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2/23

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MCE/EEC 647/747: Robot Dynamics and Control
Homework 2 - Spring 2010

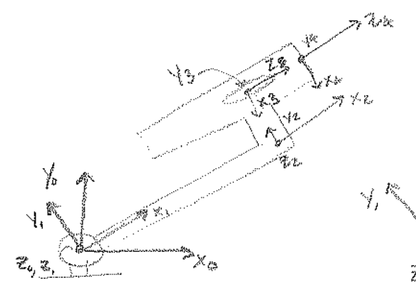
OUT: 02-10-10. DUE: 02-23-10

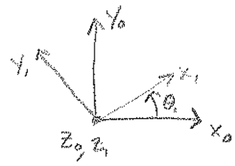
1. (20 pts ea.)
Solve the following problems from SHV:

- 3-4
- 3-6
- 4-22

2. (40 pts)
Find the Jacobian for the Puma manipulator of Fig. 3.31 (Prob. 3.10) in SHV. Use $o_n = o_9$ and frame x_9, y_9, z_9 as the end effector frame, as done in the solution of Prob. 3.10 (Fall 2008) posted on the course website. Include the details about the DH frame assignment in your homework. You must write a Matlab program that returns the Jacobian numerically, given values for the joint variables.

3-4





Base Frame / Frame 1

$O_0 = O_1 \qquad z_1 = z_0$

$a_1 = 0$

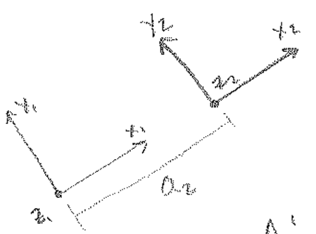
$\alpha_1 = 0$

$d_1 = 0$

$\theta_1 = \theta_1^*$

$A_1^0 = Rot_{z,\theta_1}$

Frame 2:



$a_2 = a_2$

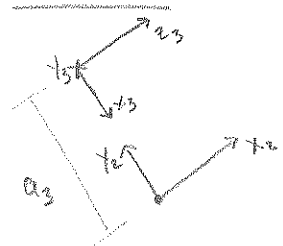
$\alpha_2 = 0$

$d_2 = 0$

$\theta_2 = 0$

$A_2^1 = Trans_{x,a_2}$

Frame 3:



$a_3 = a_3$

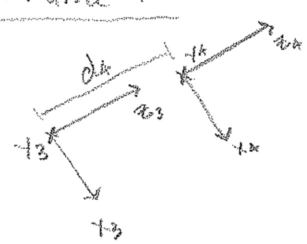
$\alpha_3 = -90°$

$d_3 = 0$

$\theta_3 = -90°$

$A_3^2 = Rot_{z,-90°}\, Trans_{x,a_3}\, Rot_{x,-90°}$

Frame 4:



$a_4 = 0$

$\alpha_4 = 0$

$d_4 = d_4^*$

$\theta_4 = 0$

$A_4^3 = Trans_{x,d_4}$

$$T_4^0 = A_1^0 A_2^1 A_3^2 A_4^3$$

$$A_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^1 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

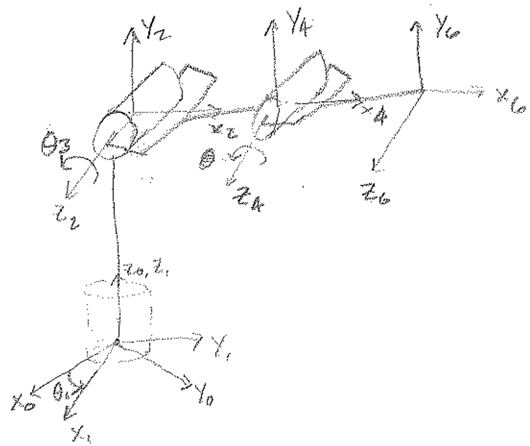
$$A_4^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
% Prob 3-4 (SHV)
% Matlab code

syms th1 a2 a3 d4
A10 = [cos(th1) -sin(th1) 0 0; sin(th1) cos(th1) 0 0; 0 0 1 0; 0 0 0 1];
A21 = [eye(3) [a2;0;0]; 0 0 0 1];
A32 = [0 1 0 0; -1 0 0 0; 0 0 1 0; 0 0 0 1]*[eye(3) [a3;0;0]; 0 0 0 1]*[1
0 0 0; 0 0 1 0; 0 1 0 0; 0 0 0 1];
A43 = [eye(3) [0;0;d4]; 0 0 0 1];

