MCE503: Modeling and Simulation of Mechatronic Systems

Lecture 4: Basic Component Models - Part II

Reading: KMR Chapter 3

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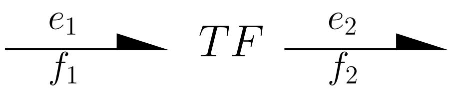
Basic 2-ports

- Only 2 types of 2-port and 2 types of 3-port, together with the 1-ports already introduced, suffice to model a large variety of engineering multiports.
- These are the <u>transformer</u> and the <u>gyrator</u>.
- Both are thru-power elements having 100% energy efficiency.
- If TP stands for “two-port”, the following must be verified:

$$e_1(t)f_1(t) = e_2(t)f_2(t)$$

The transformer



- The transformer meets power equality by the constitutive laws: (check)

$$e_1 = me_2, \; mf_1 = f_2$$

- Parameter m is the *transformer modulus*, a dimensionless quantity.
- The transformer *partially models* gearboxes, hydraulic rams, mechanical levers and electrical transformers.
- Question: For each of the above real systems, which deviations from ideal 2-port behavior are present?
- Which 1-ports must be incorporated to complete the models?

The gyrator $\rightharpoonup GY \rightharpoonup$

- The gyrator meets power equality by the constitutive laws: (check)

$$e_1 = rf_2, \; rf_1 = e_2$$

- Parameter r is the *gyrator modulus*, a quantity with dimensions of effort per unit flow.
- The gyrator *can be used to model* gyroscopes, Hall effect devices, and electromechanical conversion in motors and generators.
- The 1-port capacitor models electrical capacitors, mechanical springs (linear and torsion), and liquid tanks and gas accumulators. In mechanical systems, C is the *compliance* (reciprocal of spring constant).
- The gyrator essentially swaps the roles of effort and flow.

Notes

- Show that two gyrators in series are equivalent to a transformer and give m in terms of r.
- Show that two transformers in series are equivalent to another transformer.
- Prove the following bond graph equivalencies:

$$\rightharpoonup GY \rightharpoonup I \;\equiv \rightharpoonup C$$

$$\rightharpoonup GY \rightharpoonup C \;\equiv \rightharpoonup I$$

- Modulated transfomers and gyrators can also be introduced to account for adjustable moduli.

$$\begin{array}{c} \downarrow m \\ {}^{e_1}_{f_1} \rightharpoonup MTF \rightharpoonup {}^{e_2}_{f_2} \end{array} \qquad \begin{array}{c} \downarrow r \\ {}^{e_1}_{f_1} \rightharpoonup MGY \rightharpoonup {}^{e_2}_{f_2} \end{array} \tag{1}$$

Basic 3-port elements

- 1- and 2-ports may be directly cascaded. The 0- and 1-junctions allow the interconnection of 3 multiports.
- These junctions can be either effort-sharing (0-junction) or flow-sharing (1-junction)
- The common-effort junction is also called *flow junction* (think of water flow, the pressure is common at a tee)
- The common-flow junction is also called *effort junction* (in water flow, think of flow restrictions connected in series, forming a closed loop).

Discussion on the 0-junction and power sign convention

The basic acausal, unsigned representation of the 0-junction is

$$\begin{array}{c} e_2 \big| f_2 \\ {}^{e_1}_{f_1} - 0 - {}^{e_2}_{f_2} \end{array} \tag{2}$$

Independently of the sign convention, we must have $e_1 = e_2 = e_3 = e$. The equation indicating the distribution of flow among the three ports does depend on the sign convention. For example, we may assume an inward power convention:

$$\begin{array}{c} e_2 \downharpoonleft f_2 \\ {}^{e_1}_{f_1} \rightharpoondown 0 \leftharpoondown {}^{e_2}_{f_2} \end{array} \tag{3}$$

The 0-junction only transmits power at 100% efficiency. Therefore, if, for instance, $e > 0$, $f_1 > 0$ and $f_2 > 0$, power should be flowing *out* of port 3. For this to happen we must have $e_3 f_3 = -(e_1 f_1 + e_2 f_2) < 0$. Due to equality of effort, we arrive to the correct flow distribution equation: $f_1 + f_2 + f_3 = 0$.

