MCE 503: Modeling and Simulation of Mechatronic Systems
Case Study: Electric Vehicle with Regenerative Braking and CVT

October 9, 2006

1 Background

In this exercise, we develop a bond graph model for an electric vehicle. As shown in Fig. 1, a battery provides the necessary energy to run the DC motor. The motor is connected to a *continuously-variable transmission* or CVT, capable of adjusting the gear ratio to any value within a range. The CVT transmits torque to the wheels, which are fitted with disc brakes. Since the DC machine can operate reversibly (motor or generator), the kinetic energy from the vehicle can be converted into electrical energy and used to recharge the battery during braking. Since the brakes and the friction on the bearings dissipate energy, the battery will eventually need to be recharged.

1.1 The continuously-variable transmission

CVT has been available for quite some time. Car models like the Audi Quattro incorporate the device as a way to optimize performance and fuel economy. Among the various CVT designs, the variable-diameter pulley is simple and easy to understand. As shown in Fig. 2 (sorry for the Japanese), a belt is installed on two pairs of conical pulleys. Any transmission ratio within two limits can be obtained by varying the separation between the pulleys.

1.2 Regenerative braking

Conventional vehicles dissipate all kinetic energy as heat during braking. Electric vehicles offer the possibility of converting the kinetic energy back to electric energy and delivering it back to the battery, resulting in an increased energy efficiency. Regenerative braking alone cannot bring a vehicle to a full stop in a reasonably short time. For this reason, conventional friction brakes are maintained.

2 Vehicle operation

Vehicle velocity is assumed to be controlled from an operator stick. The stick has a -45 to 45 degrees range of motion. When the stick is moved forward (toward 45 degrees), the gear ratio increases from the minimum to the maximum value. Therefore the vehicle moves with increasing velocity and decreasing torque at the wheels. Once the vehicle is in motion, a reversal of the stick (toward -45 degrees) results in the wheels driving the motor (downshifting), which behaves as a generator and charges the battery. If the stick is moved below the neutral position, the gear ratio remains at the minimum value and the friction brakes are applied. Fig. 1 shows the gear ratio as a function