T40 = A10*A21*A32*A43
```

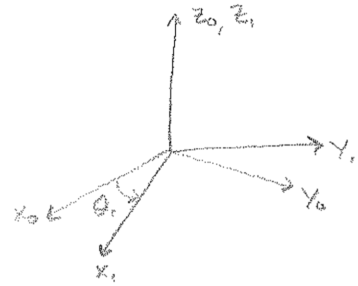
The forward kinematic equations using the DH convention are given by:

```
T40 = [  sin(th1),   0,   cos(th1),   a2*cos(th1) + d4*cos(th1) + a3*sin(th1)]
      [ -cos(th1),   0,   sin(th1),   a2*sin(th1) - a3*cos(th1) + d4*sin(th1)]
      [         0,   1,          0,                                          0]
      [         0,   0,          0,                                          1]
```

3-6



Base Frame / Frame 1



$O_0 = O_1 \qquad z_0 = z_1$

Link 1: $a_1 = 0$

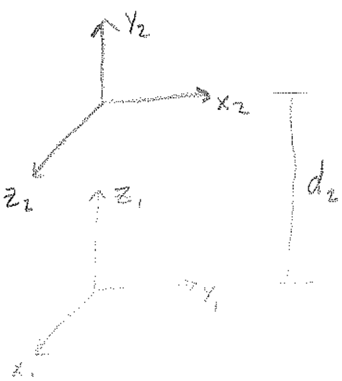
$\alpha_1 = 0$

$d_1 = 0$

$\theta_1 = \theta_1^*$

$A_1^0 = Rot_{z,\theta_1}$

Frame 2:



Link 2: $a_2 = 0$

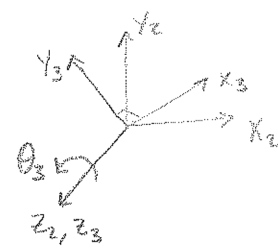
$\alpha_2 = 90°$

$d_2 = d_2$

$\theta_2 = 90°$

$A_2^1 = Rot_{z,90°} Trans_{z,d_2} Rot_{x,90}$

Frame 3:



Link 3:

$a_3 = 0$

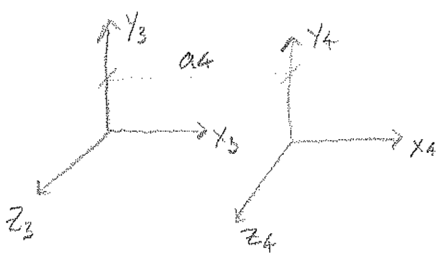
$\alpha_3 = 0$

$d_3 = 0$

$\theta_3 = \theta_3^*$

$A_3^2 = Rot_{z,\theta_3}$

Frame 4:



Link 4: $a_4 = a_4$

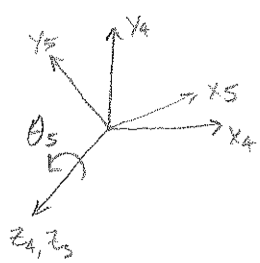
$\alpha_4 = 0$

$d_4 = 0$

$\theta_4 = 0$

$A_4^3 = Trans_{x,a_4}$

Frame 5:



Link 5: $a_5 = 0$

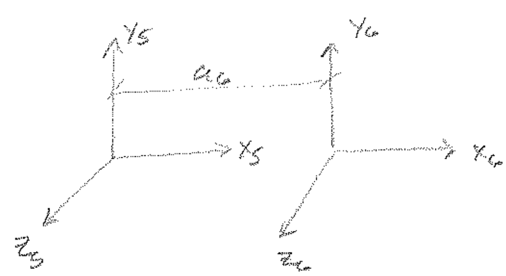
$\alpha_5 = 0$

$d_5 = 0$

$\theta_5 = \theta_5^*$

$A_5^4 = Rot_{z,\theta_5}$

Frame 6:



Link 6: $a_6 = a_6$

$\alpha_6 = 0$

$d_6 = 0$

$\theta_6 = 0$

$A_6^5 = Trans_{x,a_6}$

$$A_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4^3 = \begin{bmatrix} 1 & 0 & 0 & a_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_5^4 = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6^5 = \begin{bmatrix} 1 & 0 & 0 & a_6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
% Prob 3-6 (SHV)
% Matlab code

syms th1 d2 th3 a4 th5 a6
A10 = [cos(th1) -sin(th1) 0 0; sin(th1) cos(th1) 0 0; 0 0 1 0; 0 0 0 1];
A21 = [0 -1 0 0; 1 0 0 0; 0 0 1 0; 0 0 0 1]*[1 0 0 0; 0 1 0 0; 0 0 1 d2; 0
0 0 1]*[1 0 0 0; 0 0 -1 0; 0 1 0 0; 0 0 0 1];
A32 = [cos(th3) -sin(th3) 0 0; sin(th3) cos(th3) 0 0; 0 0 1 0; 0 0 0 1];
A43 = [eye(3) [a4;0;0]; 0 0 0 1];
A54 = [cos(th5) -sin(th5) 0 0; sin(th5) cos(th5) 0 0; 0 0 1 0; 0 0 0 1];
A65 = [eye(3) [a6;0;0]; 0 0 0 1];

T60 = A10*A21*A32*A43*A54*A65;
```

