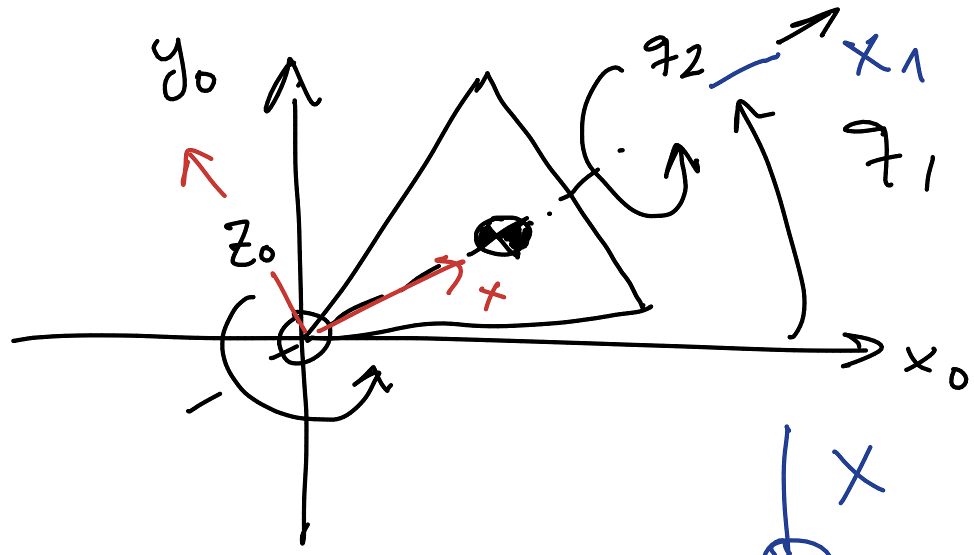
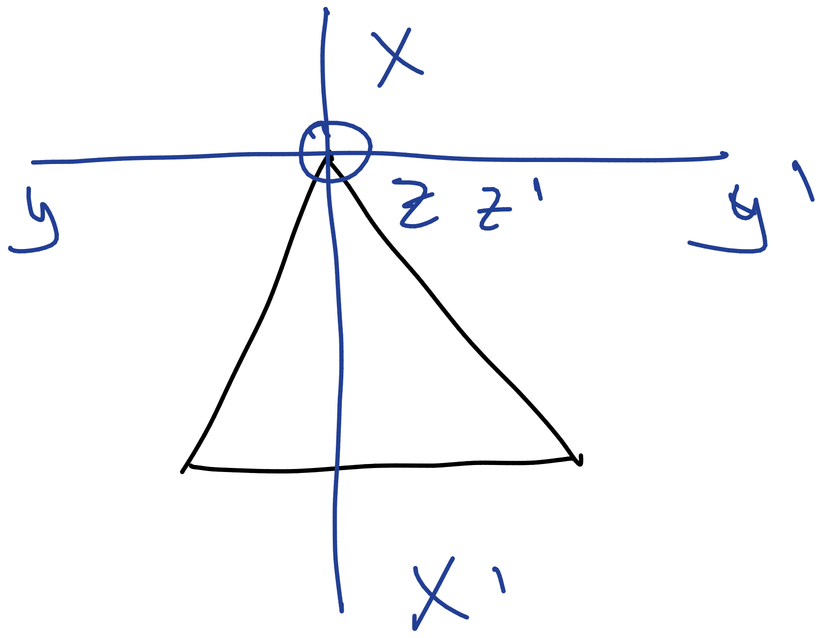
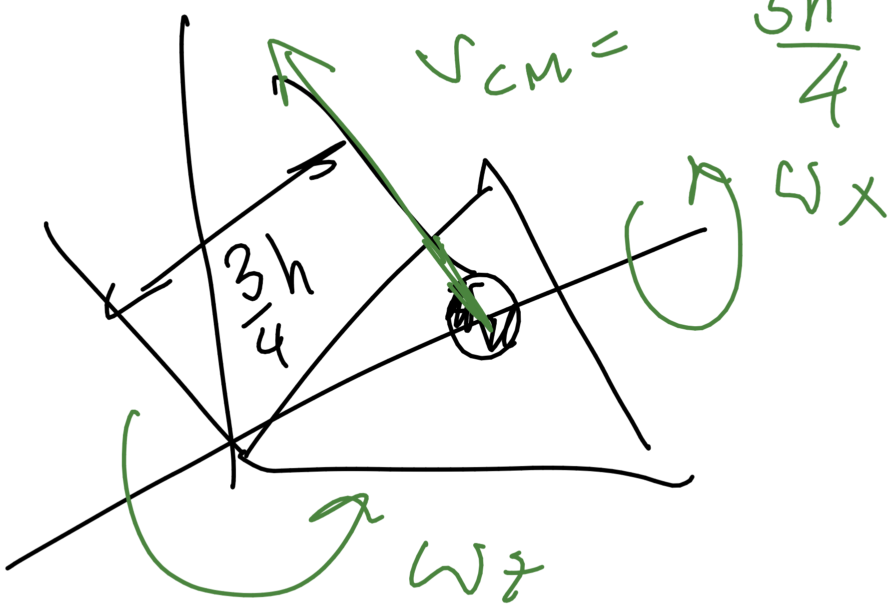