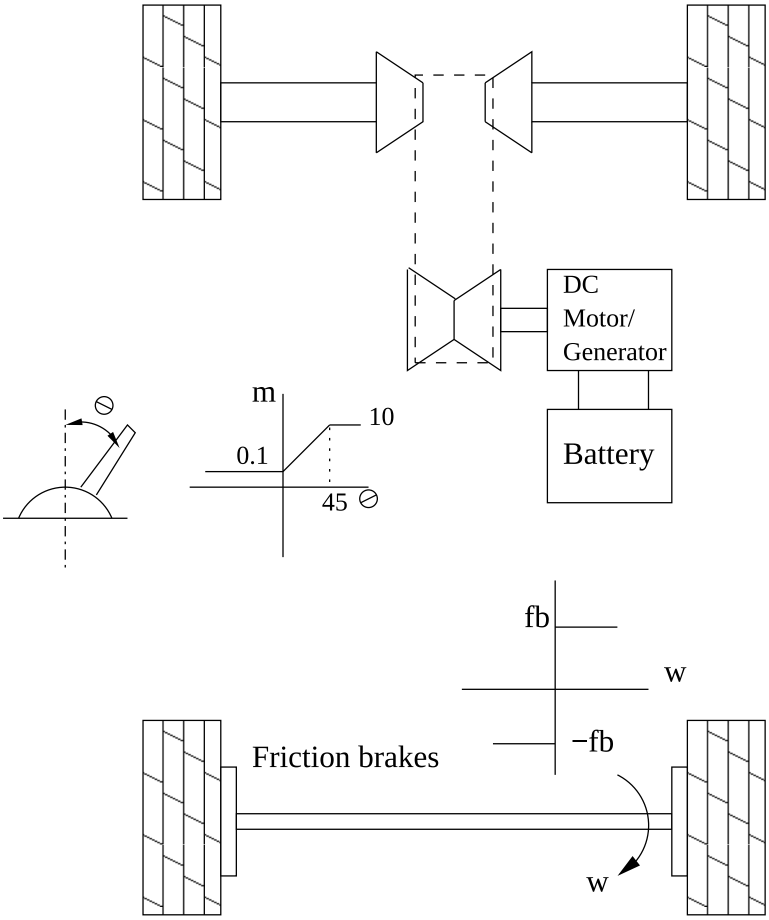

Figure 1: Electric vehicle with CVT

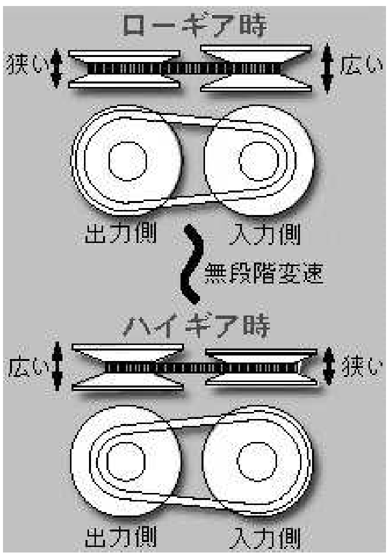

Figure 2: Schematic of the variable-diameter pulley CVT design

of θ, the stick angle. We will also consider that the connection between battery and DC machine is cutoff when $\theta = 0$.

3 Modeling guidelines

3.1 Battery

The battery will not be modeled as a source of effort. This is because we want to account for the finite charge stored in it. Instead, we model the battery as a big capacitor in series with a resistor (representing the battery's internal resistance). The capacitance can be figured out from the commercial specifications of total charge and nominal voltage. In this example we take a typical golf cart battery having 180 A-h of charge and a nominal voltage of 48 V. The internal resistance is 0.0001 Ω.

3.2 DC Machine

Model the DC machine using a gyrator. Consider the inductance in the armature circuit negligible, but do consider the armature resistance. Ignore the moment of inertia in the driving axle, including that of the motor shaft. However, do consider viscous friction.

3.3 CVT

The CVT is modeled as a modulated transformer, where the modulus is a function of the stick angle, as shown in Fig. 1.

3.4 Driven axle and brakes

Ignore the moment of inertia of the driven axle including wheels. However, do consider viscous friction in addition to the friction brakes. These are to be modeled as a nonlinear resistor. When $\theta < 0$ (brakes applied), use the standard discontinuous model for dry friction. But when $\theta >= 0$ (brakes released) use zero resistance.

3.5 Vehicle body

Assume that the wheels do not slip. Use a transformer to convert torque and angular velocity to force and mass.

4 Bond Graph and Equation Derivation

The bond graph shown in Fig. 3 is obtained by inspection. Note that the battery and armature resistances are in series. Keeping them as separate elements would result in an algebraic loop (not fully-determined causality). To avoid this, the two resistances have been combined into a single element R_{bA}. Since all energy-storage elements (in this case one C and one I) have integral causality, there will be two state variables, namely q_1 and p_{11}. There are no inputs coming from sources, since we've modelled the battery as a capacitor. Only initial conditions and parametric changes (θ) determine the evolution of the system. We begin the equation derivation process by listing all constitutive equations. The equations for the 0 and 1 junctions, however, will not be explicitly

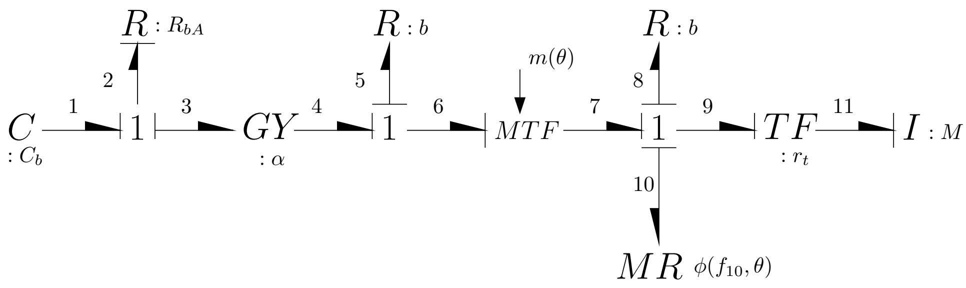

Figure 3: Completed bond graph

listed. Note that we write the equations according to causality, that is, the output variable is written on the left-hand side.

