

MCE503: Modeling and Simulation of Mechatronic Systems

Discussion on Bond Graph Sign Conventions for Electrical Systems

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1 Basic Assumption

In a physical electrical circuit, current flows from higher to lower potential when following a circuit path. Within a voltage source (say, a battery), current flows from (-) to (+). See Fig. 1. Note that we use the “conventional” or engineering interpretation of current flow. A more elaborate explanation involving movement of “vacancies” or “holes” is employed in rigorous physics.

2 Voltage and Current Sign Conventions and Interpretation

When voltages (including sources) can change polarity with time, we fix a particular set of polarities and assign voltage variables as positive for that configuration. See Fig. 2. Nothing has been assumed in the picture on the left. On the right, we have assumed that point a has higher potential than point b, which in turn has higher potential than point c. This does not mean this will be the case at all times. We indicate our assumed configuration with (+) signs at one end of each source or component and associate a voltage variable which is defined positive when measured from (+) to (-). *This applies to all components except voltage sources, for which the source voltage is positive when directed from (-) to (+) within the source.* Later, if some solution method gives, for example $e_1 > 0$ and $e_2 < 0$, we interpret this in the physical system as point a having higher potential than point b (matching the assumed configuration), but point b having lower potential than point c. For currents, the “natural” choice is to introduce variables which are positive when directed from higher to lower potential. However, one may be interested in solving for the negative variable. Referring to Fig. 3 (left), we see that if a numerical solver gives positive values for i_1 , i_2 and i_3 , the currents will be directed as shown. The solver should give $i_1 = i_2$ in any case (same sign). For

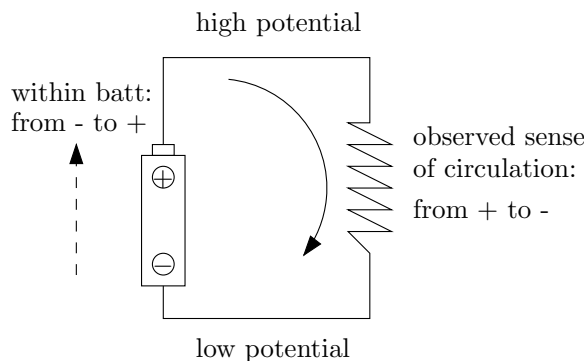


Figure 1: Conventional current circulation

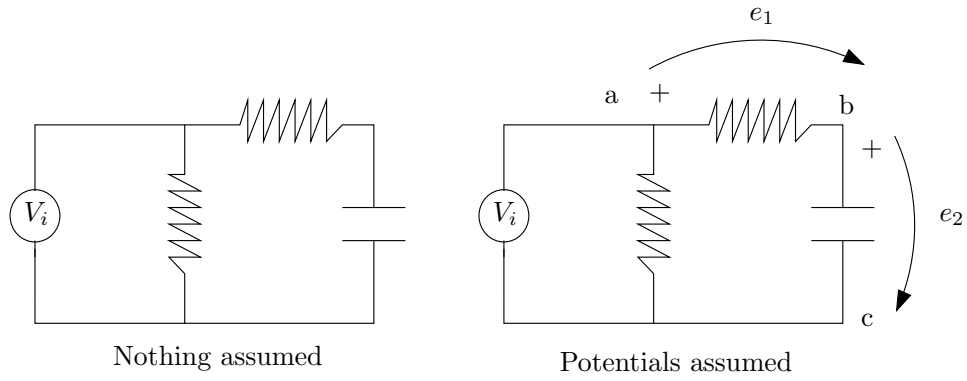


Figure 2: Voltage conventions

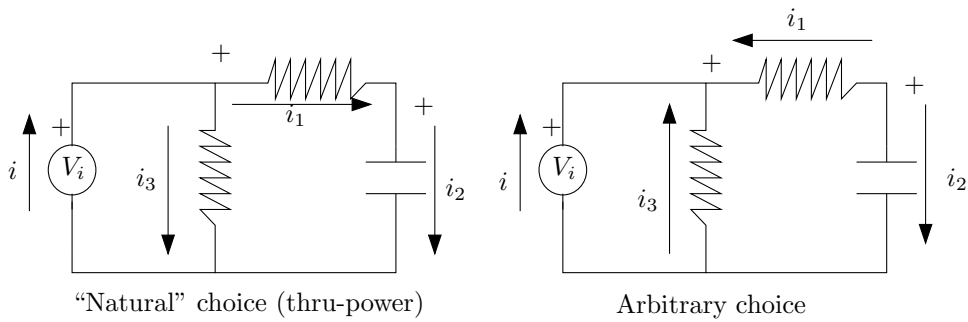


Figure 3: Current conventions

the choice on the right, if the solver gives $i_1 < 0$, then the actual current is flowing from point a to point b. The solver should always give i_1 and i_2 with opposite signs under this sign convention.

