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$$H_1^0 = \mathrm{Trans}_{z,1}\ \mathrm{Rot}_{z,-90°}\ \mathrm{Rot}_{y,90°}$$ all current

$$H_1^0 = \mathrm{Rot}_{x,90°}\ \mathrm{Trans}_{y,1}\ \mathrm{Rot}_{z,-90°}$$ all current

$$H_1^0 = \mathrm{Rot}_{z,-90} \cdot \mathrm{Trans}_{z,1} \cdot \mathrm{Rot}_{y,90}$$ all fixed

$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$

many possibilities !!

$$H_2^1 = \mathrm{Trans}_{z,-1} \cdot \mathrm{Trans}_{x,1} \cdot \mathrm{Rot}_{x,90} \cdot \mathrm{Rot}_{z,90}$$ (current)

$\vdots \qquad \vdots$

many possibilities.

Take the third H_1^0 and H_2^1 :

$$H_1^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad H_2^1 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = H_1^0 H_2^1 = \left[\begin{array}{ccc|c} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R_2^0 & O_2^0 \\ \hline 0 & 1 \end{array}\right]$$

By inspection $O_2^0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ ✓✓

Also, "2" is obtained from "0" by : (rotations only)

$Rot_{x,90}$, $Rot_{y,-90}$ (current)

so $R_2^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ +1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ ✓✓

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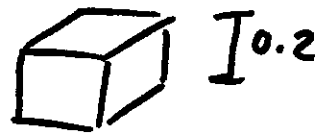
$$H_1^0 = \text{Trans}_{y,1} \cdot \text{Trans}_{z,1} = \left[\begin{array}{c|c} I & \begin{smallmatrix}0\\1\\0\end{smallmatrix} \\ \hline 0 & 1 \end{array}\right] \cdot \left[\begin{array}{c|c} I & \begin{smallmatrix}0\\0\\1\end{smallmatrix} \\ \hline 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} I & I\begin{bmatrix}0\\1\\0\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix} \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} I & \begin{bmatrix}0\\1\\1\end{bmatrix} \\ \hline 0 & 1 \end{array}\right]$$

H_2^0 : Use center of <u>cube</u> as O_2



$$H_2^0 = \text{Trans}_{y,1.5} \cdot \text{Trans}_{x,-0.5} \cdot \text{Trans}_{z,1.1}$$

$$= \left[\begin{array}{c|c} I & \begin{bmatrix}0\\1.5\\0\end{bmatrix} \\ \hline 0 & 1 \end{array}\right] \cdot \left[\begin{array}{c|c} I & \begin{bmatrix}-0.5\\0\\0\end{bmatrix} \\ \hline 0 & 1 \end{array}\right] \cdot \left[\begin{array}{c|c} I & \begin{bmatrix}0\\0\\1.1\end{bmatrix} \\ \hline 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} I & \begin{bmatrix}0\\1.5\\0\end{bmatrix} + \begin{bmatrix}-0.5\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\1.1\end{bmatrix} \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} I & \begin{bmatrix}-0.5\\1.5\\1.1\end{bmatrix} \\ \hline 0 & 1 \end{array}\right]$$

<u>Note</u>

$$\left[\begin{array}{c|c} I & a \\ \hline 0 & 1 \end{array}\right] \times \left[\begin{array}{c|c} I & b \\ \hline 0 & 1 \end{array}\right] \times \left[\begin{array}{c|c} I & c \\ \hline 0 & 1 \end{array}\right] \times \cdots\cdots$$

$$= \left[\begin{array}{c|c} I & a+b+c\ldots\ldots \\ \hline 0 & 1 \end{array}\right],$$ also product is commutative.

$$H_3^0 = \text{Trans}_{y,1.5}\,\text{Trans}_{x,-0.5}\,\text{Trans}_{z,3}\,\text{Rot}_{z,-90}\,\text{Rot}_{y,180}$$

$$= \left[\begin{array}{c|c} I & \begin{bmatrix}-0.5\\1.5\\3\end{bmatrix} \\ \hline 0 & 1 \end{array}\right] * \underbrace{\left[\begin{array}{ccc|c} 0 & 1 & 0 & \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \\ \hline & 0 & & 1 \end{array}\right] * \left[\begin{array}{ccc|c} -1 & 0 & 0 & \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \\ \hline & 0 & & 1 \end{array}\right]}_{\text{you may multiply the rotation parts separately:}}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$H_3^0 = \left[\begin{array}{c|c} I & \begin{bmatrix}-0.5\\1.5\\3\end{bmatrix} \\ \hline 0 & 1 \end{array}\right] * \left[\begin{array}{ccc|c} 0 & 1 & 0 & \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \\ \hline & 0 & & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & 1 & 0 & \begin{bmatrix}-0.5\\1.5\\3\end{bmatrix} \\ 1 & 0 & 0 & \\ 0 & 0 & -1 & \\ \hline & 0 & & 1 \end{array}\right]$$

Since $H_3^0 = H_2^0 \cdot H_3^2 \;\rightarrow\; H_3^2 = (H_2^0)^{-1} H_3^0$

$$= \underbrace{\left(\left[\begin{array}{c|c} I & \begin{bmatrix}-0.5\\1.5\\1.1\end{bmatrix} \\ \hline 0 & 1 \end{array}\right]\right)^{-1}}_{\text{use (2.60)}} \left[\begin{array}{ccc|c} 0 & 1 & 0 & \begin{bmatrix}-0.5\\1.5\\3\end{bmatrix} \\ 1 & 0 & 0 & \\ 0 & 0 & -1 & \\ \hline & 0 & & 1 \end{array}\right] = \ldots$$

$$H_3^2 = \left[\begin{array}{c|c} I^T & -I^T \begin{bmatrix} -0.5 \\ 1.5 \\ 1.1 \end{bmatrix} \\ \hline 0 & 1 \end{array}\right] \times \left[\begin{array}{ccc|c} 0 & 1 & 0 & \\ 1 & 0 & 0 & \begin{bmatrix} -0.5 \\ 1.5 \\ 3 \end{bmatrix} \\ 0 & 0 & -1 & \\ \hline & 0 & & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 0 & 1 & 0 & \\ 1 & 0 & 0 & \begin{bmatrix} -0.5 \\ 1.5 \\ 3 \end{bmatrix} - \begin{bmatrix} -0.5 \\ 1.5 \\ 1.1 \end{bmatrix} \\ 0 & 0 & -1 & \\ \hline & 0 & & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 0 & 1 & 0 & \\ 1 & 0 & 0 & \begin{bmatrix} 0 \\ 0 \\ 1.9 \end{bmatrix} \\ 0 & 0 & -1 & \\ \hline & 0 & & 1 \end{array}\right]$$

______ X ______