

# Real-Time Navigation Using the Global Positioning System

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## ABSTRACT

This paper presents the results of an investigation of the application of the Global Positioning System (GPS) to real-time integrated missile navigation. We present quantifiable measures of navigation accuracy as a function of GPS user segment parameters. These user segment parameters include antenna phase response accuracy, single versus dual frequency, and Kalman filter structure and size. We also formulate some new phase-locked loop (PLL) filter designs for application in GPS receivers, and demonstrate their superiority over more conventional filters.

## INTRODUCTION

The National Aeronautical Association describes the Global Positioning System as "The most significant development for safe and efficient navigation and surveillance of air and

spacecraft since the introduction of radio navigation 50 years ago" [5]. The successful performance of GPS in the Persian Gulf War in January 1991 resulted in an explosive demand in the military for this satellite-based navigation tool. Civilian applications of GPS are found in such areas as ship and aircraft navigation, surveying, geological studies, and the Intelligent Vehicle Highway System. A GPS satellite launch in August 1993 brought the constellation to its full operational capability of 24 healthy satellites for the first time [6]. Although the constellation has reached completion, new satellites will continue to be launched as old satellites are replaced.

The GPS concept was conceived in the early 1960s by the Aerospace Corporation as an improved navigation method for military systems (ballistic missiles and fighter aircraft). If four precisely positioned satellites transmit radio waves, then four different propagation times can be measured by a receiver. The four unknowns which need to be solved are the three components of the user's position and the time offset of the user's clock. To maintain the synchronization of the clocks on the GPS satellites, each satellite is equipped with an atomic clock which is periodically updated by a Master Control Station. Each satellite is in an inclined semi-geosynchronous (12-hour) circular orbit. In order for the receiver to solve for position and velocity, it needs to know the satellites' orbital parameters. These parameters are uplinked to the satellites by

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the Master Control Station, and transmitted by the satellites to the GPS receiver.

During the development phase of GPS, there was concern that enemies could eventually use GPS against the United States. This concern was answered by deliberately introducing errors into the satellite timing and orbital information. These errors reduce the accuracy of GPS from 10 meters (90% of the time) to 100 meters—accurate enough for civilian navigation, but not accurate enough for weapon delivery. The errors can be removed by an “authorized user” with the proper information. This scheme is called Selective Availability.

A GPS satellite transmits two sets of periodic pseudorandom (PR) binary signals—a Precision code (P code) and a Coarse/Acquisition (C/A) code. The P code has a 280-day period and a 30-meter bit length, and the C/A code has a 1-msec period and a 300-meter bit length. A GPS receiver computes its position by replicating the PR codes, shifting them in time until they correlate with the codes which are received from the satellites, and using the required time shift to compute the propagation time. This time, multiplied by the speed of light, is the measured distance between the user and the satellite. The machinery which is used to correlate the receiver’s PR code with the satellite’s PR code is called a delay lock loop (DLL). The C/A code, because of its shorter period, is much easier to lock onto than the P code; but the resultant position solution is less accurate for the C/A code because of its longer bit length.

The PR codes are modulated by two sinusoidal carriers—a 1575.42 MHz carrier called L1, and a 1227.6 MHz carrier called L2. A GPS receiver can track one of the carriers and compute the frequency. The received frequency will be different than the transmitted frequency due to the doppler effect. The frequency difference can then be used to compute the relative velocity between the user and the satellite. Four such measurements can then be used to solve for the user’s velocity. The machinery which is used to track the carrier which is received from the satellite is called a phase-locked loop (PLL).

The reason there are carriers at two frequencies is that ionospheric refraction is inversely proportional to the square of frequency. So a receiver which tracks codes modulated by two frequencies can use the difference in propagation time to very accurately correct for the ionospheric refraction. Some receivers use single frequency operation, sacrificing accuracy for the sake of simplicity and low cost.