The forward kinematic equations using the DH convention are given by:

T60 = [t11 t12 t13 t14]
[t21 t22 t23 t24]
[t31 t32 t33 t34]
[t41 t42 t43 t44]

where,

t11 = sin(th1)*sin(th3)*sin(th5) - cos(th3)*cos(th5)*sin(th1)
t12 = cos(th3)*sin(th1)*sin(th5) + cos(th5)*sin(th1)*sin(th3)
t13 = cos(th1)
t14 = a6*(sin(th1)*sin(th3)*sin(th5) - cos(th3)*cos(th5)*sin(th1)) - a4*cos(th3)*sin(th1)
t21 = cos(th1)*cos(th3)*cos(th5) - cos(th1)*sin(th3)*sin(th5)
t22 = -cos(th1)*cos(th3)*sin(th5) - cos(th1)*cos(th5)*sin(th3)
t23 = sin(th1)
t24 = a4*cos(th1)*cos(th3) - a6*(cos(th1)*sin(th3)*sin(th5) - cos(th1)*cos(th3)*cos(th5))
t31 = cos(th3)*sin(th5) + cos(th5)*sin(th3)
t32 = cos(th3)*cos(th5) - sin(th3)*sin(th5)
t33 = 0
t34 = d2 + a6*(cos(th3)*sin(th5) + cos(th5)*sin(th3)) + a4*sin(th3)
t41 = 0
t42 = 0
t43 = 0
t44 = 1

4-22)

$$J = \begin{bmatrix} z_0 \times (o_6 - o_0) & z_1 \times (o_6 - o_1) & z_2 & z_3 \times (o_6 - o_3) & z_4 \times (o_6 - o_4) & z_5 \times (o_6 - o_5) \\ z_0 & z_1 & 0 & z_3 & z_4 & z_5 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} -s\theta_1 \\ c\theta_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} c\theta_1 s\theta_2 \\ s\theta_1 s\theta_2 \\ c\theta_2 \end{bmatrix} \quad z_3 = \begin{bmatrix} c\theta_1 s\theta_2 \\ s\theta_1 s\theta_2 \\ c\theta_2 \end{bmatrix} \quad z_4 = \begin{bmatrix} -c1c2s4 - s1c4 \\ -s1c2s4 + c1c4 \\ s2s4 \end{bmatrix}$$

