

# Optimal State Estimation: Errata

Dan Simon

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This is a list of errors in the book *Optimal State Estimation*, John Wiley & Sons, 2006. The main web site for the book is at <http://academic.csuohio.edu/simond/estimation>. My email address is listed on my home page at <http://academic.csuohio.edu/simond>.

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1. For the probability distribution function, also known as the cumulative distribution function, the book should use the acronym CDF instead of PDF to avoid confusion with pdf (probability density function).
2. Use of the term "linear": Page 67 near the top refers to a "linear" transformation. Strictly speaking, this is not a linear but an affine transformation. There are two more references on the bottom of the page. Page 60 refers to a linear function (which is conventional usage). But then, to be consistent, the last sentence in the example should read "linear function" instead of "linear transformation."
3. Equation (1.11) should read

$$A^H = \begin{bmatrix} 1 & -4j \\ -2j & 5 - j \\ 3 + j & 1 + 3j \end{bmatrix}$$

4. In the fifth line after Equation (1.22) the word "invertible" should be changed to "non-invertible."
5. The discussion on pages 10 and 11 assumes that  $A$  is real. This can be corrected a couple of different ways. One way would be to explicitly state the assumption that  $A$  is real. Another way would be to allow  $A$  to be complex, in which case the word "symmetric" should be changed to "hermitian" on the 3rd and 4th lines after Equation (1.31), and all of the transpose symbols in the rest of Section 1.1.1 should be changed to hermitian transpose symbols.

6. When “singular values” are defined, it should be stated that they are positive. This clarification should be added to the text before (1.33), (6.36), and (11.113).
7. In the line above (1.49) on p. 14,  $A$  should be described as an  $n \times n$  matrix.
8. The lower right term in the matrix of (1.60) is missing a forward-slash.
9. The second equation in Equation (1.62) on p. 16 should state that the partial of  $x^T A^T$  is equal to  $A^T$ .
10.  $t$  terms are missing from the second matrix in Equation (1.73).
11. Following Equation (1.82) add the sentence, “See any linear systems text [Bay99, Che99, Kai80] for the computation of  $e^{\hat{A}t}$ .”
12. In (1.87) on p. 23,  $\tilde{x}_2$  should be changed to  $\tilde{x}_n$ .
13. The upper summation limit in Equation (1.108) should be  $L - 1$ . The two upper summation limits in Equation (1.109) should be  $n - 1$  and  $n$  respectively. The two upper summation limits in Equation (1.112) should be  $n - 1$ .
14. The caption of Figure 1.2 should say that  $x(0.5) \approx x(0) + A_1$ ,  $x(1) \approx x(0.5) + A_2$ , etc. A similar correction needs to be made to the caption of Figure 1.3.
15. All of the  $A$  terms from Equation (1.135) through Theorem 8, and also in Equation (1.142), should be changed to  $F$ .
16. In the paragraph at the top of page 41 the word “controllability” should be changed to “observability” (twice).
17. Before Equation (1.166) note that  $\bar{x}$  is defined to be equal to  $M^{-1}x$ . The  $u$  is missing from the end of the first expression in Equation (1.166).
18. An  $x$  is missing from the last line of Equation (1.167).
19. Remove the  $u$  term from the  $y$  expression in Equations (1.167) and (1.168).
20. Exercise 1.11 is stated wrong. It should read: Show that  $|e^{At}| = e^{Tr(A)t}$  for any square matrix  $A$ .
21. The equation that accompanies Exercise 1.13 should read
 
$$p = p_0 + \dot{p}_0 t + \frac{1}{2} \ddot{p}_0 t^2$$
22. In Exercise 1.19(d),  $\cos(t)$  should be  $|\cos(t)|$  (otherwise the propellant flow rate could be negative).
23. The left side of the last equation in Equation (2.14) should have  $X$  instead of  $x$ .

24. In (2.17) on p. 54, the term on the right side of the first equation should be  $f_{X_1}(x_1|X_2 = x_2)$ .
25. The last line of Equation (2.43) should have the absolute value of  $a$ .
26. The first line of Equation (2.51) should have  $f(z_1, z_2)$ .
27. Equation (2.55) on p. 62 is missing a right parenthesis.
28. The sentence following Equation (2.56) should read, “The correlation coefficient is a normalized measurement of the correlation between two RVs  $X$  and  $Y$ .”
29. Equations (2.62) and (2.63) can be derived even if  $x$  and  $y$  are not independent.
30. The left side of Equation (2.76) should be  $Y$  instead of  $y$ .
31. The last paragraph on page 70 has one too many “only”s.
32. Exercise 2.13 should read, “... The pdf’s of  $X$  and  $V$  ...” instead of “... The pdf’s of  $X$  and  $B$  ...”.
33. The top figure of Figure 2.5 should show  $\text{pdf}(x)$  equal to 0.5 instead of 1 (so that it has an area of 1).
34. On page 81, two lines below Equation (3.7), “greater than” should be changed to “greater than or equal to”.
35. In Equation (3.15)  $\partial J/\partial \hat{x}$  should be multiplied by 2.
36. On page 84 the second paragraph of Section 3.3 should read, “In this section, we show how to recursively compute the weighted least squares estimate of a constant *vector*.”
37. On page 84, “estimation error mean” just before Equation 3.21 should be hyphenated to be consistent with “error-estimation covariance” on page 85.
38. The first line of Equation (3.22) has a couple of extra right parentheses,  $\hat{x}_1$  should be changed to  $\hat{x}_{1k}$ , and  $\hat{x}_n$  should be changed to  $\hat{x}_{nk}$ . Also it should be noted that the cost function in this section is different than the cost function in the previous section, and we are summing quantities that may have different units.
39. Page 84, 7th line from the bottom, should read “...equal to  $x$  for all  $k$ ...” instead of “equal to  $x_k$  for all  $k$ .”
40. The  $\epsilon$  in Equation (3.24) is missing a transpose.
41. At the bottom of page 85, the phrase “it will also be consistently close to zero” should be changed to “it will also be consistently as close as possible to zero.”
42. On page 89 following Equation (3.49),  $P(0)$  should be changed to  $P_0$  (twice).