## **AEROSPACE APPLICATIONS OF THE GLOBAL POSITIONING SYSTEM**

There are at least two ways in which GPS can benefit an aerospace program. One way is that GPS can be used to obtain a reference trajectory for the vehicle. The GPS-indicated trajectory will be different than that indicated by the vehicle’s Inertial Navigation System (INS). A Kalman filter can process the two trajectories and form optimal estimates of the INS errors. This information can then be used to quantify the

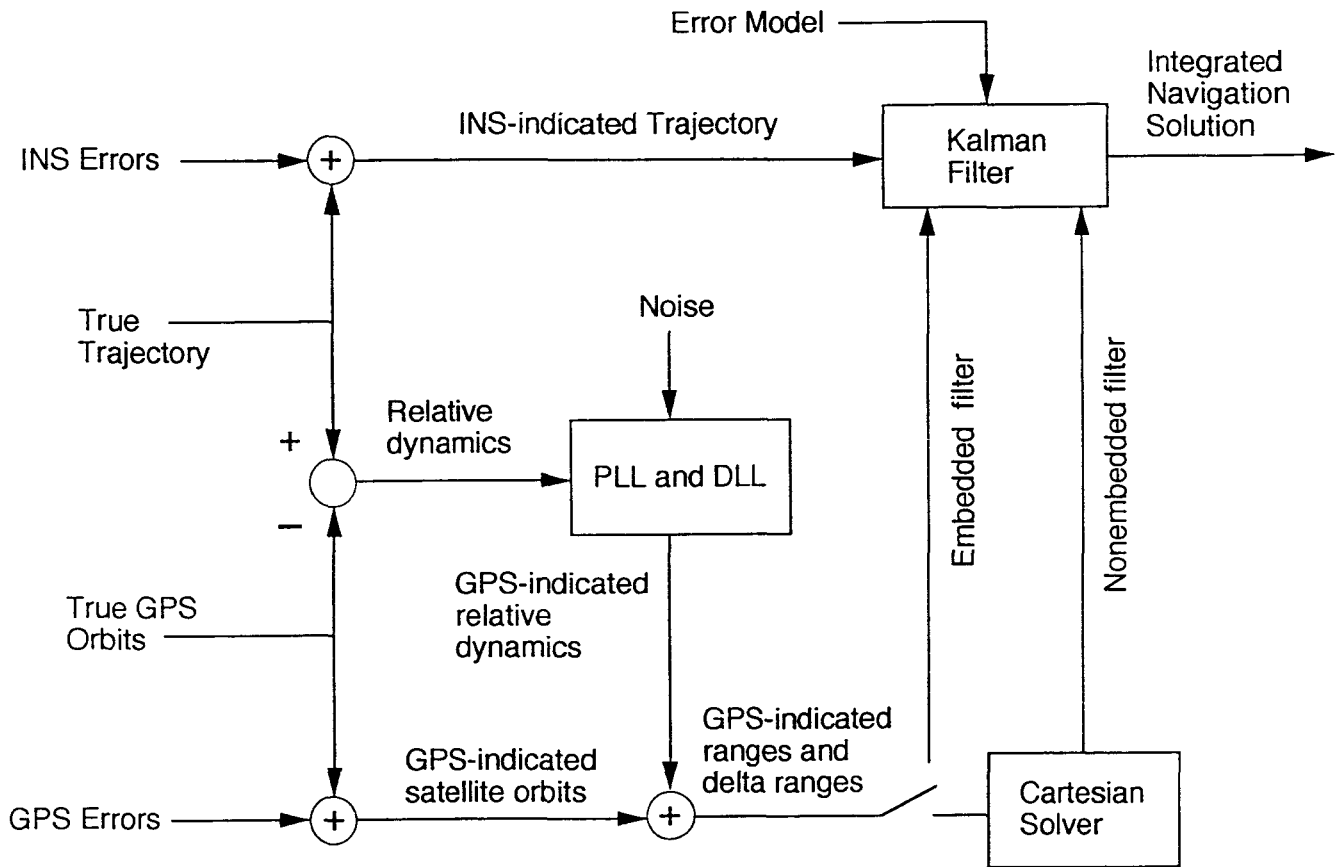
accuracy of the vehicle’s INS. This process is referred to as post flight analysis (PFA) [3]. The GPS-indicated trajectory can be obtained using either a receiver or a translator. If a receiver is used, the GPS-indicated trajectory is obtained on-board the vehicle and either recorded on the vehicle or telemetered to the ground. If a translator is used, the GPS signals are received by the translator and retransmitted to the ground where they are recorded for later processing. Translator processing offers the advantages of low cost, low power consumption, and simplicity. Furthermore, ground processing of the signals allows for the use of accurate weather information, GPS satellite orbital parameters, and other data which is not available in real-time and which improves the accuracy of the GPS-indicated trajectory.