$$z_5 = \begin{bmatrix} c1c2c4s5 - s1s4s5 + c1s2c5 \\ s1c2c4s5 + c1s4s5 + s1s2c5 \\ -s2c4s5 + c2c5 \end{bmatrix}$$

$$o_0 = o_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad o_3 = o_4 = o_5 = \begin{bmatrix} c1s2d3 - s1d2 \\ s1s2d3 + c1d2 \\ c2d3 \end{bmatrix}$$

$$o_6 = \begin{bmatrix} c1s2d3 - s1d2 + d6(c1c2c4s5 + c1c5s2 - s1s4s5) \\ s1s2d3 - c1d2 + d6(c1s4s5 + c2c4s1s5 + c5s1s2) \\ c2d3 + d6(c_2c_5 - c_4s_2s_5) \end{bmatrix}$$

```
% Prob 4-22 (SHV)
% Matlab code

syms s1 c1 s2 c2 c4 s4 c5 s5 d2 d3 d6
z0 = [0;0;1];
z1 = [-s1; c1; 0];
z2 = [c1*s2; s1*s2; c2];
z3 = z2;
z4 = [-c1*c2*s4-s1*c4; -s1*c2*s4+c1*c4; s2*s4];
z5 = [c1*c2*c4*s5-s1*s4*s5+c1*s2*c5; s1*c2*c4*s5+c1*s4*s5+s1*s2*c5; -
s2*c4*s5+c2*c5];
o0 = [0;0;0];
o1 = o0;
o3 = [c1*s2*d3-s1*d2; s1*s2*d3+c1*d2; c2*d3];
o4 = o3;
o5 = o3;
o6 = [c1*s2*d3-s1*d2+d6*(c1*c2*c4*s5+c1*c5*s2-s1*s4*s5); s1*s2*d3-
c1*d2+d6*(c1*s4*s5+c2*c4*s1*s5+c5*s1*s2); c2*d3+d6*(c2*c5-c4*s2*s5)];

J = [cross(z0,o6-o0) cross(z1,o6-o1) z2 cross(z3,o6-o3) cross(z4,o6-o4)
cross(z5,o6-o5); z0 z1 [0;0;0] z3 z4 z5];

Jv1 = J(1:3,1)
Jv2 = J(1:3,2)
Jv3 = J(1:3,3)
Jv4 = J(1:3,4)
Jv5 = J(1:3,5)
Jv6 = J(1:3,6)
Jw1 = J(4:6,1)
Jw2 = J(4:6,2)
Jw3 = J(4:6,3)
Jw4 = J(4:6,4)
Jw5 = J(4:6,5)
Jw6 = J(4:6,6)
```

The linear and angular velocity components of the Jacobian are given by:

Jv1 = c1d2 - d6*(c5s1s2 + c1s4s5 + c2c4s1s5) - d3s1s2
d6*(c1c5s2 - s1s4s5 + c1c2c4s5) - d2s1 + c1d3s2
0

Jv2 = c1*(c2d3 + d6*(c2c5 - c4s2s5))
s1*(c2d3 + d6*(c2c5 - c4s2s5))
-c1*(d6*(c1c5s2 - s1s4s5 + c1c2c4s5) - d2s1 + c1d3s2) - s1*(d6*(c5s1s2 + c1s4s5 + c2c4s1s5) - c1d2 + d3s1s2)

Jv3 = c1s2
s1s2
c2

Jv4 = c2*(2*c1d2 - d6*(c5s1s2 + c1s4s5 + c2c4s1s5)) + d6s1s2*(c2c5 - c4s2s5)
c2d6*(c1c5s2 - s1s4s5 + c1c2c4s5) - c1d6s2*(c2c5 - c4s2s5)
-c1s2*(2*c1d2-d6*(c5s1s2+c1s4s5+c2c4s1s5)) - d6s1s2*(c1c5s2 - s1s4s5 + c1c2c4s5)