- 43. On page 92, the  $P(2, 2)$  legend in Figure 3.1(b) is missing a parenthesis.
- 44. On page 96, the footnote should read “...present and *past* inputs...”
- 45. Exercises 3.4(b) and 3.5(b) are missing the word “the.”
- 46. Exercise 3.4(c) should read, “... what is the variance of the voltage estimation error?”
- 47. The equation in Exercise 3.8(d) should read

$$\hat{x} = \operatorname{argmin}_y \max_x |x - y|$$

- 48.  $Q_k$  should be replaced with  $\tilde{Q}_k$  in Equation (4.14).
- 49. Note that Equation (4.18) only holds if  $A$  is constant between sample times [Bay99, pp. 252ff.].
- 50. The footnote at the bottom of page 111 should refer to zero-order holds rather than first-order holds.
- 51. Problem 4.13(b) does not make sense.
- 52. Item 3(b) on page 124 is wrong.
- 53. The first paragraph on page 126 and the last paragraph on page 127 refer to  $P$  as the covariance of the estimate. It should instead refer to  $P$  as the covariance of the estimation *error*. This mistake may be repeated elsewhere in the text also.
- 54. Exercise 5.9(a) (page 147) should read “... $P_0^+ = 1$ ...” instead of “... $E(x_0^2) = 1$ ...”
- 55. In the third line of Equation (6.22),  $y_{33}$  should be changed to  $y_{31}$ .
- 56. On page 157 on the line after Equation (6.34), the word “and” should be changed to “an”.
- 57. The text following Equation (6.36) says that  $P^T P$  is symmetric, and the eigenvalues of a symmetric matrix are always real and nonnegative. Instead of “symmetric” it should say “symmetric positive definite.”
- 58. In Equation (7.1),  $E[w_k v_j^T] = M_k \delta_{k-j+1}$  should be changed to  $E[w_{k-1} v_j^T] = M_k \delta_{k-j}$  (same thing in Equation (7.12)).
- 59. The  $[ 1 \ 1 ]^T$  array in Equation (7.46) should be changed to  $[ 0 \ 1 ]^T$ .
- 60. In the 2nd expression in Equation (7.34), the last  $\hat{x}_k^+$  should instead be  $\hat{x}_k^-$ . This changes the experimental results in Table 7.2 so that the Measurement Differencing column becomes identical to the Augmented Filter column.

61. In the  $P_{k+1}^-$  expression in Equation (7.34), the last  $M_k$  should be changed to  $M_k^T$ , and the last  $M_k^T$  should be changed to  $M_k$ .

62.  $P_0$  following Equation (7.42) should be  $P_0^+$ .

63. Equation (7.59) should read

$$\hat{x}_k^+ = \frac{1}{2}\hat{x}_{k-1}^+ + \frac{3}{4}y_k$$

64. The 3rd and 4th expressions of Equation (7.69) should read

$$P_{11}^- = \frac{K_1 R}{1 - K_1}$$

$$P_{12}^- = \frac{K_2 R}{1 - K_1}$$

65. Equation (7.70) should read

$$\lambda = \frac{\sigma_w T^2}{R}$$

66. The discussion of Equations (7.108) and (7.109) has some conceptual errors and needs to be rewritten. The discussion starting at Equation (7.110) is fine, but the material leading up to it is wrong.

67. On page 212, on the 4th line before Section 7.5.1, “this will presented” should be replaced by “this will be presented.”

68. Equation (7.158) should read

$$\rho_k = e_k^T - \sum_{i=1}^{k-1} (\rho_i e_k) \rho_i$$

Equation (7.160) should read

$$\rho_k = e_1^T - \sum_{i=1}^{k-1} (\rho_i e_1) \rho_i$$

69. In Equation (7.165) on page 220,  $d_i$  should be changed to  $d_{ki}$ .

70. In Equation (8.22),  $\Lambda$  should be deleted. In Equation (8.23) the definition of  $\Lambda$  should be deleted, and the  $N$  in  $v_k \sim N(0, R)$  should be deleted.