Another way that GPS can be used is for real-time integrated navigation. As with PFA, the trajectory information of GPS is combined with that of the INS in a Kalman filter to obtain optimal estimates of the INS errors. The error estimates are then used to obtain an optimal estimate of the error of the INS-indicated trajectory. The INS-indicated trajectory is then adjusted by the error estimate to obtain a more accurate trajectory. Integrated navigation is an effective way to obtain a highly accurate navigation solution with a relatively inexpensive INS.

## **INTEGRATED NAVIGATION USING AN INERTIAL NAVIGATION SYSTEM AND THE GLOBAL POSITIONING SYSTEM**

In the years following World War II, Inertial Navigation System (INS) technology was refined to the point that it could guide a ballistic missile from launch to target over a range of thousands of miles. An INS consists of accelerometers and gyroscopes to measure changes in position and attitude. However, a highly accurate INS can be prohibitively expensive. In addition, INSs have low frequency drifts which result in accuracy which degrades over time. On the other hand, the strength of GPS is its relatively good long term accuracy. In the short term, an INS is more accurate than GPS; but in the long term, GPS is more accurate. This combination of the high frequency accuracy of INSs and the low frequency accuracy of GPS makes Kalman filter-based integrated GPS/INS navigation a natural application [9]. The measurements of the Kalman filter consist of the difference between the INS and GPS navigation solutions, and the states of the Kalman filter consist of INS errors and GPS errors. The Kalman filter uses the estimates of the INS errors to refine the INS’s navigation solution. Due to throughput constraints on the flight computer, the Kalman filter may be reduced order; that is, only the most significant INS and GPS errors are included as states.

In designing an integrated navigation system, a choice must be made between embedded filtering and nonembedded filtering. Nonembedded filtering is accomplished by combining the navigation solution of a GPS receiver with that of an INS. Embedded filtering is accomplished by combining



The Phase-locked loop (PLL) solves for the vehicle-to-satellite range rates.  
The Delay-lock loop (DLL) solves for the vehicle-to-satellite ranges.

**Fig. 1. Integrated GPS / INS Navigation Block Diagram**

the INS navigation solution with GPS-indicated range and delta range information *without the GPS receiver directly computing a navigation solution*. The difference between embedded and nonembedded navigation is depicted in Figure 1. Since the GPS receiver computes its own navigation solution in nonembedded filtering, satellite-specific errors cannot be modeled in the Kalman filter. As we shall see later, this degrades the accuracy of nonembedded filtering relative to embedded filtering.

Navigation performance can be quantified by Position and Velocity Spherical Error Probable (PSEP and VSEP). SEP is defined as the 50th percentile probability radius of the navigation solution. That is, there is a 50% probability that the navigation solution is within PSEP (VSEP) of the true position (velocity). PSEP and VSEP can be computed using Monte Carlo simulations. It can be shown [2] that ballistic missile weapon delivery accuracy depends more strongly on velocity accuracy than on position accuracy, so in this paper VSEP is of greater interest than PSEP. Although PSEP and VSEP are functions of time, we will use their values at payload deployment as a scalar measure of navigation performance.

## GPS/INS NAVIGATION PERFORMANCE CURVES

Suppose a user is given a low-cost IMU, a typical trajectory, and an integrated navigation accuracy requirement. He then needs to be able to choose a suitable GPS receiver to integrate with the INS. Does he need a dual frequency receiver, or will single frequency suffice? How good does the antenna need to be? How many states does he need to include in the Kalman filter? Our objective was to develop a methodology by which such questions could be answered. The availability of such a methodology avoids the problem of choosing a receiver which is either over- or under-designed.