3 Effect of the Conventions on the Equations

Suppose that the convention shown in Fig. 3 (left) is used. For node a we write

$$i = i_1 + i_3$$

while for node b we have

$$i_1 = i_2$$

For voltages we write

$$e_1 + e_2 = V_i$$

Finally, for the components we write

$$V_i = i_3 R_1, \quad V_1 = i_1 R_2, \quad i_2 = C \dot{e}_2$$

Now suppose the convention shown on the right is used. For node a we write

$$i = -i_1 - i_3$$

while for node b we have

$$i_1 = -i_2$$

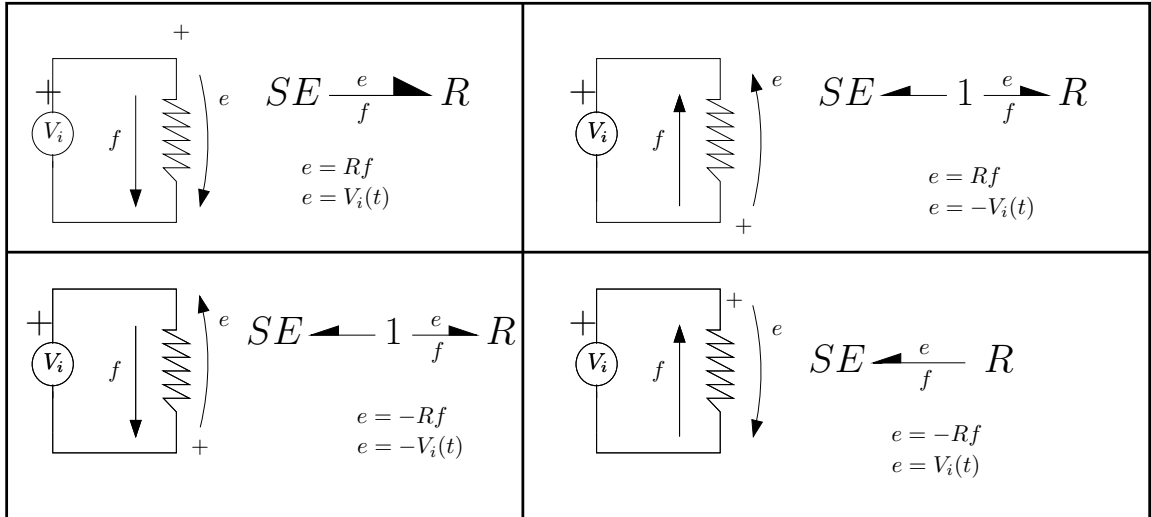


Figure 4: Bond Graph sign conventions

For voltages we write

$$e_1 + e_2 = V_i$$

and for components we have

$$V_a - V_c = V_i = -i_3 R_1, \quad V_a - V_b = e_1 = -i_1 R_2, \quad C \dot{e}_2 = i_2$$

4 Sign Conventions in Bond Graphs and Physical Interpretation

Examine Fig. 4. If a 1-port (except sources) has the half-arrow directed towards it, we write the constitutive equations without negative signs. If the half arrow is directed away from the 1-port, we need to introduce a minus sign in the constitutive equation. For example, consider the bottom-right case. If the solver gives $e > 0$ and $f < 0$ at some instant, we interpret this physically in the circuit as the top being at higher potential than the bottom and therefore the current circulating clockwise. The bond graph should confirm these orientations. In fact, the product ef is negative, therefore power flows from the source to the R element.

5 Effect of Bond Graph Sign Conventions on the Equations

Figure 4 shows the equations that need to be written for each sign convention. The same applies to C and I elements. Note that, for R , the equations and power interpretation of the half-arrow forbid the situation of power flowing from R into the source. In the case of 2-ports (TF and GY), the basic principle is that the 2-port passes power thru with 100% efficiency. Regarding sign convention, if power enters a port, then it must leave the other. For transformers, refer to Fig. 5. In the top figure, we see from the equations that if e_1 and f_1 are positive, then so must be e_2 and f_2 . Thus, power flows from left to right. If either port variable changes sign, then the equations dictate that the corresponding variable in the opposite port must do so. If, for instance, $e_1 < 0$ and $f_1 > 0$, then $e_2 < 0$ and $f_2 > 0$ and power flows from right to left. If we assume the same for the bottom figure, however, we get $e_2 < 0$ and $f_2 < 0$, but power still enters from the right and leaves to the left. Work out the meaning of conventions for GY yourself.