Jv5 = d6*(c1c4 - c2s1s4)*(c2c5 - c4s2s5) + s2s4*(2*c1d2 - d6*(c5s1s2 + c1s4s5 + c2c4s1s5))
d6*(c4s1 + c1c2s4)*(c2c5 - c4s2s5) + d6s2s4*(c1c5s2 - s1s4s5 + c1c2c4s5)
(2*c1d2 - d6*(c5s1s2 + c1s4s5 + c2c4s1s5))*(c4s1 + c1c2s4) - d6*(c1c4 - c2s1s4)*(c1c5s2 - s1s4s5 + c1c2c4s5)

Jv6 = (2*c1d2 - d6*(c5s1s2 + c1s4s5 + c2c4s1s5))*(c2c5 - c4s2s5) + d6*(c2c5 - c4s2s5)*(c5s1s2 + c1s4s5 + c2c4s1s5)
0
-(2*c1d2 - d6*(c5s1s2 + c1s4s5 + c2c4s1s5))*(c1c5s2 - s1s4s5 + c1c2c4s5) - d6*(c1c5s2 - s1s4s5 + c1c2c4s5)*(c5s1s2 + c1s4s5 + c2c4s1s5)

Jw1 = 0
0
1

Jw2 = -s1
c1
0

Jw3 = 0
0
0

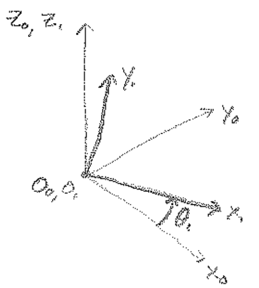
Jw4 = c1s2
s1s2
c2

Jw5 = -c4s1 - c1c2s4
c1c4 - c2s1s4
s2s4

Jw6 = c1c5s2 - s1s4s5 + c1c2c4s5
c5s1s2 + c1s4s5 + c2c4s1s5
c2c5 - c4s2s5

PROBLEM #2

Base Frame / Frame 1:

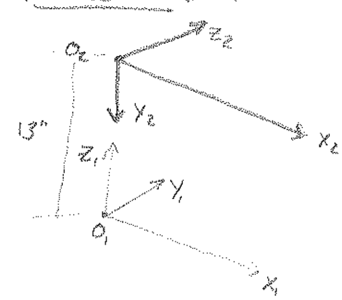


$o_1 = o_0$; $z_1 = z_0$

$a_1 = 0$
$\alpha_1 = 0$
$d_1 = 0$
$\theta_1 = \theta_1^*$

$A_1^0 = Rot_{z,\theta_1}$

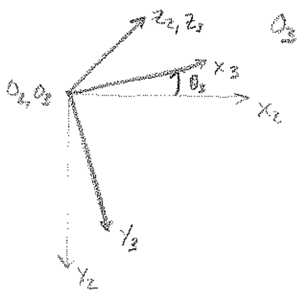
Frame 2: (AUX)



$a_2 = 0$
$\alpha_2 = -90°$
$d_2 = 13$
$\theta_2 = 0$

$A_2^1 = Trans_{z,13}\ Rot_{x,-90°}$

Frame 3:

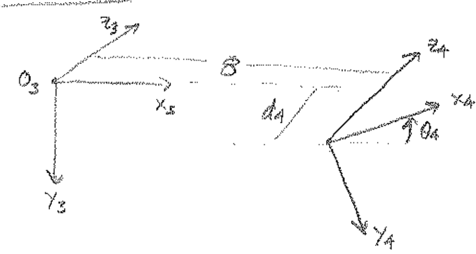


$o_3 = o_2$ $z_3 = z_2$

$a_3 = 0$
$\alpha_3 = 0$
$d_3 = 0$
$\theta_3 = \theta_3^*$

$A_3^2 = Rot_{z,\theta_3}$

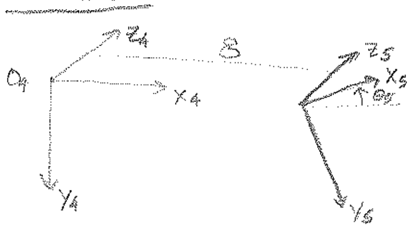
Frame 4:



