Lecture 12: Multivariable Control of Robotic Manipulators
Part II

Reading: SHV Ch.8

Mechanical Engineering

Hanz Richter, PhD

Robust vs. Adaptive Control

A *robust* control system uses a fixed controller capable of "performing well" (guaranteed stability+performance) despite uncertainties in plant model parameters.
An *adaptive* system, in contrast, attempts to obtain estimates of uncertain/unknown plant parameters and contains self-adjusting control parameters, which are calculated on the basis of the current estimates of the plant parameters. That is, the structure of the controller is fixed, but the control parameters (gains) are self-adjusted.

Adaptive systems were used in the 1950's before they were fully understood. Stability proofs and systematic procedures became available as late as the 1980's. Kumpati Narendra and Karl Åström are among the most recognized researchers in this field (see textbooks by each).

Robot Adaptive Inverse Dynamics

Consider the manipulator dynamics again:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{q} + g(q) = u$$

We know that an inner-loop feedback linearizing input of the form

$$u = M(q)a_q + C(q,\dot{q})\dot{q} + g(q)$$

results in a double integrator plant $\ddot{q} = a_q$ which is easily stabilized by decoupled PD loops.

Now suppose $M(q)$, $C(q,\dot{q})$ and $g(q)$ are not known exactly. We only have current estimates $\hat{M}$, $\hat{C}$ and $\hat{q}$ which will be adjusted in real time by the adaptive system.

Take the control law (drop the arguments q and $\dot{q}$ for notational simplicity):

$$u = \hat{M}a_q + \hat{C}\dot{q} + \hat{g}$$

Adaptive Inverse Dynamics...

Take the virtual control a_q to be

$$a_q = \ddot{q}^d - K_0\tilde{q} - K_1\dot{\tilde{q}}$$

where K_0 and K_1 are positive-definite diagonal matrices and $\tilde{q} = q - q^d$ is the tracking error. In class, we carry the full details of the derivation of the adaptation law (how to update $\hat{M}$, $\hat{C}$ and $\hat{q}$).

First, we recall the linear parameterization property:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})\Theta$$

This allows us to isolate all parameters in vector Θ. Defining the parameter estimation error as $\tilde{\Theta} = \hat{\Theta} - \Theta$, we show that the dynamics of the tracking error are given by

$$\dot{e} = Ae + B\Phi\tilde{\Theta}$$

where A is made Hurwitz by choosing K_0 and K_1 to be diagonal with positive nonzero entries and $\Phi = \hat{M}^{-1}Y$.

Adaptive Inverse Dynamics...

Since A is Hurwitz, we can find a symmetric and positive-definite matrix as the solution to the Lyapunov equation

$$A^T P + PA = -Q$$

where Q is an arbitrary symmetric, positive-definite matrix.
Then we form a Lyapunov function for the adaptive system as

$$V = \frac{1}{2} e^T P e + \frac{1}{2} \tilde{\Theta}^T \Gamma \tilde{\Theta}$$

We compute its derivative and force it to be negative-semidefinite by choosing the parameter adaptation law as follows

$$\dot{\hat{\Theta}} = -\Gamma^{-1} \Phi^T B^T P e$$

where Γ is a symmetric positive-definite matrix and $\Phi = \hat{M}^{-1} Y$.

Adaptive Inverse Dynamics...

In summary, the control system is formed by a control law and a parameter adaptation law:

$$\begin{aligned} u &= \hat{M}(\ddot{q}^d - K_0 \tilde{q} - K_1 \dot{\tilde{q}}) + \hat{C}\dot{q} + \hat{g} \\ \dot{\hat{\Theta}} &= -\Gamma^{-1} \Phi^T B^T P e \end{aligned}$$

Notes:

Since $\dot{V} = -\frac{1}{2} e^T Q e$ (independent of $\tilde{\Theta}$), it follows that it is zero at non-zero values of $[e^T | \tilde{\Theta}^T]^T$. This implies $\dot{V}$ is really negative-semidefinite. Therefore, it can only be said that e is and $\tilde{\Theta}$ are bounded. We can actually prove $e \to 0$ by taking extra steps (details developed in class). The parameter estimation error can only be guaranteed to be bounded (sometimes good enough). A more serious problem is the need to measure $\ddot{q}$ in realtime and that $\hat{M}$ must be invertible at all times.

Passivity-Based Robot Control

The inverse dynamics approaches seen before are characterized by their attempt to cancel plant nonlinearities and transform the control problem into a linear one.
An alternative is to apply direct nonlinear control techniques to guarantee that the closed-loop system will be stable, without attempting to linearize it.
Take the control input to be

$$u = M(q)a + C(q,\dot{q})v + g(q) - Kr$$

where

$$\begin{aligned} v &= \dot{q}^d - \Lambda\tilde{q} \\ a &= \dot{v} \\ r &= \dot{q} - v \end{aligned}$$

with K and Λ being diagonal matrices with positive nonzero entries.

