MCE503: Modeling and Simulation of Mechatronic Systen

Lecture 8: Fields and Junction Structures - Part II

Reading: KMR Chapter 7

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Resistive Fields

- A resistive field is a multiport generalization of the basic one-port R.
- An R-field has a constitutive law relating the efforts and flows by means of algebraic functions.
- Note that simple 0- and 1-junctions are included in this definition. For instance,

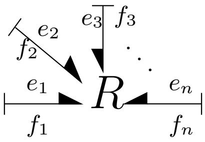
$$e_1 = e_2 = ...e_n$$

$$f_1 = \pm f_2 \pm f_3 ... \pm f_n$$

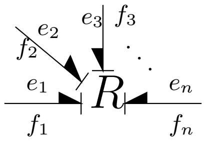
  is indeed a set of algebraic equations relating the efforts and flows.
- Likewise, sources can be contained in an implicit R-field.
- Implicit and explicit R-fields are possible. In the implicit case, any interconnection of sources, resistors, transformers, gyrators and 0- and 1-junctions can be represented as an R-field between defined ports.

Causality in R-Fields

- Causality can be fully integral, fully derivative, or a mix.
- The symbol for *resistance* causality is



while for *conductance* causality it is



Many forms of mixed causality are also possible. The only limitation is the existence of single-valued functions returning the value of the output.

Onsager Reciprocal Form

- Implicit, linear R-fields contaning *no gyrators or sources* will result in symmetric constitutive matrices.
- The resulting symmetric conductance and resistance forms are called *Onsager* forms, in analogy to similar relations found in irreversible thermodynamics.
- Any implicit R-field containing only 0- and 1-junctions, transformers and linear 1-port resistances will obey the Onsager form.
- If a given constitutive law in Onsager form is altered to have mixed causality, anti-symmetric terms appear. This form is called *Casimir* form.
- If gyrators are present in the system, the R-field may not be representable in either Onsager or Casimir form.

Examples

KMR Fig. 7.16 (4th ed.)

Energy dissipation in R-fields

- A general R-field can dissipate, conserve or create power (with active elements).
- In the linear case, we can check if the field is dissipative by looking at the *sign-definiteness* of the constitutive matrix.
- The power dissipation is $P = e^T f$. For the conductance form we have

$$f = Ke$$

where $K = R^{-1}$. Therefore

$$P = e^T Ke$$

- *The R-field is dissipative if K is positive-definite*, that is, $x^T Kx > 0$ for all $x \neq 0$ and $x^T Kx = 0$ if $x = 0$.
- The sign-definiteness of a matrix is checked with its eigenvalues. If all eigenvalues are positive, then the matrix is positive-definite.
- Alternatively, check all sub-determinants along the diagonal. They have to be all positive.

Junction Structures

- A junction structure contains only 0-, 1-. TF and GY. Therefore it is energy-conserving.
- As KMR says, they are energy switchyards whose role is to enforce constraints among parts of the system.
- As seen earlier, junction structures will fall in the category of R-fields, with the restriction that power is conserved.
- Like R-fields with mixed causality, the Onsager or Casimir forms may not appear. Moreover, some mixed causalities will not occur.
- There is no special symbol for a junction structure. We can represent it with the R symbol, or leave it in implicit form.

Example

KMR 7.21 (4th ed)

Multiport Transformers

- A multiport transformer generalizes the notions of transformer and gyrator in a unified way.
- A multiiport transformer is a special kind of junction structure with constitutive law of the form

$$e_1 = Me_2$$

$$M^T f1 = f2$$

  where e_1, e_2, f_1 and f_2 are the vectors of efforts and flows at both sides of the transformer, and M is the modulus matrix.
- For a system to behave implicitly as a multiport transformer, the modulus matrix has to satisfy the transpose form restriction shown above.
- The symbol for a 2-dimensional transformer is

Example

Study the rectangular-to-polar coordinate transformation example in KMR, Fig. 7.24 (4th ed). Then extend the results to the rectangular-to-spherical coordinate transformation