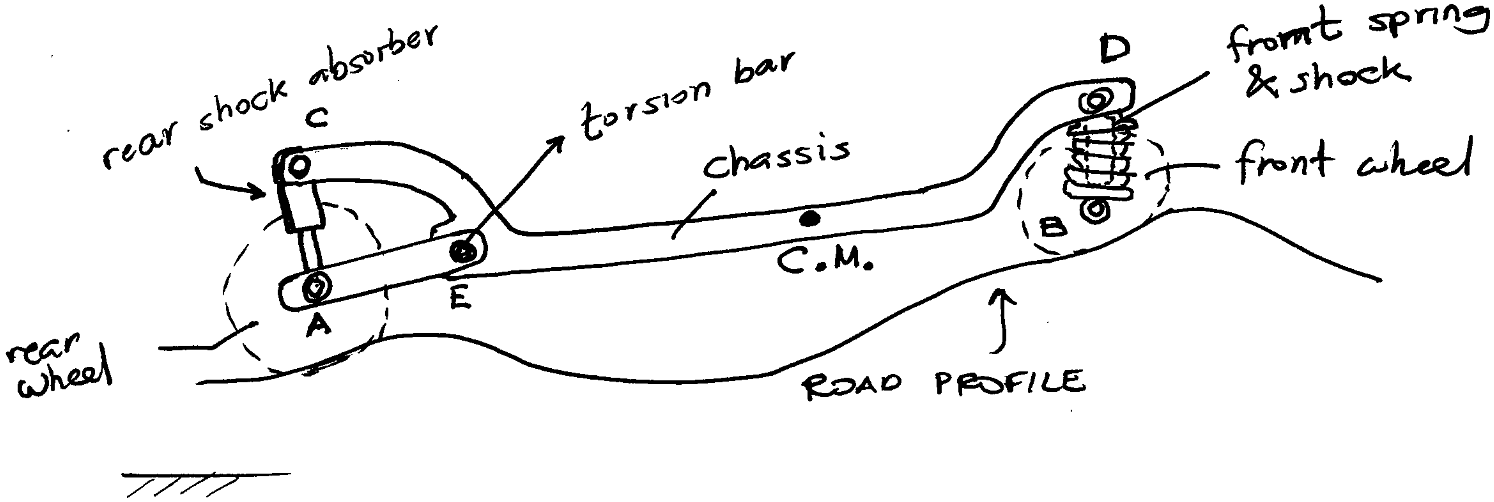
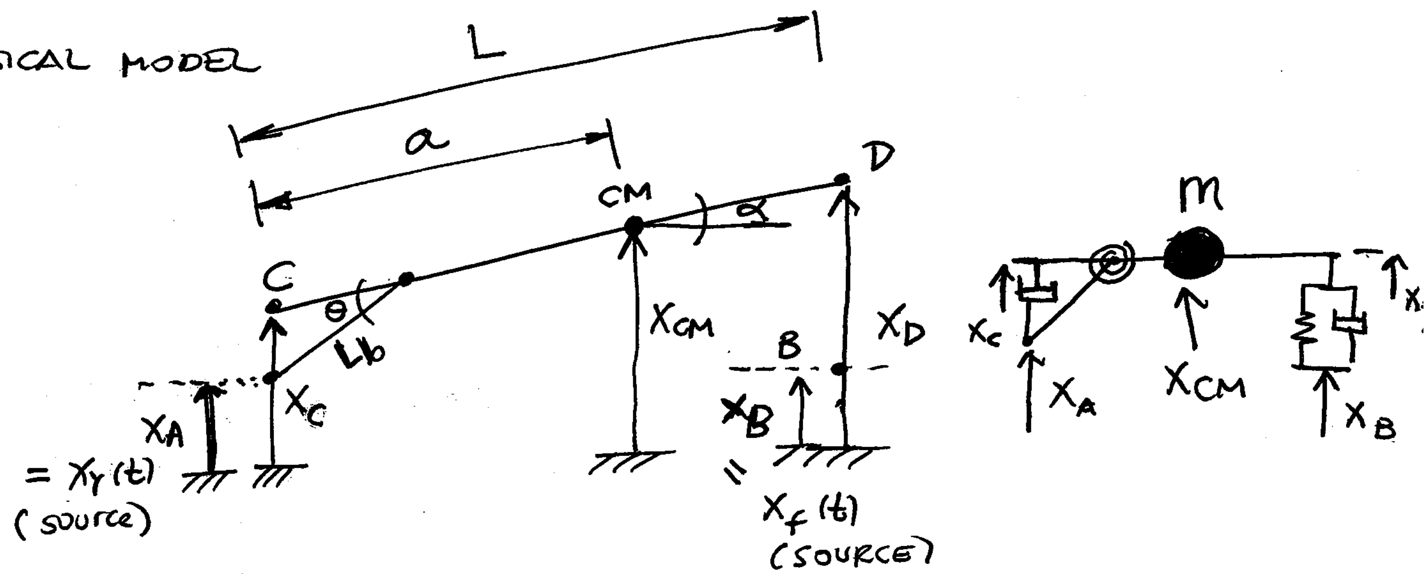
CSU – MECH. ENGINEERING

