Output Feedback Control

■ Basic state-space control methods assume that all states are available for measurement, so that the control law can be calculated.

■ In practice, this rarely happens. Only a set of outputs, assumed linear functions of the states, are measured and controlled. The output equation has the form

\[ y = Cx \]

■ Since \( C \) is generally not an invertible matrix, \( x \) cannot be obtained from \( y \).

■ Output feedback refers to a control law calculation based on \( y \) only. Output feedback stabilization is possible when the plant is at least detectable.

■ That is, the unobservable states decay naturally, while \( y \) contains enough information on the remaining states.
Observers

- A state estimator or observer is an artificial dynamic system that evolves simultaneously with the plant. Its function is to estimate the state of the plant using input and output information.

- The observer has two inputs: \( u \) and \( y \) from the plant, and its output is the estimate of the plant state vector \( \hat{x} \).

- How are observers built? Why not just
  \[
  \dot{\hat{x}} = A\hat{x} + Bu
  \]

- The Luenberger observer solves the initial condition mismatch problem by introducing feedback.

- Add an error-correcting term \( H(y - \hat{y}) = H(y - C\hat{x}) \) to obtain
  \[
  \dot{\hat{x}} = (A - HC)\hat{x} + Bu + Hy
  \]

- Matrix \( H \) is the observer gain. It must be chosen so that \( A - HC \) has eigenvalues on the left half of the complex plane for \( \hat{x} \) to converge to \( x \).

Separation Principle and Dynamic Compensator

- What happens when the state estimates \( \hat{x} \) are used in the computation of the state feedback law \( (u = -K\hat{x}) \)?

- The closed-loop system can be reduced to
  \[
  \dot{x} = (A - BK)x + BK\bar{x}
  \]
  \[
  \dot{\bar{x}} = (A - HC)x
  \]
  where \( \bar{x} = x - \hat{x} \) is the estimation error.

- Note that the convergence of the estimator and the stability of the plant can be enforced separately, by placing the poles of \( (A - BK) \) and \( (A - HC) \) on the left half of the complex plane.

- It is good practice to tune the observer to converge faster than the plant, by choosing faster poles in the observer.

- The transfer function of the observer-controller combination can be shown to be
  \[
  K(s) = \frac{U(s)}{Y(s)} = -K[(sI - A + BK + HC)^{-1}]H
  \]
Design an observer-based state feedback controller to stabilize the double integrator plant. Use LQR tuning for both the observer and the controller. Simulate.