Department of Mechanical Engineering
Cleveland State University

MCE/EEC 647/747: Robot Dynamics and Control
Homework 4 - Fall 2008

OUT: 10-09-08. DUE: 10-23-08

1. (100 pts.)
Consider the following nonlinear system with two inputs and one output:

$$\begin{aligned} \dot{x}_1 &= x_1 x_2 - u_1 x_3 + u_2 \\ \dot{x}_2 &= -x_1^2 + (1 - x_2)u_1 + x_2 x_3 u_2 \\ \dot{x}_3 &= -2x_1 + 3x_2 + u_1 - u_2 \\ y &= x_1 + x_2 + x_3 \end{aligned}$$

1. Find all equilibrium points for $u_0 = [1; 0]^T$.

2. Linearize the system about the equilibrium point x_0 having all positive components. Represent the linearized system as
$$\dot{\Delta x} = A\Delta x + B\Delta u$$

3. Determine the stability of the linearized system about x_0. Also determine controllability and observability.

4. Choose a feedback gain K so that the closed-loop poles of the linearized system under the control $\Delta u = -K\Delta x$ are $s = -25$ and $s = -7.5 \pm 4.375$.

5. Build a Simulink diagram that simultaneously applies the control $u = u_0 + \Delta u = u_0 - K(x - x_0)$ to the nonlinear system and the control $\Delta u = -K\Delta x$ to the linearized system. Simulate using consistent initial conditions (use $(\Delta x)_0$ in the linearized system and $x_0 + (\Delta x)_0$ in the nonlinear system). Show that the nonlinear system can be unstable when operated away from x_0.

6. Find the state equations for the nonlinear system under the control $u = u_0 - K(x - x_0)$. Verify that x_0 is still an equilibrium point.

7. Consider the quadratic Lyapunov function $V = (x - x_0)^T P(x - x_0)$ with
$$P = \begin{bmatrix} 0.1921 & -0.1690 & 0.1458 \\ -0.1690 & 0.2053 & -0.1355 \\ 0.1458 & -0.1355 & 0.1555 \end{bmatrix} \quad (1)$$
Write a program that scans a neighborhood of x_0 using a spherical sweep to determine the sign of $\dot{V} = \dot{x}^T P(x - x_0) + (x - x_0)^T P\dot{x}$. Run the program to determine a radius of attraction for x_0 as determined by V (expected to be small). Test for $\dot{V} > 0$ using a small tolerance (take $\dot{V} > 0.001$, for example). Pick a point just outside the radius of attraction and use it as an initial condition for simulation. Does the point belong to the actual region of attraction?

How to turn in the homework:

Any hand calculations must be explained. Please scan any handwritten sheets. Email the instructor, attaching the hand calculations, m-file(s) and Simulink files (.mdl). Please save the Simulink files as version 6.4 or earlier (Matlab version 2006a or earlier). You are encouraged to consult the instructor on issues related to Matlab/Simulink.