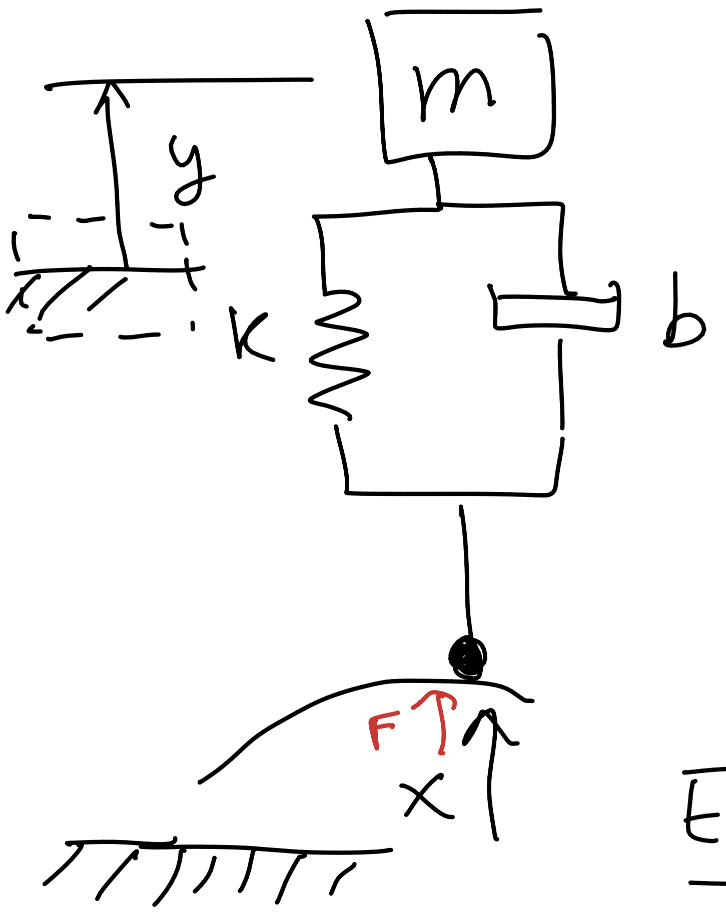


$$T = \frac{1}{2} m \dot{y}^2$$

$$U = mgy + \frac{1}{2} k (x-y)^2$$

$$\mathcal{R} = \frac{1}{2} b (\dot{x}-\dot{y})^2$$

$$\mathcal{L} = T - U = \frac{1}{2} m \dot{y}^2 - mgy - \frac{1}{2} k (x-y)^2$$

E-L equations:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) - \frac{\partial \mathcal{L}}{\partial y} + \frac{\partial \mathcal{R}}{\partial \dot{y}} = 0$$

because F is an internal (constraint) force

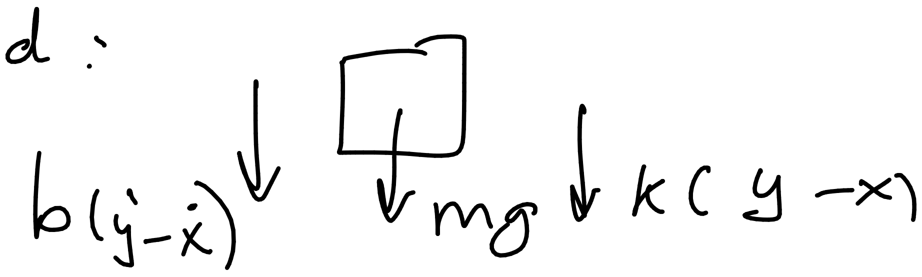
$$\frac{\partial \mathcal{L}}{\partial \dot{y}} = m\dot{y} \quad , \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) = m\ddot{y}$$

$$\frac{\partial \mathcal{L}}{\partial y} = -mg + k(x-y) \quad , \quad \frac{\partial \mathcal{R}}{\partial \dot{y}} = -b(\dot{x}-\dot{y})$$