Discussion on the 1-junction and power sign convention

The basic acausal, unsigned representation of the 1-junction is

$$\begin{array}{c} e_2 \big| f_2 \\ {}^{e_1}_{f_1} - 1 - {}^{e_2}_{f_2} \end{array} \tag{4}$$

Independently of the sign convention, we must have $f_1 = f_2 = f_3 = f$. The equation indicating the distribution of effort among the three ports does depend on the sign convention. For example, we may assume an inward power convention:

$$\begin{array}{c} e_2 \downharpoonleft f_2 \\ {}^{e_1}_{f_1} \rightharpoondown 1 \leftharpoondown {}^{e_2}_{f_2} \end{array} \tag{5}$$

The 1-junction only transmits power at 100% efficiency. Therefore, if, for instance, $f > 0$, $e_1 > 0$ and $e_2 > 0$, power should be flowing *out* of port 3. For this to happen we must have $e_3 f_3 = -(e_1 f_1 + e_2 f_2) < 0$. Due to equality of flow, we arrive to the correct effort distribution equation: $e_1 + e_2 + e_3 = 0$.

Notes

- Normally, the thru-power convention will be assumed (0-junction example shown):

$$\begin{array}{c} e_2 \upharpoonleft f_2 \\ {}^{e_1}_{f_1} \rightharpoonup 0 \rightharpoonup^{e_2}_{f_2} \end{array} \quad (6)$$

  Under this convention we have: $e_1(t) = e_2(t) = e_3(t)$ and $f_1(t) - f_2(t) - f_3(t) = 0$.

- Note that generalization to $n-$ port junctions is straightforward.
- Mutilated 0- and 1-junctions will arise: $\rightharpoonup 0 \rightharpoonup \equiv \rightharpoonup$ and the same with 1.
- With opposite power directions, a 2-port 0- or 1-junction can be used to reverse the signs of effort or flow.

The fbw source and the effort source

$$S_f \rightharpoonup \quad \text{and} \quad S_e \rightharpoonup$$

The sign convention shown reflects the idea of sources providing power to a subsystem.

- The ideal flow source can provide a prescribed flow at any effort.
- The ideal effort source can provide a prescribed effort at any flow.
- What is the only possible causality for S_e?
- What is the only possible causality for S_f?
- These sources represent devices like batteries, current sources and mechanical actuators.
- A very powerful motor used in a machine tool maintains the velocity regardless of the torque arising from cutting tool - workpiece interaction. One can model the vibrations of the workpiece using a fi xed-frequency excitation and an appropriate expression for the cutting force.

Examples: KMR Fig. 3.12

Discussion on allowable causalities: 1-ports

- S_e and S_f have only one meaningful causality: $S_e \rightharpoonup |$ and $S_f| \rightharpoonup$.
- R allows any of the two possible causalities: $\rightharpoonup |R$ or $| \rightharpoonup R$. If one of Φ_R or Φ_R^{-1} is single-valued (a function) while the other is multiple-valued, the causality associated with the single-valued case is preferred.
- C allows, in principle, any of the two possible causalities: $\rightharpoonup |C$ or $| \rightharpoonup C$, leading to different forms of writing the constitutive equations:

$$e = \Phi_C^{-1} \int f dt, \text{ known as integral causality}$$

or

$$f = \frac{d}{dt}\left(\Phi_C(e)\right), \text{ known as derivative causality}$$

- The same applies to I (derive the expressions for both integral and derivative causality).
- The choice of causality will become important. Integral causality is preferrable for numerical reasons (derivatives amplify high-frequency errors (noise), integrals attenuate (filter) them).

Discussion on allowable causalities: 2-ports

- TF allows two possible causalities: $\rightharpoonup |TF \rightharpoonup |$ and $| \rightharpoonup TF| \rightharpoonup$. Show why.
- GY allows two possible causalities: $\rightharpoonup |GY| \rightharpoonup$ and $| \rightharpoonup TF \rightharpoonup |$. Show why.
- The 0- and 1- junctions have three ports, each one with two candidate causalities. Of the 2^3 permutations, however, only three are valid. Write down the equations and corresponding bond graph representations.