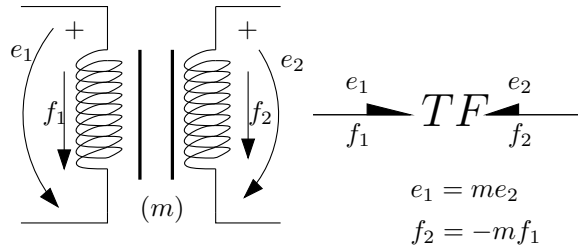
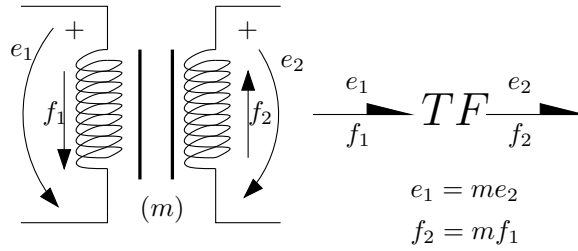


Figure 5: Conventions for Transformers

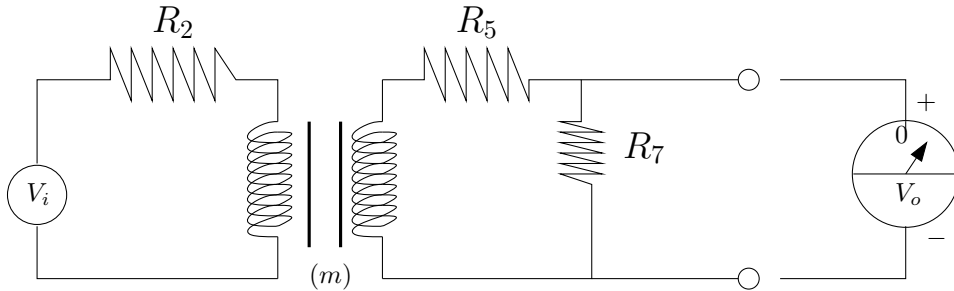


Figure 6: Example circuit

6 Example

Obtain a bondgraph for the circuit of Fig. 6. Assign arbitrary power directions and obtain a valid set of equations. Then interpret in the physical diagram. The bondgraph with arbitrary conventions is shown in Fig. 7. The following is the list of correct equations:

$$\begin{aligned}
 e_1 &= V_i(t) \\
 f_1 &= f_2 = f_3 \\
 e_1 + e_2 + e_3 &= 0 \\
 e_2 &= -R_2 f_2
 \end{aligned}$$

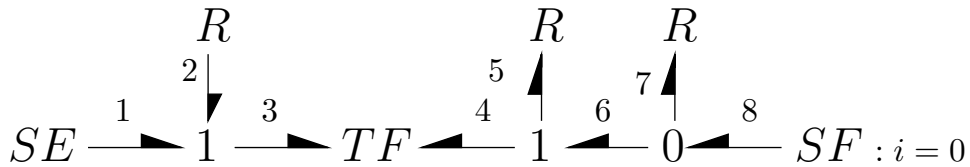


Figure 7: Bondgraph for example

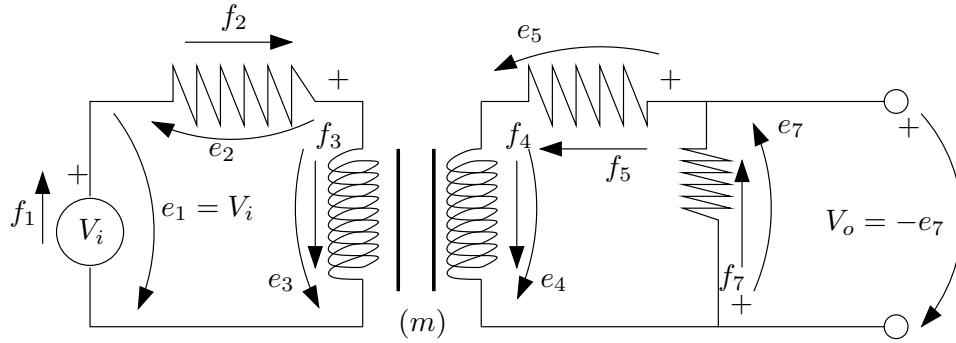


Figure 8: Interpretation of sign conventions

$$\begin{aligned}
 e_3 &= me_4 \\
 f_4 &= f_5 = f_6 \\
 e_5 &= f_5 R_5 \\
 -e_4 - e_5 + e_6 &= 0 \\
 e_6 &= e_7 = e_8 \\
 -f_6 - f_7 + f_8 &= 0 \\
 e_7 &= R_7 f_7 \\
 f_8 &= 0
 \end{aligned}$$

The physical interpretation of our signed bondgraph is shown in Fig. 8. If $V_i(t)$ is given and the solver gives, for example, $e_7 < 0$ at some instant, we anticipate that the meter will have a positive reading at that instant.

7 Laboratory Demonstration

A simple circuit will be brought to class. A bondgraph will be obtained and signed. Then the equations will be solved by computer using some $V_i(t)$ input. The sign convention will be interpreted with the physical schematic and verified with the experimental setup.

8 Conclusions

- Sign conventions in a bond graph cannot be wrong until the wrong equations are written.
- A consistent bond graph-equations combination has to be interpreted properly in a physical situation.
- The entire line of reasoning can be applied to thermal, mechanical and hydraulic systems, if the abstract notions of effort and flow are retained.
- The power signs are arbitrary, but the thru-power convention is more intuitive.