$a_4 = 8$
$\alpha_4 = 0$
$d_4 = d_4$
$\theta_4 = \theta_4^*$

$A_4^3 = Rot_{z,\theta_4}\ Trans_{z,d_4}\ Trans_{x,8}$

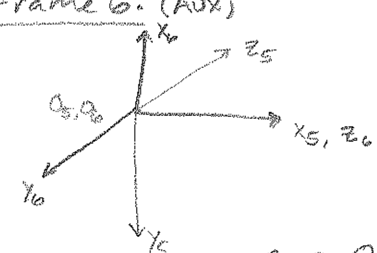
Frame 5:



$a_5 = 8$
$\alpha_5 = 0$
$d_5 = 0$
$\theta_5 = \theta_5^*$

$A_5^4 = Rot_{z,\theta_5}\ Trans_{x,8}$

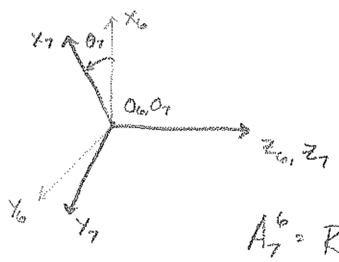
Frame 6: (AUX)



$a_6 = 0$
$\alpha_6 = -90°$
$d_6 = 0$
$\theta_6 = -90°$

$A_6^5 = Rot_{z,-90°}\ Rot_{x,-90°}$

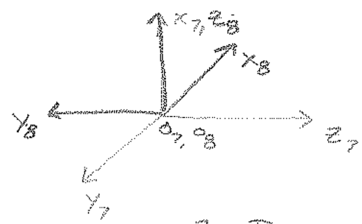
Frame 7:



$a_7 = 0$
$\alpha_7 = 0$
$d_7 = 0$
$\theta_7 = \theta_7^*$

$A_7^6 = Rot_{z, \theta_7}$

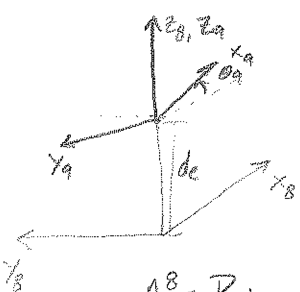
Frame 8: (AUX)



$a_8 = 0$
$\alpha_8 = -90°$
$d_8 = 0$
$\theta_8 = -90°$

$A_8^7 = Rot_{z,-90} \; Rot_{x,-90°}$

Frame 9:



$a_9 = 0$
$\alpha_9 = 0$
$d_9 = d_e$
$\theta_9 = \theta_9^*$

$A_9^8 = Rot_{z,\theta_9} \; Trans_{z, d_e}$

	a_i	α_i	d_i	θ_i
Frame 1	0	0	0	θ_1^*
Frame 2	0	-90°	13	0
Frame 3	0	0	0	θ_3^*
Frame 4	8	0	d_4	θ_4^*
Frame 5	8	0	0	θ_5^*
Frame 6	0	-90°	0	-90°
Frame 7	0	0	0	θ_7^*
Frame 8	0	-90°	0	-90°
Frame 9	0	0	d_e	θ_9^*

$$J = \begin{bmatrix} z_0 \times (o_9 - o_0) & z_2 \times (o_9 - o_2) & z_3 \times (o_9 - o_3) & z_4 \times (o_9 - o_4) & z_6 \times (o_9 - o_6) & z_8 \times (o_9 - o_8) \\ z_0 & z_2 & z_3 & z_4 & z_6 & z_8 \end{bmatrix}$$

