

Lecture 3: Basic Component Models - Part I

Reading: KMR Chapter 3

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Mechanical Engineering

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Overview

- A set of building blocks is introduced: 1-port capacitor, resistor and inertia, 2-port transformer and gyrator and 0- and 1- port junctions.
- These multiports can be used to represent models from various energy domains.
- In many cases, the multiports directly represent idealized physical devices (resistors, masses, etc.)
- In some cases, the multiports do not correspond to a particular component of the real system, but still serve to model system behavior. Example: using a network of springs to model a vibrating beam.

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The 1-port resistor

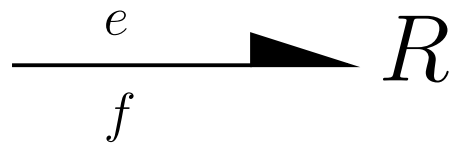
- The 1-port resistor relates e and f through a static function:

$$e = \Phi_R(f), \quad f = \Phi_R^{-1}(e)$$

- The constitutive relation Φ_R can be linear or nonlinear.
- Linear form: $e = Rf$, where R is the *resistance*, $1/R$ is the *conductance*.
- The 1-port resistor models electrical resistors, dashpots (mechanical dampers) and pressure drops in fluid lines.
- Resistors usually dissipate energy (passive resistors), but some active elements exist that behave as resistors supplying power.
- The resistor is dissipative if Φ_R lies in the first and third quadrants of the e, f plane.

Notes

1-port resistor and physical elements



	General Relation	Linear Relation	SI Units, Linear Case
Generalized variables	$e = \Phi_R(f)$	$e = Rf$	$R = e/f$
Mechanical translation	$F = \Phi(V)$	$F = bV$	$b : \text{N-s/m}$
Mechanical rotation	$\tau = \Phi(w)$	$\tau = cw$	$c : \text{N-m-s}$
Hydraulic systems	$P = \Phi(Q)$	$P = RQ$	$R : \text{N-s/m}^5$
Electrical systems	$e = \Phi(i)$	$e = Ri$	$R : \text{V/A} = \Omega$

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The 1-port capacitor

- The 1-port capacitor relates e and q through a static function:

$$q = \Phi_C(e), \quad e = \Phi_C^{-1}(q)$$

- The constitutive relation Φ_C can be linear or nonlinear.
- Linear form: $q = Ce$, where C is the *capacitance*.
- The 1-port capacitor models electrical capacitors, mechanical springs (linear and torsion), and liquid tanks and gas accumulators. In mechanical systems, C is the *compliance* (reciprocal of spring constant).
- Capacitors store and release energy with no loss (see KMR, notes).

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Notes

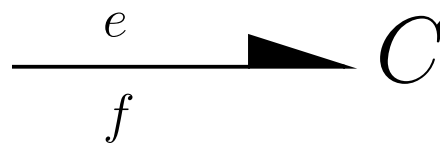
Write the energy

$$E(t) = \int_0^t e(t)f(t)dt$$

as a function of the displacement q . Use the resulting expression and a graph of Φ_C^{-1} in showing how energy is conserved by the 1-port capacitor.

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1-port capacitor and physical elements



	General Relation	Linear Relation	SI Units, Linear Case
Generalized variables	$q = \Phi_C(e)$	$q = Ce$	$C = q/e$
Mechanical translation	$X = \Phi(F)$	$X = CF$	$C : \text{m/N}$
Mechanical rotation	$\theta = \Phi(\tau)$	$\theta = C\tau$	$C : \text{rad/N-m}$
Hydraulic systems	$V = \Phi(P)$	$V = CP$	$C : \text{m}^5/\text{N}$
Electrical systems	$q = \Phi(e)$	$q = Ce$	$C : \text{A-s/V}=\text{farad}$

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The 1-port inertia

- The 1-port inertia relates p and f through a static function:

$$p = \Phi_I(f), \quad f = \Phi_I^{-1}(p)$$

- The constitutive relation Φ_I can be linear or nonlinear.
- Linear form: $p = If$, where I is the *inertia parameter*.
- The 1-port inertia models electrical inductance and mass and inertia effects in mechanical and fluid systems.
- Inertias store and release energy with no loss (see KMR, notes).

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Notes

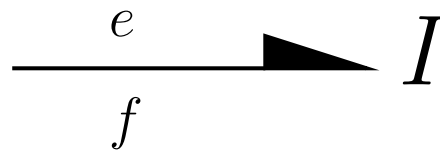
Write the energy

$$E(t) = \int_0^t e(t)f(t)dt$$

as a function of the momentum p . Use the resulting expression and a graph of Φ_I^{-1} in showing how energy is conserved by the 1-port inertia.

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1-port inertia and physical elements



	General Relation	Linear Relation	SI Units, Linear Case
Generalized variables	$p = \Phi_I(f)$	$p = If$	$I = p/f$
Mechanical translation	$p = \Phi(V)$	$p = mV$	$m : \text{N}\cdot\text{s}^2/\text{m}=\text{kg}$
Mechanical rotation	$p_\tau = \Phi(w)$	$p_\tau = Jw$	$J : \text{N}\cdot\text{m}\cdot\text{s}^2$
Hydraulic systems	$p_P = \Phi(Q)$	$p_P = IQ$	$I : \text{N}\cdot\text{s}^2/\text{m}^5$
Electrical systems	$\lambda = \Phi(i)$	$\lambda = Li$	$L : \text{V}\cdot\text{s}/\text{A}=\text{henry}$

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Notes

- Regardless of the variables involved in the static relationships, bonds are always labeled with e and f .
- The energy associated with the capacitor is usually called *potential energy*.
- The energy associated with the inertia is usually called *kinetic energy*.
- In electrical systems, the terms are *electric* and *magnetic* energy, respectively.

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Examples

KMR 3-1, 3-4, 3-17, 3-21.