71. In Equation (8.94) on page 245,

$$\begin{bmatrix} S_{11}^T \\ S_{21}^T \\ S_{31}^T \end{bmatrix}$$

in the second line should be changed to  $\begin{bmatrix} S_{11}^T & S_{21}^T & S_{31}^T \end{bmatrix}$ . A similar change needs to be made to the fourth line of the equation.

72. There are several places in Section 8.4.2 where  $C$  should be changed to  $\tilde{C}$ : The  $K$  expression in Equation (8.118), Equation (8.125), the  $\dot{K}$  expression in Equation (8.126), and Equation (8.127).
73. On page 252, three lines below Equation (8.129), “sdomain” should be changed to “s domain.”
74. In the first sentence in the second paragraph on p. 260 there is one too many “the”s.
75. In the line following Equation (9.57),  $x_f$  should be  $\hat{x}_f$ , and  $x_b$  should be  $\hat{x}_b$ .
76. The 2nd sentence before Example 10.1 on page 303 should read, “For example,  $\max(1 - x^2) = 1$  because the maximum of  $(1 - x^2)$  is 1, but  $\operatorname{argmax}_x(1 - x^2) = 0$  because  $(1 - x^2)$  attains its maximum value when  $x = 0$ .”
77. The fourth line after Equation (10.24) needs a space before the word “we.”
78. In Equation (10.35) the two  $\Lambda$  terms should be deleted. The last expression in Equation (10.37) should read  $Q = BQ_cB^TT$ .
79. The text prior to Equation (10.73) says that it is a Riccati equation, but it is actually a Lyapunov equation.
80. On page 315 in the paragraph below Equation (10.79), “process noise” should be changed to “measurement noise,” and vice versa.
81. On page 315 in the line above Equation (10.82) a right parenthesis is missing.
82. On page 316, the paragraph following Equation (10.86) indicates that the derivative of  $J$  with respect to  $K$  must be computed numerically, but it can probably be computed analytically.
83. On page 321 in Equation (10.107),  $\hat{x}^-(k)$  should be changed to  $\hat{x}(k)$ .
84. On page 321 in Equation (10.112), there should be a comma after  $x(k)$  in the first two expressions, and the tranpose symbols should be dropped. In the first expression, the right parenthesis before  $H(k)$  should be a right bracket.
85. In the fourth line of Equation (11.6) and the second line of Equation (11.7) there is a superscript minus sign missing from one of the  $P_k$  terms.
86. In the last paragraph on page 343, the word “pdf” should be changed to “statistics” (three times).
87. Equation (11.25) should specify that  $x_0$  is constant but unknown. This gives the additional term  $\lambda_0^T(x_0 - k_0)$  in Equation (11.27), which in turn affects the following three equations, and makes the final result of Equation (11.33) correct as written.

88. In Equation (11.38),  $< 1$  should be changed to  $< 0$ .
89. The line prior to Equation (11.47) contains the phrase “(with respect to  $x_0$  and  $w_k$ )”. That phrase should be deleted.
90. The phrase “to the cost function” should be replaced with “from the cost function” prior to Equation (11.73).
91. In the second line of Equation (11.79) the pre and post  $P_k^{-1}$  terms can be omitted to make the derivation simpler.
92. In Equation (11.93)  $K_k$  should be  $K$ .
93. A right parenthesis is missing from footnote 2 on page 355.
94. In Equation (12.2) the first term in the denominator should be removed.
95. On page 383 in Equation (12.40), the subscripts of the terms on the left of two of the equations should be  $k + 1$  instead of  $k$ .
96. In the first paragraph of Section 13.1, “are” should be changed to “is”.
97. It would make more sense to switch the order of Sections 13.2.2 and 13.2.3.
98.  $h_k(\hat{x}_k^-, \cdot, t_k)$  should be changed to  $h_k(\hat{x}_k^-, \cdot)$  in Equations (13.29) and (13.33).
99. In the line following Equation (13.32),  $F$  should be changed to  $A$ .
100. In Equation (13.37),  $1/2000$  should be changed to  $2000$ ,  $1/2500$  should be changed to  $2500$ , and  $1/250000$  should be changed to  $250000$ .
101. The plus superscripts on the right side of Equation (13.52) should both be changed to minus superscripts.
102. Equation (14.7) should read  $\bar{y}_1 = 0$ , and (14.9) should read  $\bar{y}_2 = E(\cos \tilde{\theta})$ . The equations as written in the book are correct but confusing. It could also be clearer if it is stated before (14.7) and (14.9) that  $\tilde{r}$  and  $\tilde{\theta}$  are zero mean.
103.  $y_u$  should be changed to  $\bar{y}_u$  in Section 14.2.2.
104. Just before Equation (15.20),  $y_k$  should be changed to  $Y_k$ . The same change should also be made in Equation (15.21), in the text before Equation (15.21), and in step (3e) on page 468. Similarly,  $y_{k-1}$  should be changed to  $Y_{k-1}$  twice in the first paragraph of Section 15.3.1.
105. In the [Bat82] reference on page 503, the quotation mark is incorrect.
106. On p. 507 [Gre01] should include “2nd edition” to be more precise.
107. On p. 516 [Sea67] is out of order.