

EEC 693/793, ESC 794
Final Exam – 13 Problems
Fall 2008

Solution

1. The child of a non-engineer has a 90% chance of getting a non-engineering degree and a 10% chance of getting an engineering degree. The child of an engineer has an 80% chance of getting an engineering degree and a 20% chance of getting a non-engineering degree.
 - a. Write a Markov transition matrix Q for this process.
 - b. What is the steady-state percentage of engineers in the population?
 - c. What is the limit of Q^k as $k \rightarrow \infty$?

Solution:

$$Q = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$$

Eigenvalues $\lambda = 0.7, 1$

The eigenvector corresponding to $\lambda = 1$ is $[2/3 \ 1/3]^T$, so the steady-state percentage of engineers in the population is 33%

The columns of Q^∞ are equal to the eigenvector corresponding to $\lambda = 1$, so

$$Q^\infty = \begin{bmatrix} 2/3 & 2/3 \\ 1/3 & 1/3 \end{bmatrix}$$

2. Prove that all Markov transition matrices have an eigenvalue of 1.

Solution:

The elements in each column of a Markov transition Q matrix add up to 1. Therefore the elements in each column of $(I-Q)$ add up to 0. Using the notation $A=I-Q$, we have

$$A_{11} + A_{21} + \dots + A_{n1} = 0$$

...

$$A_{1n} + A_{2n} + \dots + A_{nn} = 0$$

Now add up all of the above equations to get

$$[A_{11} \ \dots \ A_{1n}] + \dots + [A_{n1} \ \dots \ A_{nn}] = [0 \ \dots \ 0]$$

In other words, the rows of A are linearly dependent. That means the rows of $I-Q$ are linearly dependent. That means $|I-Q| = 0$, which means that $|\lambda I-Q| = 0$ for $\lambda = 1$, which means that 1 is an eigenvalue of Q .

3. A Markov matrix Q is called *regular* or *primitive* if some power of Q has all nonzero elements. A Markov matrix is called *ergodic* or *irreducible* if it is possible to reach every state from every other state (not necessarily in one move). Give an example of a two-state Markov matrix that is ergodic but not regular.

Solution:

$$Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

4. Suppose that a population of N individuals, each with m bits, is randomly generated, each bit with an equal probability of being 0 or 1.
- What is the probability that there are no duplicates in the population?
 - What is the probability that the population contains at least one 0 and one 1 at each bit position?

Solution:

(a) The probability that two bits are the same is $1/2$. The probability that two individuals are the same is $(1/2)^m$. The probability that two individuals are not the same is $1 - (1/2)^m$. In an N -member population, there are $N(N-1)/2$ pairs of individuals. The probability that no two individuals are the same is therefore $[1 - (1/2)^m]^{N(N-1)/2}$.

(b) The probability that all bits at some bit position are the same is $(1/2)^{N-1}$. The probability that all bits at some bit position are not the same is $1 - (1/2)^{N-1}$. The probability that all bits at all bit positions are not the same is $[1 - (1/2)^{N-1}]^m$.

5. In biogeography, species are more likely to migrate to nearby islands than to distant islands. How could this be implemented in BBO? Answer in 30 words or less.

Solution:

Nearness in BBO could be quantified by the similarity between solutions (for example, Hamming or Euclidean distance). Migration probability between solutions could be made proportional to the distance between solutions.

6. Give an example of a computer intelligence paradigm that is based on natural processes that occur over a period of:
- Milliseconds – **Solution:** neural networks
 - Weeks or months – **Solution:** ant colony optimization, particle swarm optimization, adaptive cultural model
 - Centuries – **Solution:** BBO

d. Eons – **Solution:** GA

7. What is the advantage of gray coding in evolutionary algorithms?

Solution: Gray coding allows solutions with similar phenotypes to have similar genotypes. That is, it allows similar solutions to remain similar to each other even after encoding.

8. What is elitism?

Solution: Keep the n best individuals in the population from one generation to the next.

9. What is the difference between a strict tournament and a soft tournament?

Solution: A strict tournament always chooses the winning individual as a parent, while a soft tournament chooses parents based on a probability that is derived from their placement in the tournament.

10. Suppose that q -point crossover is used in a GA with m -bit chromosomes. What is the probability that a schema with defining length d survives crossover? Assume that crossover cannot occur at the end points of a chromosome.

Solution: The probability that one-point crossover occurs within the schema is $d / (m-1)$. The probability that one-point crossover does not occur within the schema is $1-d / (m-1)$. The probability that none of the crossover points in q -point crossover occurs within the schema is $[1-d / (m-1)]^q$.

Note: This assumes that the crossover points are chosen independently (for example, if $d \ll m$). Also, this is actually a lower bound, because even if crossover occurs within the schema, the schema could still survive, depending on the bit values of the other parent.

11. Suppose that N samples of a random variable x are taken.

- What is the estimate of the mean of x ?
- What is the unbiased estimate of the variance of x ?

Solution:

$$E(x) = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\text{var}(x) = \frac{1}{N-1} \sum_{i=1}^N (x_i - E(x))^2$$

12. What is a Pareto-optimal set? What is a Pareto-optimal front?

Solution: A Pareto-optimal set is the set of all solutions to a multiobjective optimization problem that are not dominated by any other solutions. A Pareto-optimal front is the set of all objective function values corresponding to the Pareto-optimal set.

13. Suppose you have a one-bit GA with fitness values 1 and 2 respectively. The first individual has a 10% chance of mutating to the second individual, and the second individual has a 30% chance of mutating to the first individual.

- What is the mutation matrix?
- Find the fixed points of the infinite-population GA under selection and mutation.
- Sketch the unit simplex that corresponds to this problem. Which of the fixed points are inside the unit simplex?

Solution:

$$U = \begin{bmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{bmatrix}$$

The fixed points are the eigenvectors of $U \text{diag}(f)$, that is, the eigenvectors of

$$\begin{bmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.6 \\ 0.1 & 1.4 \end{bmatrix}$$

The eigenvalues are $4/5$ and $3/2$, and the eigenvectors are $[6/5 \ -1/5]^T$ and $[1/2 \ 1/2]^T$. The second eigenvector (which corresponds to the largest eigenvalue) is the one inside the unit simplex.

