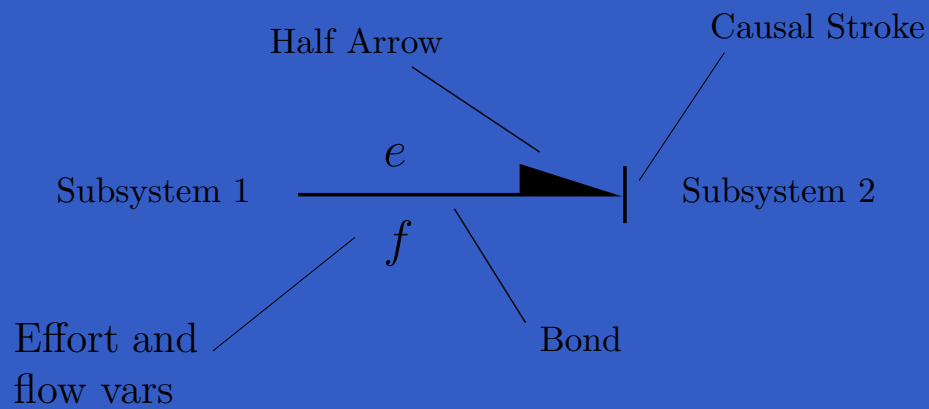
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■ Eat Pizza, Feel Queasy

■ Counterclockwise, integrate

Graphical format for bonds



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- More about signs as we go along.

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- Why can't both e and f be simultaneously prescribed?

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Exercises: Word bond graphs

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