MCE 503 – MODELING OF SUSPENSION



PHYSICAL MODEL



<u>Assumptions</u>

i) θ and α are small, so that $\begin{cases} \theta L_b = X_c - X_A \\ \alpha a = X_{CM} - X_c \end{cases}$

ii) Chassis has mass m and moment of inertia I_c about CM. (or $\alpha L = X_D - X_c$)

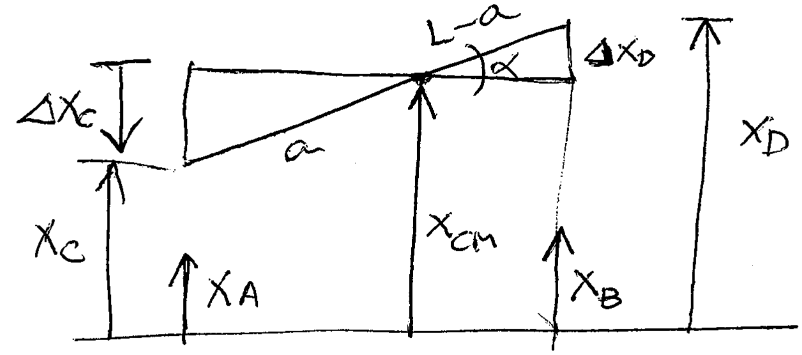
iii) Front spring & shock assembled in parallel

iv) $\overline{AE}$ is a rigid link

v) Torsion bar is linearly elastic : $\Delta\theta = \frac{M \Delta x}{G I_b}$ but a distributed parameter model is desired