$$e_1 = \frac{q_1}{C_b} \tag{1}$$
$$f_2 = \frac{e_2}{R_{bA}} \tag{2}$$
$$e_4 = \alpha f_3 \tag{3}$$
$$e_3 = \alpha f_4 \tag{4}$$
$$e_5 = b f_5 \tag{5}$$
$$f_6 = \frac{f_7}{m(\theta)} \tag{6}$$
$$e_7 = \frac{e_6}{m(\theta)} \tag{7}$$
$$e_8 = b f_8 \tag{8}$$
$$e_{10} = \phi(f_{10}, \theta) \tag{9}$$
$$f_9 = \frac{f_{11}}{r_t} \tag{10}$$
$$e_{11} = \frac{e_9}{r_t} \tag{11}$$
$$f_{11} = \frac{p_{11}}{M} \tag{12}$$

Now we are in a position to derive the required state derivative expressions.

$$\dot{q}_1 = f_1 = f_2 = \frac{e_2}{R_{bA}} = \frac{e_1 - e_3}{R_{bA}} = \frac{q_1}{C_b R_{bA}} - \frac{\alpha f_4}{R_{bA}} = \frac{q_1}{C_b R_{bA}} - \frac{\alpha f_7}{R_{bA} m(\theta)}$$

where Eqs.(1),(4) and (6) have been used directly (having the outputs on the left-hand side helps). But $f_7 = f_9 = f_{11}/r_t$ and $f_{11} = p_{11}/M$ from (10) and (12), which gives the required state equation:

$$\dot{q}_1 = \frac{q_1}{C_b R_{bA}} - \frac{\alpha p_{11}}{R_{bA} m(\theta) r_t M} \tag{13}$$

The derivation of the expression for $\dot{p}_{11}$ is somewhat lenghtier but equally straightforward. Verify that the correct expression is

$$\dot{p}_{11} = \left(\frac{\alpha}{R_{bA} C_b m(\theta) r_t} \right) q_1 - \left(\frac{\alpha^2}{M R_{bA} m^2(\theta) r_t^2} + \frac{b}{M m^2(\theta) r_t^2} + \frac{b}{M r_t^2} \right) p_{11} - \frac{1}{r_t} \phi \left(\frac{p_{11}}{M r_t}, \theta \right) \tag{14}$$

Eqs. (13) and (14) are the two required state equations.

4.1 Battery charge

Note that the current through the capacitor must be thought of as coming from a source. Therefore, it should be positive when flowing from low to high potential, as explained earlier in the course. But f_2, the current flowing through the armature resistance is assumed positive when going from a higher to a lower potential. This current equals the battery current f_1 in our bond graph. The correct physical interpretation for the battery charge history must take this into account. We use $-f_1$ to represent the physical current flowing through the battery. Therefore the battery charge will be given by:

$$i_{\text{batt}} = \frac{dq_{\text{batt}}}{dt} = -f_1$$

$$dq_{\text{batt}} = -f_1 dt = -\frac{dq_1}{dt} dt$$

$$q_{\text{batt}}(t) - q_{10} = -q_1(t) + q_{10}$$

$$q_{\text{batt}}(t) = 2q_{10} - q_1(t)$$

where q_{10} represents the initial battery charge.

4.2 Battery voltage

This voltage can be obtained from the constitutive equation

$$e_1 = \frac{q_{\text{batt}}}{C_b}$$

4.3 Battery charging/discharging

This can be determined by looking at the sign of f_1, which is numerically equal to the right-hand side of Eq. (13).

4.4 Direction of power flow

Since we simulate only for positive battery voltage, power flow can be determined from f_1 alone. In general, to obtain the numerical value of the power as well, we use $e_1 f_1$.