$$m\ddot{y} + mg - k(x-y) - b(\dot{x}-\dot{y}) = 0$$

$$\boxed{m\ddot{y} + b\dot{y} + ky = b\dot{x} + kx - mg}$$

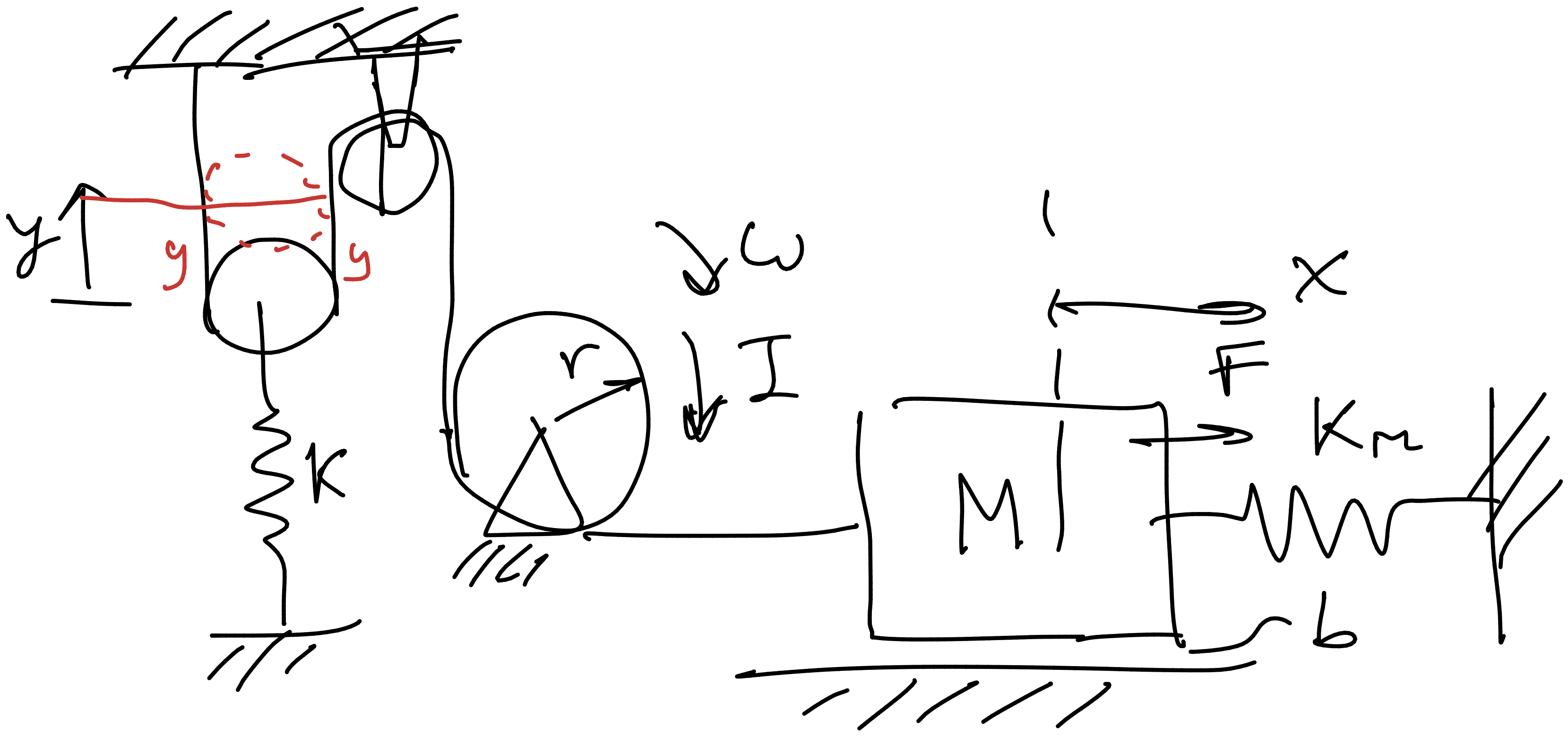
'Conventional' method:



$$\sum F = m\ddot{y}$$

$$-b(\dot{y}-\dot{x}) - mg - k(y-x) = m\ddot{y}$$

$$\boxed{m\ddot{y} + b\dot{y} + ky = b\dot{x} + kx - mg}$$



F : input

X : output

$$\begin{cases} T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} I \omega^2 \\ U = \frac{1}{2} k_M x^2 + \frac{1}{2} k y^2 \\ R = \frac{1}{2} b \dot{x}^2 \end{cases}$$

$$\omega = \frac{\dot{x}}{r}$$

$$x = 2y$$

$$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} \frac{I}{r^2} \dot{x}^2$$

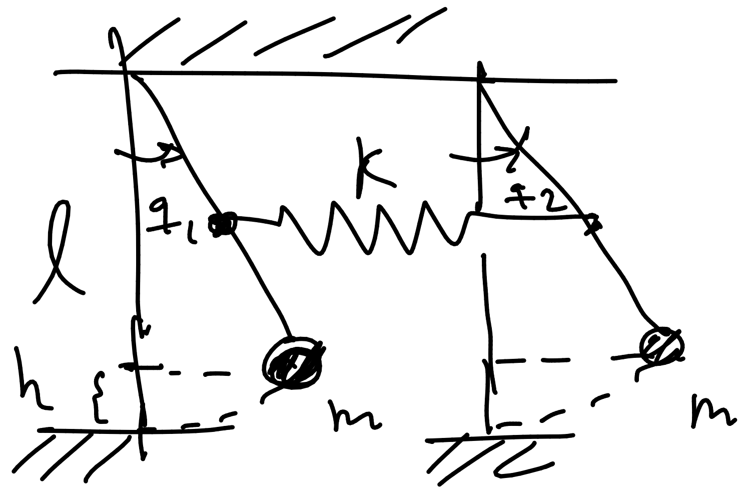
$$U = \frac{1}{2} K_M x^2 + \frac{1}{8} K x^2$$