$0 \to 1 : \mathrm{Rot}_{z, q_1} \ \mathrm{Rot}_{x, q_2}$



From tables

$$I = \begin{bmatrix} I_{xx} & & \\ & I_{yy} & \\ & & I_{zz} \end{bmatrix}$$



$$v_{CM} = \frac{3h}{4}\omega_z = \dot{q}_1 \left(\frac{3h}{4}\right)$$

$$K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_{z,CM} \omega_z^2$$

2- link planar robot

$$D(q) = \begin{bmatrix} m_1 l_{c_1}^2 + I_1 + I_2 + m_2 (l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2} \cos q_2) & m_2 (l_{c_2}^2 + l_1 l_{c_2} \cos q_2) + I_2 \\ // & m_2 l_{c_2}^2 + I_2 \end{bmatrix}$$

$l_{c_1} = l_1/2 \; ; \; l_{c_2} = l_2/2$

$$\theta_1 = m_1 \frac{l_1^2}{4} + m_2 \left(l_1^2 + \frac{l_2^2}{4} \right) + I_1 + I_2$$

$$\theta_2 = m_2 l_1 l_2 / 2$$

$$\theta_3 = m_2 l_2^2 / 4 + I_2$$

$$D(q) = \begin{bmatrix} \theta_1 + 2\theta_2 \cos q_2 & \theta_3 + \theta_2 \cos q_2 \\ \theta_3 + \theta_2 \cos q_2 & \theta_3 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} h \dot{q}_2 & h \dot{q}_2 + h \dot{q}_1 \\ -h \dot{q}_1 & 0 \end{bmatrix}$$

$$h = \underbrace{-m_2 l_1 l_{c_2}}_{-\theta_2} \sin(q_2)$$ $\rightarrow$ can write C in parametric form.

$-\theta_2 \sin q_2$

$$g(q) = \begin{bmatrix} (m_1 l_{c_1} + m_2 l_1) g \cos(q_1) + m_2 l_{c_2} g \cos(q_1 + q_2) \\ m_2 l_{c_2} g \cos(q_1 + q_2) \end{bmatrix}$$

$$\theta_4 = m_1 \frac{l_1}{2} + m_2 l_1$$

(l_{c_1})

$$\theta_5 = m_2 l_2 = 2 m_2 l_{c_2}$$

$Y(\ddot{q}, \dot{q}, q) =$ ✓

(2-link planar)

<u>Check</u>

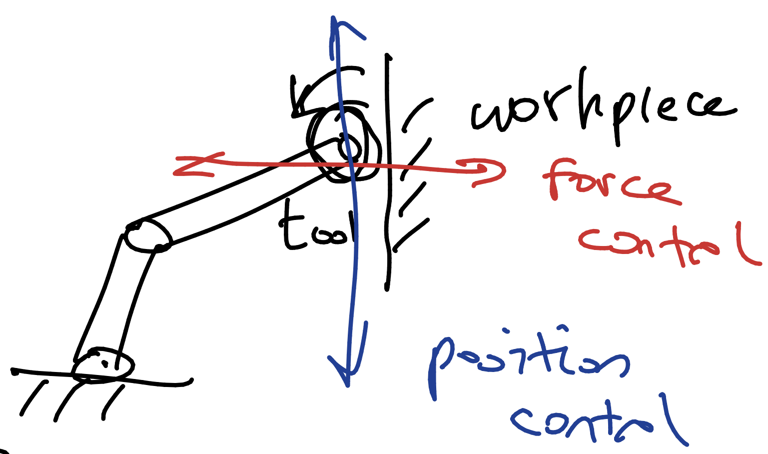
$$Y(\ddot{q}, \dot{q}, q)\,\theta = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

Robot Dynamics (plant to be controlled)

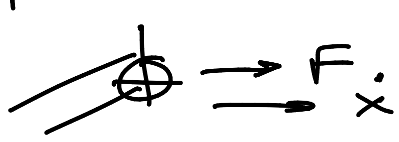
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

Control objectives:

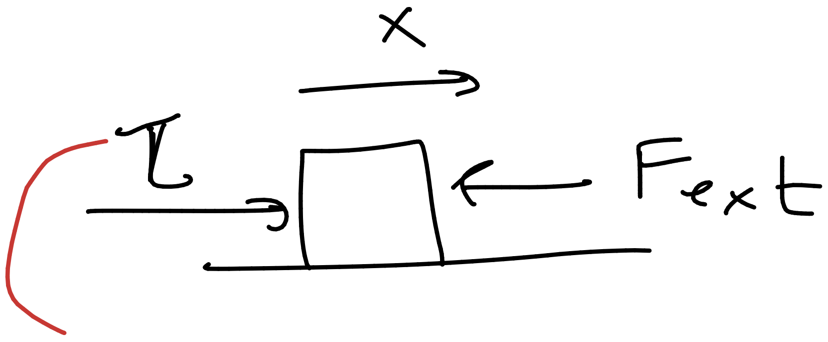
- Trajectory tracking : $q(t) \to q^d(t)$ (joint space)
- Setpoint regulation: $x_e(t) \to x_e^d(t)$ end effector (task space)
  - special case: $q^d(t) = \text{const}$, $x_e(t) = \text{const}$
- Force control: maintain specified force btw. robot & environment.
- Hybrid force/position control



