Basic 2-ports

- Only 2 types of 2-port and 2 types of 3-port, together with the 1-ports already introduced, suffice to model a large variety of engineering multiports.
- These are the **transformer** and the **gyrator**.
- Both are thru-power elements having 100% energy efficiency.
- If $TP$ stands for “two-port”, the following must be verified:

$$e_1(t)f_1(t) = e_2(t)f_2(t)$$
The transformer

\[ \frac{e_1}{f_1} \xrightarrow{T \! F\!} \frac{e_2}{f_2} \]

- The transformer meets power equality by the constitutive laws: (check)
  \[ e_1 = me_2, \; mf_1 = f_2 \]
- Parameter \( m \) is the transformer modulus, a dimensionless quantity.
- The transformer partially models gearboxes, hydraulic rams, mechanical levers and electrical transformers.
- Question: For each of the above real systems, which deviations from ideal 2-port behavior are present?
- Which 1-ports must be incorporated to complete the models?

The gyrator \( \rightarrow GY \rightarrow \)

- The gyrator meets power equality by the constitutive laws: (check)
  \[ e_1 = rf_2, \; rf_1 = e_2 \]
- Parameter \( r \) is the gyrator modulus, a quantity with dimensions of effort per unit flow.
- The gyrator can be used to model gyroscopes, Hall effect devices, and electromechanical conversion in motors and generators.
- The gyrator essentially swaps the roles of effort and flow.
Show that two gyrators in series are equivalent to a transformer and give $m$ in terms of $r$.

Show that two transformers in series are equivalent to another transformer.

Prove the following bond graph equivalencies:

\[ \rightarrow GY \rightarrow I \equiv \rightarrow C \]

\[ \rightarrow GY \rightarrow C \equiv \rightarrow I \]

Modulated transformers and gyrators can also be introduced to account for adjustable moduli.

\[ \begin{align*}
\downarrow m \\
\overset{e_1}{f_1} & \rightarrow MTF \rightarrow \overset{e_2}{f_2} \\
\downarrow r \\
\overset{e_1}{f_1} & \rightarrow MGY \rightarrow \overset{e_2}{f_2}
\end{align*} \]  

Basic 3-port elements

1- and 2-ports may be directly cascaded. The 0- and 1-junctions allow the interconnection of 3 multiports.

These junctions can be either effort-sharing (0-junction) or flow-sharing (1-junction)

The common-effort junction is also called \textit{flow junction} (think of water flow, the pressure is common at a tee)

The common-flow junction is also called \textit{effort junction} (in water flow, think of flow restrictions connected in series, forming a closed loop).
Discussion on the 0-junction and power sign convention

The basic acausal, unsigned representation of the 0-junction is

\[
e_2 \mid f_2 \\
\frac{e_1}{f_1} - 0 - \frac{e_3}{f_3}
\]  

(2)

Independently of the sign convention, we must have \( e_1 = e_2 = e_3 = e \). The equation indicating the distribution of flow among the three ports does depend on the sign convention. For example, we may assume an inward power convention:

\[
e_2 \downarrow f_2 \\
\frac{e_1}{f_1} \rightarrow 0 \leftarrow \frac{e_3}{f_3}
\]  

(3)

The 0-junction only transmits power at 100% efficiency. Therefore, if, for instance, \( e > 0 \), \( f_1 > 0 \) and \( f_2 > 0 \), power should be flowing out of port 3. For this to happen we must have \( e_3 f_3 = -(e_1 f_1 + e_2 f_2) < 0 \). Due to equality of effort, we arrive to the correct flow distribution equation: \( f_1 + f_2 + f_3 = 0 \).

Discussion on the 1-junction and power sign convention

The basic acausal, unsigned representation of the 1-junction is

\[
e_2 \mid f_2 \\
\frac{e_1}{f_1} - 1 - \frac{e_3}{f_3}
\]  

(4)

Independently of the sign convention, we must have \( f_1 = f_2 = f_3 = f \). The equation indicating the distribution of effort among the three ports does depend on the sign convention. For example, we may assume an inward power convention:

\[
e_2 \downarrow f_2 \\
\frac{e_1}{f_1} \rightarrow 1 \leftarrow \frac{e_3}{f_3}
\]  

(5)

The 1-junction only transmits power at 100% efficiency. Therefore, if, for instance, \( f > 0 \), \( e_1 > 0 \) and \( e_2 > 0 \), power should be flowing out of port 3. For this to happen we must have \( e_3 f_3 = -(e_1 f_1 + e_2 f_2) < 0 \). Due to equality of flow, we arrive to the correct effort distribution equation: \( e_1 + e_2 + e_3 = 0 \).
Normally, the thru-power convention will be assumed (0-junction example shown):

\[
\begin{align*}
e_2 & \uparrow f_2 \\
e_1 & \leftarrow 0 \rightarrow e_3 \\
f_1 & \leftarrow 0 \rightarrow f_3
\end{align*}
\] (6)

Under this convention we have: \(e_1(t) = e_2(t) = e_3(t)\) and \(f_1(t) - f_2(t) - f_3(t) = 0\).

Note that generalization to \(n\)-port junctions is straightforward.

Mutilated 0- and 1-junctions will arise: \(\leftarrow 0 \rightarrow \equiv \rightarrow 0\) and the same with 1.

With opposite power directions, a 2-port 0- or 1-junction can be used to reverse the signs of effort or flow.

The flow source and the effort source

\[ S_f \rightarrow \quad \text{and} \quad S_e \rightarrow \]

The sign convention shown reflects the idea of sources providing power to a subsystem.

- The ideal flow source can provide a prescribed flow at any effort.
- The ideal effort source can provide a prescribed effort at any flow.
- What is the only possible causality for \(S_e\)?
- What is the only possible causality for \(S_f\)?
- These sources represent devices like batteries, current sources and mechanical actuators.
- A very powerful motor used in a machine tool maintains the velocity regardless of the torque arising from cutting tool - workpiece interaction. One can model the vibrations of the workpiece using a fixed-frequency excitation and an appropriate expression for the cutting force.
Discussion on allowable causalities: 1-ports

- $S_e$ and $S_f$ have only one meaningful causality: $S_e \rightarrow |$ and $S_f | \rightarrow$.

- $R$ allows any of the two possible causalities: $\rightarrow |R$ or $| \rightarrow R$. If one of $\Phi_R$ or $\Phi_R^{-1}$ is single-valued (a function) while the other is multiple-valued, the causality associated with the single-valued case is preferred.

- $C$ allows, in principle, any of the two possible causalities: $\rightarrow |C$ or $| \rightarrow C$, leading to different forms of writing the constitutive equations:

\[ e = \Phi_C^{-1} \int f dt, \text{ known as integral causality} \]

or

\[ f = \frac{d}{dt} (\Phi_C(e)), \text{ known as derivative causality} \]

- The same applies to $I$ (derive the expressions for both integral and derivative causality).

- The choice of causality will become important. Integral causality is preferrable for numerical reasons (derivatives amplify high-frequency errors (noise), integrals attenuate (filter) them).
Discussion on allowable causalities: 2-ports

- **TF** allows two possible causalities: \(\rightarrow |TF\rightarrow| \) and \(\rightarrow|TF|\leftarrow\). Show why.

- **GY** allows two possible causalities: \(\rightarrow |GY|\leftarrow\) and \(\rightarrow|TF|\leftarrow\). Show why.

- The 0- and 1- junctions have three ports, each one with two candidate causalities. Of the \(2^3\) permutations, however, only three are valid. Write down the equations and corresponding bond graph representations.