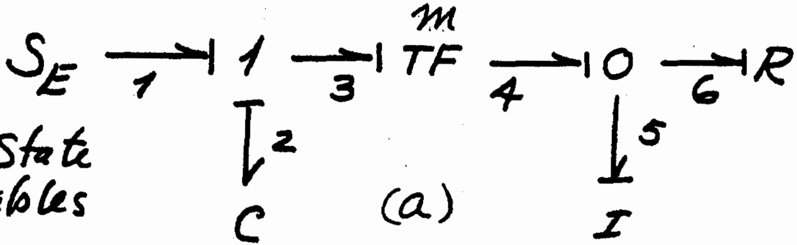
5-1 30



(a)

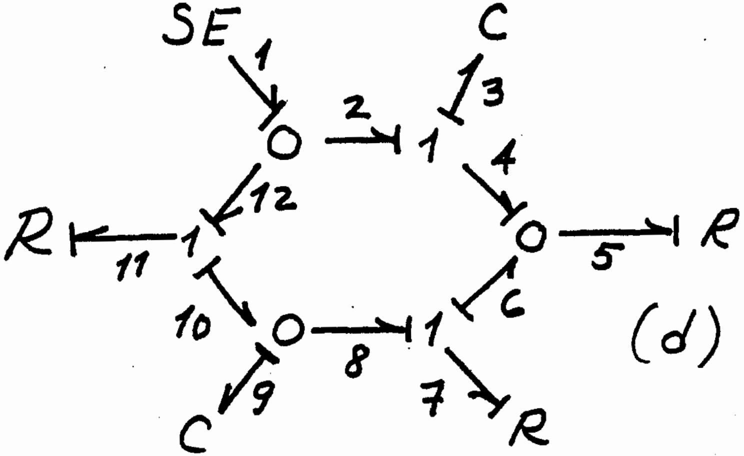
Two State Variables

$$\dot{p}_5 = e_4 = m e_3 = m(E_1(t) - e_2) = \underline{m(E_1(t) - q_2/C_2)}$$

$$\dot{q}_2 = f_3 = m f_4 = m(f_5 + f_6) = m(p_5/I_5 + e_6/R_6)$$

$$= m(p_5/I_5 + m e_3/R_6) = m\left[\frac{p_5}{I_5} + \frac{m(E_1 - e_2)}{R_6}\right]$$

$$= \underline{m\left[\frac{p_5}{I_5} + \frac{m(E_1(t) - q_2/C_2)}{R_6}\right]}.$$



(d)

Two State Variables

$$\dot{q}_3 = f_4 = f_5 - f_6 = \frac{e_5}{R_5} - f_7 = \frac{e_4}{R_5} - \frac{e_7}{R_7} = \frac{e_2 - e_3}{R_5} - \left(\frac{e_8 - e_6}{R_7}\right)$$

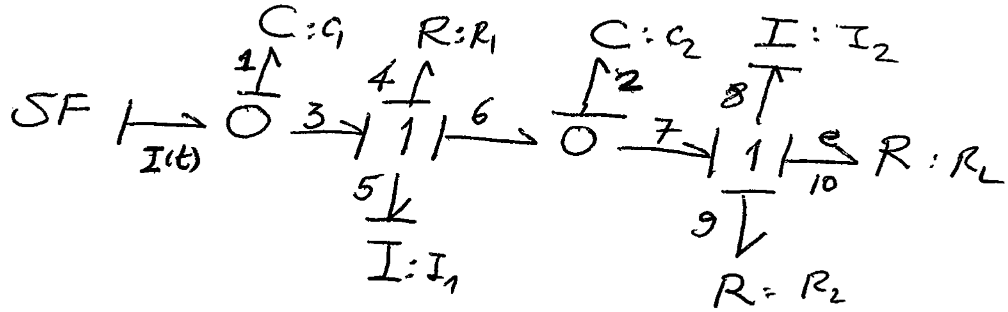
$$= \frac{E_1(t) - q_3/C_3}{R_5} - \left(\frac{e_9 - e_4}{R_7}\right) = \frac{E_1 - q_3/C_3}{R_5} - \left(\frac{q_9/C_9 - e_2 + e_3}{R_7}\right)$$

$$= \frac{E_1(t) - q_3/C_3}{R_5} - \left(\frac{q_9/C_9 - E_1(t) + q_3/C_3}{R_7}\right)$$

$$\dot{q}_9 = \frac{E_1(t) - q_9/C_9}{R_{11}} - \left(\frac{q_9/C_9 - E_1(t) + q_3/C_3}{R_7}\right)$$

5-4

By inspection :



Since all C and I elements have integral causality, there are 4 states : q_1, p_5, q_2, p_8.