- Impedance control



} specify desired $F, \dot{x}$ relationship



Control input

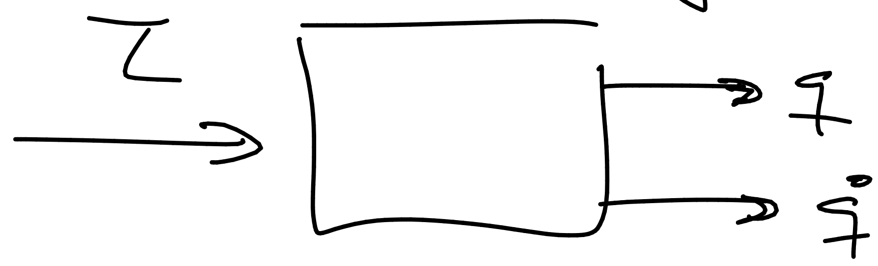
"Native dynamics"

$$M\ddot{x} + C\dot{x} + g = \tau - F_{ext}$$

"Target impedance"

$$I\ddot{x} + B\dot{x} + Kx = -F_{ext}$$

Robot Dynamics



(nonlinear) state-space:

Define $z_1 \triangleq q$.
$z_2 \triangleq \dot{q}$

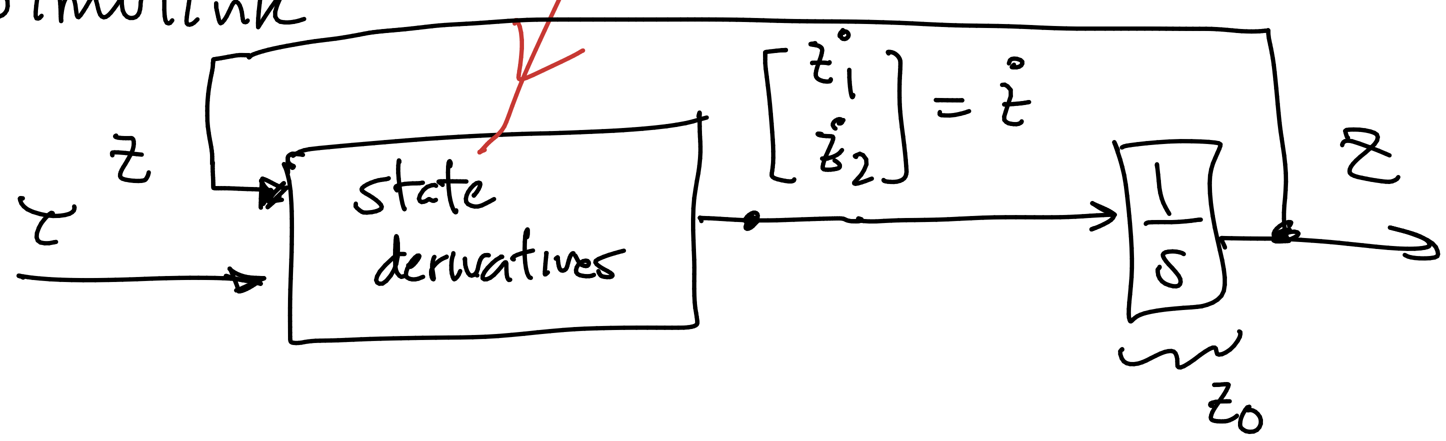
$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = M^{-1}(z_1)\left[\tau - c(z_1, z_2)z_2 - g(z_1)\right] \end{cases}$$

Simulink

$z_1 : n \times 1$

$z_2 : n \times 1$

$z = 2n \times 1$



$X_{n \times n}$, invertible

$f(X) = X^{-1}$

dX ?

$$X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$$

$$f(X) = \begin{bmatrix} x_{22} & -x_{12} \\ -x_{21} & x_{11} \end{bmatrix}\left(\frac{1}{\det(X)}\right)$$

$$f_{11}(X) = \frac{x_{22}}{x_{11}x_{22} - x_{12}x_{21}}$$

$$df_{11} = \frac{\partial f_{11}(X)}{\partial x_{11}} dx_{11} + \frac{\partial f_{11}(X)}{\partial x_{12}} dx_{12} + \cdots \frac{\partial f_{11}(X)}{\partial x_{22}} dx_{22}$$

$$df(X) = \begin{bmatrix} df_{11} & df_{12} \\ df_{21} & df_{22} \end{bmatrix}$$

$$df(x) = -X^{-1} dX X^{-1}$$

$$dX = \begin{bmatrix} dx_{11} & dx_{12} \\ dx_{21} & dx_{22} \end{bmatrix}$$

$$\frac{d}{dt}\left(D(q)\ddot{q}\right) = \dot{D}(q)\ddot{q} + D(q)\dddot{q}$$

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$