z: $z_0, z_2, z_3, z_4, z_6, z_8$

O: $o_0, o_2, o_3, o_4, o_6, o_8, o_9$

$o_0 = o_1$
$o_2 = o_3$
o_4
$o_5 = o_6 = o_7 = o_8$
o_9

$$A_1^0 = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} c3 & -s3 & 0 & 0 \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4^3 = \begin{bmatrix} c4 & -s4 & 0 & 0 \\ s4 & c4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^4 = \begin{bmatrix} c5 & -s5 & 0 & 0 \\ s5 & c5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_6^5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_7^6 = \begin{bmatrix} c7 & -s7 & 0 & 0 \\ s7 & c7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_8^7 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_9^8 = \begin{bmatrix} c9 & -s9 & 0 & 0 \\ s9 & c9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

z_2 = A30(1:3, 3) $\qquad z_4$ = A50(1:3, 3)

z_3 = A40(1:3, 3) $\qquad z_6$ = A70(1:3, 3)

z_8 = A90(1:3, 3)

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

o_2 = A20 (1:3, 4) $\qquad o_6$ = A60 (1:3, 4)

o_3 = A30(1:3, 4) $\qquad o_8$ = A80(1:3, 4)

o_4 = A40(1:3, 4) $\qquad o_9$ = A90(1:3, 4)

$$A_9^0 = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5 A_7^6 A_8^7 A_9^8$$

```
% Problem 2 (PUMA)
% Unknowns: th1 th3 th4 th5 th7 th9 d4 de

A10 = [cos(th1) -sin(th1) 0 0; sin(th1) cos(th1) 0 0; 0 0 1 0; 0 0 0 1];
A21 = [eye(3) [0;0;13]; 0 0 0 1]*[1 0 0 0; 0 0 1 0; 0 -1 0 0; 0 0 0 1];
A32 = [cos(th3) -sin(th3) 0 0; sin(th3) cos(th3) 0 0; 0 0 1 0; 0 0 0 1];
A43 = [cos(th4) -sin(th4) 0 0; sin(th4) cos(th4) 0 0; 0 0 1 0; 0 0 0
1]*[eye(3) [0;0;d4]; 0 0 0 1]*[eye(3) [8;0;0]; 0 0 0 1];
A54 = [cos(th5) -sin(th5) 0 0; sin(th5) cos(th5) 0 0; 0 0 1 0; 0 0 0
1]*[eye(3) [8;0;0]; 0 0 0 1];
A65 = [0 1 0 0; -1 0 0 0; 0 0 1 0; 0 0 0 1]*[1 0 0 0; 0 0 1 0; 0 -1 0 0; 0
0 0 1];
A76 = [cos(th7) -sin(th7) 0 0; sin(th7) cos(th7) 0 0; 0 0 1 0; 0 0 0 1];
A87 = A65;
A98 = [cos(th9) -sin(th9) 0 0; sin(th9) cos(th9) 0 0; 0 0 1 0; 0 0 0
1]*[eye(3) [0;0;de]; 0 0 0 1];

A20 = A10*A21;
A30 = A10*A21*A32;
A40 = A10*A21*A32*A43;
A50 = A10*A21*A32*A43*A54;
A60 = A10*A21*A32*A43*A54*A65;
A70 = A10*A21*A32*A43*A54*A65*A76;
A80 = A10*A21*A32*A43*A54*A65*A76*A87;
A90 = A10*A21*A32*A43*A54*A65*A76*A87*A98;

z0 = [0;0;1];
z2 = A30(1:3,3);
z3 = A40(1:3,3);
z4 = A50(1:3,3);
z6 = A70(1:3,3);
z8 = A90(1:3,3);

o0 = [0;0;0];
o2 = A20(1:3,4);
o3 = A30(1:3,4);
o4 = A40(1:3,4);
o6 = A60(1:3,4);
o8 = A80(1:3,4);
o9 = A90(1:3,4);

J = [cross(z0,o9-o0) cross(z2,o9-o2) cross(z3,o9-o3) cross(z4,o9-o4)
cross(z6,o9-o6) cross(z8,o9-o8); z0 z2 z3 z4 z6 z8];
```