$$\dot{q}_1 = f_1 = I(t) - \overset{= f_5}{f_3} = I(t) - \frac{p_5}{I_1}$$

$$\dot{p}_5 = e_5 = e_3 - e_4 - e_6 = e_1 - e_4 - e_2 = \frac{q_1}{C_1} - R_1 \overset{= f_5}{f_4} - \frac{q_2}{C_2}$$

$$\dot{p}_5 = \frac{q_1}{C_1} - R_1 \frac{p_5}{I_1} - \frac{q_2}{C_2}$$

$$\dot{q}_2 = f_2 = \overset{= f_5}{f_6} - \underset{= f_8}{f_7} = \frac{p_5}{I_1} - \frac{p_8}{I_2}$$

$$\dot{p}_8 = e_8 = e_7 - e_9 - e_{10} = e_2 - R_2 \overset{= f_8}{f_9} - R_L \overset{= f_8}{f_{10}}$$

$$\dot{p}_8 = \frac{q_2}{C_2} - (R_2 + R_L) \frac{p_8}{I_2}$$

Output: $e = e_{10} = R_L f_{10} = \dfrac{R_L p_8}{I_2}$

5.16 state variables p_5, q_7, p_{10}, q_{12}

$$\dot{p}_5 = \frac{T}{R_2}\left(e - \frac{T}{I_5} p_5\right) - \frac{q_7}{C_7}$$

$$\dot{q}_7 = \frac{p_5}{I_5} - \frac{1}{R}\frac{p_{10}}{I_{10}}$$

$$\dot{p}_{10} = -\frac{q_{12}}{C_{12}} - \frac{R_{11}}{I_{10}} p_{10} + \frac{1}{R}\frac{q_7}{C_7}$$

$$\dot{q}_{12} = p_{10}/I_{10}$$

$$\frac{d}{dt}\begin{bmatrix} p_5 \\ q_7 \\ p_{10} \\ q_{12} \end{bmatrix} = \left[\begin{array}{c|c|c|c} -T^2/R_2 I_5 & -1/C_7 & 0 & 0 \\ 1/I_5 & 0 & -1/R I_{10} & 0 \\ 0 & 1/R C_7 & -R_{11}/I_{10} & -1/C_{12} \\ 0 & 0 & 1/I_{10} & 0 \end{array}\right] \begin{bmatrix} p_5 \\ q_7 \\ p_{10} \\ q_{12} \end{bmatrix} + \begin{bmatrix} T/R_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} e$$

output eqns:

$$e_4 = \frac{T}{R_2}\left(e - \frac{T}{I_5} p_5\right)$$

$$f_7 = \frac{p_5}{I_5} - \frac{1}{R}\frac{p_{10}}{I_{10}}$$

$$e_2 = e - \frac{T}{I_5} p_5$$

5.17 state variables p_5, p_{10}, q_{12}

To handle derivative causality, $p_8 = I_8\left[\frac{R}{I_5}p_5 + \frac{p_{10}}{I_{10}}\right]$

$$\dot{p}_8 = e_8 = R\frac{I_8}{I_5}\dot{p}_5 + \frac{I_8}{I_{10}}\dot{p}_{10}$$

state eqns:

$$\dot{p}_5 = \frac{T}{R_2}\left(e - \frac{T}{I_5}p_5\right) - R\,\dot{p}_8$$

$$\dot{p}_{10} = \frac{q_{12}}{C_{12}} - \dot{p}_8$$

$$\dot{q}_{12} = v_i - \frac{p_{10}}{I_{10}}$$

substitute for $\dot{p}_8$

$$\dot{p}_5\left[1 + R^2\frac{I_8}{I_5}\right] + R\frac{I_8}{I_{10}}\dot{p}_{10} = \frac{T}{R_2}e - \frac{T^2}{R_2 I_5}p_5$$

$$R\frac{I_8}{I_5}\dot{p}_5 + \dot{p}_{10}\left[1 + \frac{I_8}{I_{10}}\right] = q_{12}/C_{12}$$

$$\dot{q}_{12} = v_i - \frac{p_{10}}{I_{10}}$$

Can write as,

$$\begin{bmatrix} 1+R^2\frac{I_8}{I_5} & R\frac{I_8}{I_{10}} & 0 \\ RI_8/I_5 & 1+\frac{I_8}{I_{10}} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}\dot{p}_5\\ \dot{p}_{10}\\ \dot{q}_{12}\end{bmatrix} = \begin{bmatrix} -T^2/R_2I_5 & 0 & 0 \\ 0 & 0 & 1/C_{12} \\ 0 & -1/I_{10} & 0 \end{bmatrix}\begin{bmatrix}p_5\\ p_{10}\\ q_{12}\end{bmatrix} + \begin{bmatrix}T/R_2\\0\\0\end{bmatrix}e + \begin{bmatrix}0\\0\\1\end{bmatrix}v_i$$

↖ final eqns. requires inversion of this matrix