BOND GRAPH

Use the following information

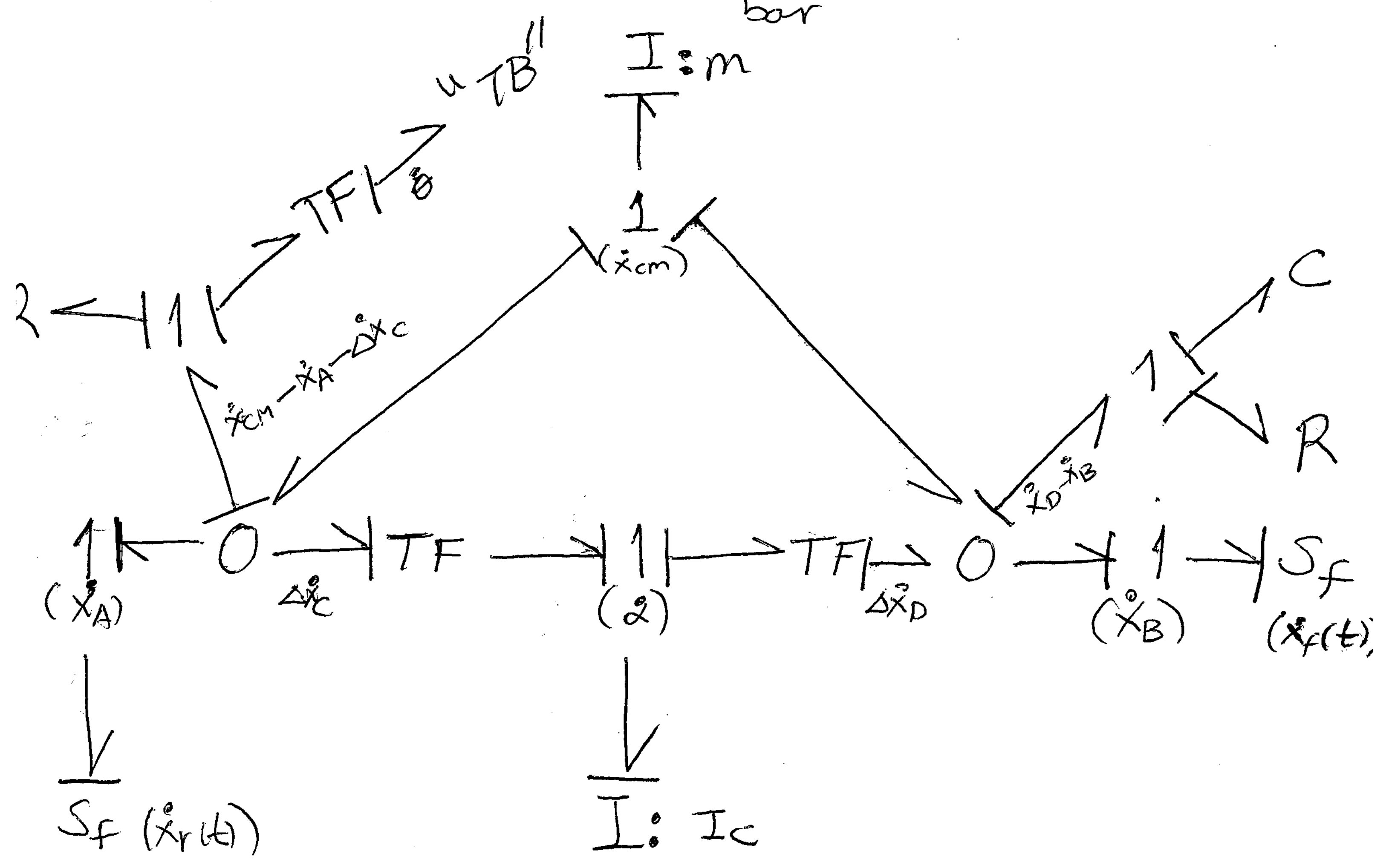


$X_D = X_{CM} + \Delta X_D$

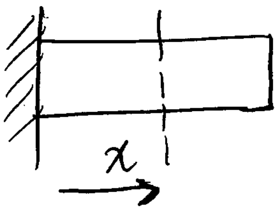
$\underbrace{X_D - X_B}_{\text{used for C and R}} = X_{CM} + \Delta X_D - X_B$

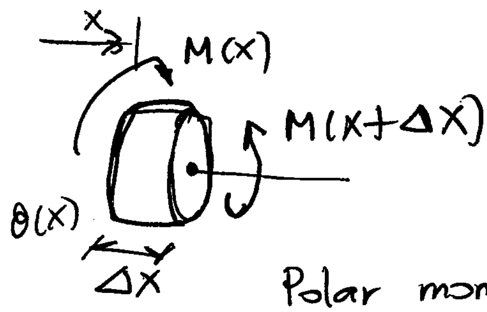
$X_C = X_{CM} - \Delta X_C$

$\underbrace{X_C - X_A}_{\text{used for R and torsion bar}} = X_{CM} - \Delta X_C - X_A$



TORSION BAR MODELING.





Polar moment of inertia : J

Density : ρ

Mass moment of inertia : $J\rho\Delta x$

Newton's law : $$M(x+\Delta x) - M(x) = J\rho\Delta x \frac{\partial^2\theta}{\partial t^2}$$

Constitutive equation for linear elasticity :

$$\theta(x) - \theta(x-\Delta x) = \frac{M(x)\Delta x}{GJ}$$

G is the shear modulus of the material.