Passivity-Based Control...

Substitution results in the closed-loop dynamics

$$M(q)\dot{r} + C(q,\dot{q})r + Kr = 0$$

The closed-loop system is nonlinear and coupled. Consider the Lyapunov function

$$V = \frac{1}{2}r^T M(q)r + \tilde{q}^T \Lambda K \tilde{q}$$

As we show in class, the derivative computes to

$$\dot{V} = -e^T Q e$$

where Q is symmetric and positive-definite, making $\dot{V}$ negative-definite. The error dynamics are therefore globally asymptotically stable.

Note that we still require exact knowledge of the inertia matrix, the C matrix and the gravity term. The real advantage of the passivity method comes when considering the robust and adaptive cases.

Robust Passivity-Based Control

Assuming only estimates of the robot matrices are available, we take

$$u = \hat{M}(q)a + \hat{C}(q,\dot{q})v + \hat{g}(q) - Kr$$

Using the linear parameterization we can write the control as

$$u = Y\hat{\Theta} - Kr$$

As we show in class, we take $\hat{\Theta} = \Theta_0 + \delta\Theta$, where Θ_0 is our best estimate of the parameters (nominal values) and $\delta\Theta$ is a new control. The uncertainity in Θ must be bounded by a known constant ρ:

$$||\tilde{\Theta}|| = ||\Theta - \Theta_0|| \leq \rho$$

Then we use the *variable-structure control* or *discontinuous control* of Eq.(8.98) to force the previous Lyapunov function to be negative-definite.

Robust Passivity-Based Control...

Note that this powerful control strategy requires only position and velocity feedback (easily obtained from encoders) and an estimate of parameter uncertainty $(\rho$ bound).

In the final project, you will be implementing this controller assuming imperfect knowledge of the manipulator parameters.

Adaptive Passivity-Based Control

In this case, our estimates of M, C and g are not frozen, but will be continuously adjusted by the system. The form of the control law is the same as in the robust passivity-based approach:

$$u = \hat{M}(q)a + \hat{C}(q,\dot{q})v + \hat{g}(q) - Kr$$

We again obtain

$$M(q)\dot{r} + C(q,\dot{q})r + Kr = Y\tilde{\Theta}$$

The derivation of an adaptation law is done on the basis of the Lyapunov function

$$V = \frac{1}{2}r^T M(q)r + \tilde{q}^T \Lambda K \tilde{q} + \frac{1}{2}\tilde{\Theta}^T \Gamma \tilde{\Theta}$$

As shown in class, the parameter adaptation law

$$\dot{\hat{\Theta}} = -\Gamma^{-1} Y^T(q,\dot{q},a,v)r$$

results in a negative-semidefinite derivative $\dot{V}$.

Adaptive Passivity-Based Control...

Since $\dot{V} = -e^T Qe \leq 0$, we can only conclude non-asymptotic stability at this point. However, e is a *square-integrable* function, since the integral of a quadratic function of e is a finite number, computable in closed form as:

$$-\int_0^t e^T(\tau)Qe(\tau)d\tau = V(t) - V(0)$$

This implies that $\tilde{q}$ itself and $\dot{\tilde{q}}$ are themselves square-integrable (components of e). Also, $\dot{V} \leq 0$ and the fact that V contains only non-negative terms must mean that r, $\tilde{q}$ and $\tilde{\Theta}$ remain bounded. Therefore, since $\dot{\tilde{q}} = r - \Lambda\tilde{q}$, it must also be bounded, which concludes boundedness of $\tilde{q}$.

Adaptive Passivity-Based Control...

These two facts ($\tilde{q}$ square integrable and $\dot{\tilde{q}}$ bounded can be used in *Barbalat's Lemma* (see p.311 in SHV) to conclude $\tilde{q} \to 0$ as $t \to \infty$.
We can go further and use a similar argument to conclude that $\dot{\tilde{q}} \to 0$.
To use Barbalat's Lemma, we need to establish that $\ddot{\tilde{q}}$ is bounded, which follows from the closed-loop equation

$$M(q)\dot{r} + C(q,\dot{q})r + Kr = Y\tilde{\Theta}$$

if we assume that $\ddot{q}^d$ is bounded.

Example: Two-Link Planar Robot

We design each of the above controllers to the 2-link planar manipulator. The regressor $Y(q,\dot{q},\ddot{q})$ and the parameter vector Θ are listed in SHV, p.271. We assume that the true value of each one of the parameters Θ_i is unity. For robust schemes, we take a random parameter deviation of 10$\%$ in each parameter. For adaptive schemes, we take a deviation of 90$\%$ as initial guess for the adaptation laws.
We first design an inverse dynamics controller to track a circular trajectory using joint-space feedback, then using task-space feedback.
We examine the performance of the controller under off-nominal conditions.

We the tune the controllers to track the same circular trajectory using adaptive inverse dynamics, robust passivity-based control and adaptive passivity-based control.