4.5 Vehicle speed

This can be obtained from

$$f_{11} = \frac{p_{11}}{M}$$

4.6 Vehicle range

The distance traveled can be obtained from

$$q_{11} = \int f_{11} dt = \frac{1}{M} \int p_{11} dt$$

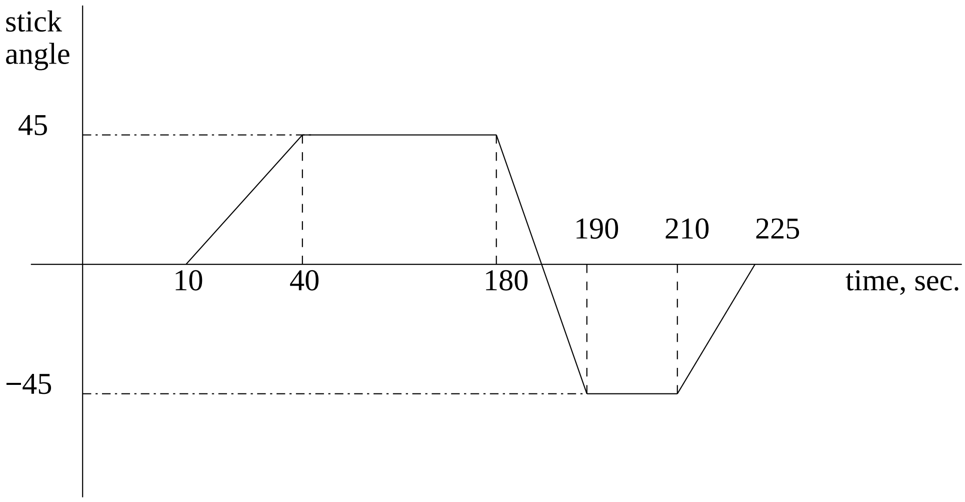

Figure 4: Stick input profile

5 Simulation

The resulting bond graph should have all integral and fully-determined causality (case I). Derive the state equations and write output equations for the charge in the battery, vehicle velocity, vehicle position and torque on the wheels. Use the stick motion profile shown in Fig. 4 to perform the simulation. Analyze the results to see if they are meaningful. Point out any shortcomings of the model and suggest ways to overcome them.

5.1 Finite-slope approximation to signum function

As it can be seen in Fig. 5, the signum function defined by

$$\text{sgn}\ (x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$$

This function has a sharp discontinuity at zero, which creates difficulties with numerical solvers. Instead, use the saturation function shown in Fig. 5, defined as

$$\text{sat}\ (\frac{x}{\delta}) = \begin{cases} 1, & x > \delta \\ -1, & x < -\delta \\ \frac{x}{\delta}, & |x| \leq \delta \end{cases}$$

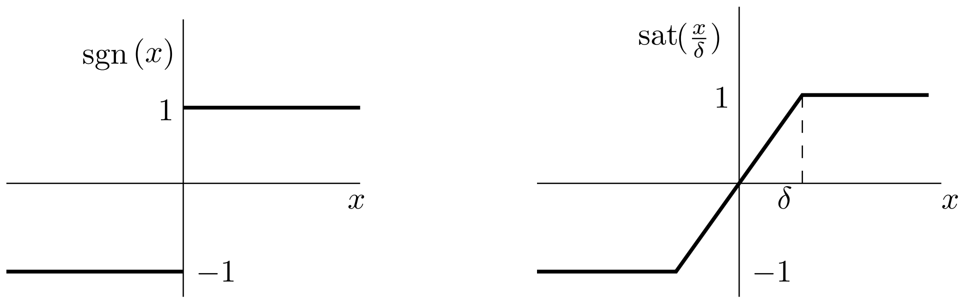

Figure 5: Finite-slope approximation to signum function

6 Suggested numerical values

Parameter	Value	Units
Battery capacity	180	A-h
Battery resistance	0.0001	Ω
Battery nominal voltage	48	V
Vehicle mass	200	kg
Tire radius	0.3	m
Gear ratio range	0.1 to 10	n.a.
Damping coefficient	0.1	N-m-s
Braking torque	± 200	N-m
Torque Constant	10	N-m/A
Armature resistance	1.5	Ω