The momentum of the i^{th} element is $$p_i = J\rho\Delta x \frac{\partial\theta_i}{\partial t}$$

and the relative rotation angle is $$q_i = \theta_i - \theta_{i-1}$$

We can write : $$M_{i+1} - M_i = \frac{dp_i}{dt}$$

$$\theta_i - \theta_{i-1} = q_i = \frac{M_i\Delta x}{JG}$$

so $$\dot{q}_i = \frac{p_i - p_{i-1}}{J\rho\Delta x}$$

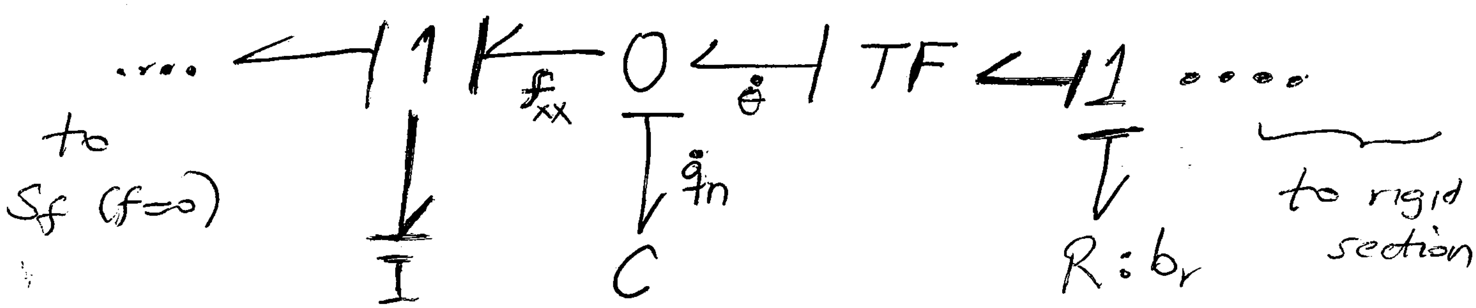
$$M_i = \frac{JG}{\Delta x} q_i$$

The state equations for the intermediate elements are

$$\begin{cases} \dot{q}_i = \dfrac{p_i - p_{i-1}}{J\rho\Delta x} \\[2ex] \dot{p}_i = \dfrac{JG}{\Delta x}\left(q_{i+1} - q_i\right) \end{cases}$$

