Adaptive inverse dynamics

Plant dynamics $\quad M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \quad$ ---(1)

Adaptive inverse dynamics control:

$$u = \hat{M}(q)a_q + \hat{C}(q,\dot{q})\dot{q} + \hat{g}(q) \quad \text{-----(2)}$$

$\hat{M}, \hat{C}, \hat{g}$ will be adapted (updated continously)

Virtual accel $\quad a_q = \ddot{q}^d - K_1\dot{\tilde{q}} - K_0\tilde{q} \quad$ ---(3)

$$\tilde{q} = q - q^d$$

K_1, K_0 : diagonal, positive-definite

Linear parameter property : $\quad Y(q,\dot{q},\ddot{q})\theta = u \quad$ ----(4)

Equate (1) and (2): (omit arguments in M, C, g)

$$M\ddot{q} + C\dot{q} + g = \hat{M}(\ddot{q}^d - K_1\dot{\tilde{q}} - K_0\tilde{q}) + \hat{C}\dot{q} + \hat{g}$$

add and subtract $\hat{M}\ddot{q}$

$$M\ddot{q} - \hat{M}\ddot{q} + \hat{M}\ddot{q} - \hat{M}\ddot{q}^d + \hat{M}(K_1\dot{\tilde{q}} + K_0\tilde{q}) + C\dot{q} - \hat{C}\dot{q} + g - \hat{g} = 0$$

$$M\ddot{q} + C\dot{q} + g - (\hat{M}\ddot{q} + \hat{C}\dot{q} + \hat{g}) + \hat{M}(K_1\dot{\tilde{q}} + K_0 q + \ddot{\tilde{q}})$$

$$\underbrace{M\ddot{q} + C\dot{q} + g}_{Y\theta} - (\underbrace{\hat{M}\ddot{q} + \hat{C}\dot{q} + \hat{g}}_{Y\hat{\theta}}) + \hat{M}(K_1\dot{\tilde{q}} + K_0 q + \ddot{\tilde{q}})$$

$$\ddot{\tilde{q}} + K_1\dot{\tilde{q}} + K_0\tilde{q} = \hat{M}^{-1} Y(\hat{\theta} - \theta) = \hat{M}^{-1} Y \tilde{\theta}$$

--- (5)

Define $e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix}$

Then $\dot{e} = Ae + B\Phi\tilde{\theta}$ ...... (6)

where $A = \begin{bmatrix} 0 & I \\ -K_0 & -K_1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$

$\Phi = \hat{M}^{-1} Y$ (A is Hurwitz)

Choose $P = P^T > 0$ so that

$A^T P + PA = -Q < 0$ (always feasible)

Let $V = \frac{1}{2} e^T P e + \frac{1}{2} \tilde{\theta} \Gamma \tilde{\theta}$, $\Gamma = \Gamma^T > 0$

$$V = \frac{1}{2} e^T P e + \frac{1}{2} \tilde{\theta} \Gamma \tilde{\theta}$$

$$\dot{V} = \frac{1}{2} \dot{e}^T P e + \frac{1}{2} e^T P \dot{e} + \frac{1}{2} \dot{\tilde{\theta}}^T \Gamma \tilde{\theta} + \frac{1}{2} \tilde{\theta}^T \Gamma \dot{\tilde{\theta}}$$

$\tilde{\theta} = \hat{\theta} - \underset{\text{true}}{\theta}$ (assumed constant)

$\dot{\tilde{\theta}} = \dot{\hat{\theta}}$

from (6)

$$\dot{V} = \frac{1}{2}\left[e^T A^T + \tilde{\theta}^T \phi^T B^T\right] P e + \frac{1}{2} e^T P \left[A e + B \phi \tilde{\theta}\right] + \frac{1}{2} \dot{\hat{\theta}}^T \Gamma \tilde{\theta} + \frac{1}{2} \tilde{\theta}^T \Gamma \dot{\hat{\theta}}$$

$$\dot{V} = \frac{1}{2} e^T [\underbrace{A^T P + P A}_{-Q}] e + \underbrace{\frac{1}{2} \tilde{\theta}^T \phi^T B^T P e + \frac{1}{2} e^T P B \phi \tilde{\theta}}_{\text{scalars}} + \underbrace{\frac{1}{2} \dot{\hat{\theta}}^T \Gamma \tilde{\theta} + \frac{1}{2} \tilde{\theta}^T \Gamma \dot{\hat{\theta}}}_{\text{scalars}}$$

$$\dot{V} = -\frac{1}{2} e^T Q e + \tilde{\theta}^T \phi^T B^T P e + \tilde{\theta}^T \Gamma \dot{\hat{\theta}}$$

$$\dot{V} = -\frac{1}{2} e^T Q e + \tilde{\theta}^T \left[\phi^T B^T P e + \Gamma \dot{\hat{\theta}}\right]$$

choose $\dot{\hat{\theta}} = -\Gamma^{-1} \phi^T B^T P e$

(adaptation law)

$$\dot{V} = -\frac{1}{2} e^T Q e \leq 0 \qquad (\text{n.s.d})$$

conclude stability

We need Barbalat's lemma to conclude asymptotic convergence of $\tilde{q} \to 0$

$$\dot{V} = -e^T Q e$$

$$\int_0^t \frac{dV}{dt}\, dt = -\int_0^t e^T Q e\, dt$$

$$V(t) - V(0) = -\int_0^t e^T(t)\, Q\, e(t)\, dt$$

$V(0)$ is finite

$V(t)$ is non-increasing and non-negative

$\rightarrow V(t) - V(0) < \infty \implies \lim_{T\to\infty} \int_0^T e^T Q e\, dt < \infty$

e is sq. integrable

$\rightarrow e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} \rightarrow \tilde{q}$ and $\dot{\tilde{q}}$ must be square integrable

$\dot{V} \le 0,\ V \ge 0 \rightarrow e$ must be bounded

$\rightarrow \tilde{q}, \dot{\tilde{q}}$ must be bounded

Apply Barbalat's lemma to $\tilde{q}$

$\tilde{q} \to 0$ as $t \to \infty$

No guarantee that $\tilde{\theta} \to 0$