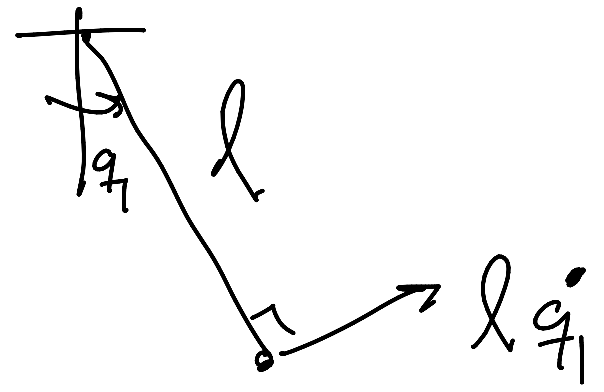
$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = M\dot{x} + \frac{I}{r^2}\dot{x} = \left(M + \frac{I}{r^2}\right)\dot{x}, \quad \frac{d}{dt}(\cdot) = \left(M + \frac{I}{r^2}\right)\ddot{x}$$

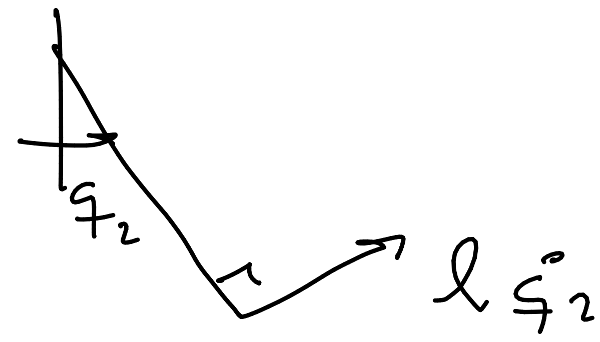
$$\frac{\partial \mathcal{L}}{\partial x} = -K_M x - \frac{1}{4} K x = -\left(K_M + \frac{1}{4} K\right) x$$

$$\frac{\partial R}{\partial \dot{x}} = b\dot{x}$$

$$\boxed{\left(M + \frac{I}{r^2}\right)\ddot{x} + \left(K_M + \frac{1}{4}K\right)x + b\dot{x} = F}$$







$h = l - l\cos q_1$

$$T = \frac{1}{2} m (l\dot{q}_1)^2 + \frac{1}{2} m (l\dot{q}_2)^2$$

2-DOF

$\approx \frac{l}{2} q_1$

$\frac{l}{2}\sin q_1$

$$U = mgl\left[(1-\cos q_1) + (1-\cos q_2)\right] + \frac{1}{2} k \left(\frac{l}{2} q_1 - \frac{l}{2} q_2\right)^2$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = ml^2(\dot{q}_1 + \dot{q}_2) \; ; \quad \frac{d}{dt}(\cdot) = ml^2(\ddot{q}_1 + \ddot{q}_2)$$

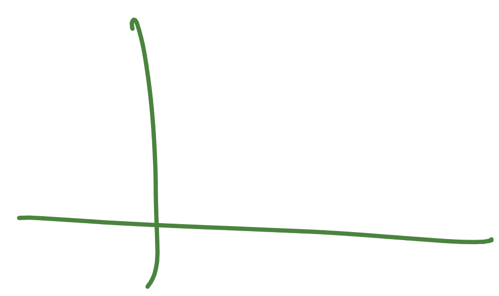
$$\frac{\partial \mathcal{L}}{\partial q_1} = -mgl(\sin q_1 + \sin q_2) - \frac{l^2}{8} k (q_1 - q_2)$$

1st eq.:

$$ml^2(\ddot{q}_1 + \ddot{q}_2) + \frac{l^2}{8} k (q_1 - q_2) + mgl\underbrace{(q_1 + q_2)}_{\sin q \approx q} = 0$$

$$\int x^T Q x + u^T R u$$

$$\dot{x} = Ax + Bu$$
$$y = Cx$$



$$Q = C^T C$$

$$x^T Q x \; : \; (x^T C)(Cx) = y^T y$$