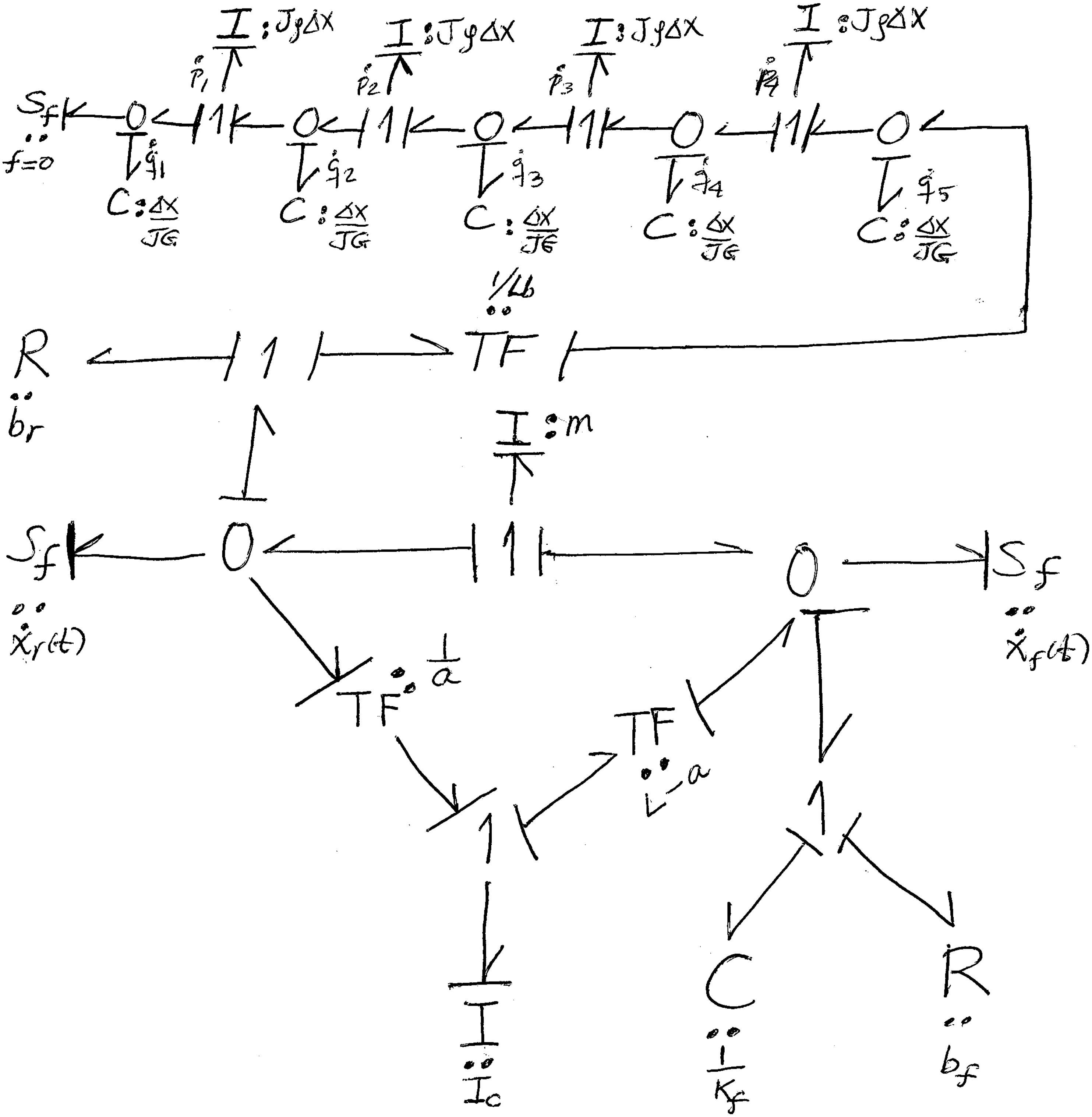
For the clamped end: $\dot{q}_1 = \dfrac{p_1}{J\rho\Delta x}$ $\left(p_0 = J\rho\Delta x\,\cancelto{0}{\dot{q}_0}\right)$

For the other end (flow imposed by TF) $\dot{q}_n = \dot{\theta} - f_{xx}$



Note that attaching the torsion bar by a 1-junction would force derivative causality on the I.

FINAL BONDGRAPH (4 ELEMENTS)



VALUES FOR SIMULATION

$\Delta x = 0.1\ m$

AISI/SAE 4130 Steel $G = 80\ GPa$

$\rho = 7850\ kg/m^3$

$L = 3.5\ m$, $L_b = 0.5\ m$, $a = 2\ m$, $b_f = b_r = 1500\ \frac{N\text{-}s}{m}$

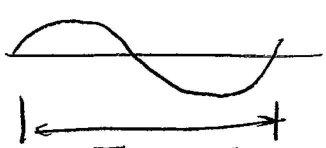
$K_f = 50000\ N/m$; $J = 1.57\times10^{-8}\ kg\text{-}m^2$ $(r = 1\ cm)$

$I_c = 1600\ kg\ m^2$ $m = 700\ kg$

$J\rho\Delta x = 1.233\times10^{-5}$; $\frac{\Delta x}{JG} = 7.962\times10^{-5}$

Road profile

$\longrightarrow v = 40\ mph \simeq 18\ m/s$



$\updownarrow A = 0.05\ m$

$T = 0.25\ m$

$$x_f = 0.05 \sin\left(\frac{2\pi}{0.25} \times 18 \times t\right) = 0.05 \sin(452.39t)$$

$$x_r = 0.05 \sin\left(\frac{2\pi}{0.25} \times 18(t - \Delta t)\right)$$

$$\Delta t = \frac{L}{v} = \frac{3.5}{18} = 0.194$$

$$x_r = 0.05 \sin(452.39t - 88)$$

Then
$$\begin{cases} \dot{x}_f = 22.62 \cos(452.39t) \\ \dot{x}_r = 22.62 \cos(452.39t - 88) \end{cases}$$