There are many errors which corrupt the GPS navigation solution. Some of the errors are beyond the control of the user, while others can be minimized by suitable receiver design. Some of the errors are biases, some are markov processes, and some are white noise. The primary GPS error contributors are the following:

- Errors in the orbital parameters which are transmitted by the satellite

- Errors in the satellite's clock phase and frequency
- Errors in the correlation between the receiver's PR code and the satellite's PR code (DLL tracking errors)
- Errors in the estimation of the received carrier frequency (PLL tracking errors)
- Ionospheric refraction errors
- Tropospheric refraction errors
- Uncertainties in the location of the phase center of the receiver's antenna
- Uncertainties in the phase response pattern of the receiver's antenna.

These errors have all been modeled in our simulation software using values and dynamics derived from the literature [7, 8] and TRW's flight test experience [4].

Integrated GPS/INS navigation is a complex, nonlinear problem of high dimensionality. As such, it is not feasible to use analytic methods to estimate integrated navigation accuracy. Therefore, in order to quantify integrated navigation performance as a function of GPS receiver design, we conducted sets of Monte Carlo simulations for various key design parameters. Each Monte Carlo simulation consisted of 100 samples. These 100 samples were then used to statistically compute PSEP and VSEP. The parameters which we varied were the following:

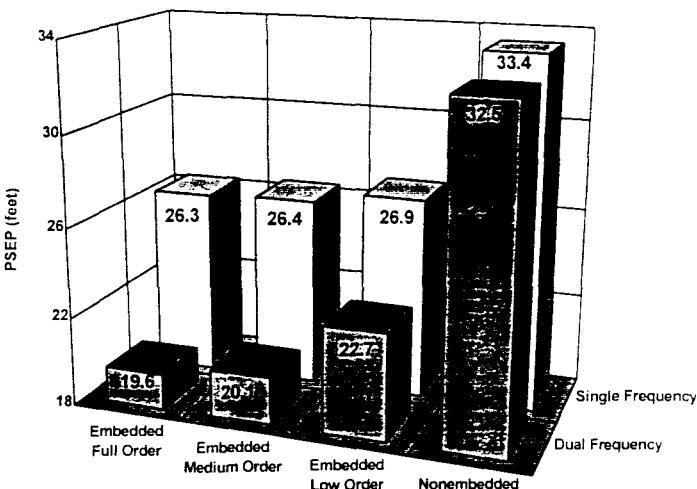


Fig .3.

- Single frequency operation versus dual frequency operation
- Integrated navigation filter size
- Antenna phase response error
- Integrated navigation filter structure (embedded versus nonembedded).

Our simulation study was based on a typical ballistic missile test trajectory where payload deployment occurred 184 seconds following launch. The INS which was simulated had three accelerometers, three gyros, and 76 errors. The INS errors included things like initial position errors, initial attitude errors, accelerometer scale factor errors, gyro drift, etc. The simulated GPS receiver tracked six satellites, and the Kalman filter in the simulated flight computer computed an integrated navigation solution once per second.

Figure 2 shows the number of states and CPU time for embedded and nonembedded filtering. The CPU time was measured for a 184-second simulation run on a Sun-10 workstation. The reduced order filters in the embedded filter were obtained by ignoring (in the Kalman filter) those INS and GPS errors which were second and third order. For the nonembedded filter, it was found that increasing the number of INS error states did not improve accuracy. This is because the effect of neglecting the GPS errors states is greater than the effect of neglecting the second order INS errors.

Figure 3 shows the effect of filter structure and single versus dual frequency on integrated navigation PSEP. It is seen that the use of dual frequency significantly improves the position accuracy relative to single frequency, and the use of embedded filtering increases the position accuracy relative to nonembedded filtering.

Figure 4 shows the effect of filter structure, single versus dual frequency, and antenna phase accuracy on integrated navigation VSEP. It was found that antenna accuracy had a negligible effect on VSEP for single frequency operation. This is because the ionospheric error swamps the antenna error for single frequency operation. On the other hand, as seen in Figure 4, antenna accuracy has a strong effect on VSEP for

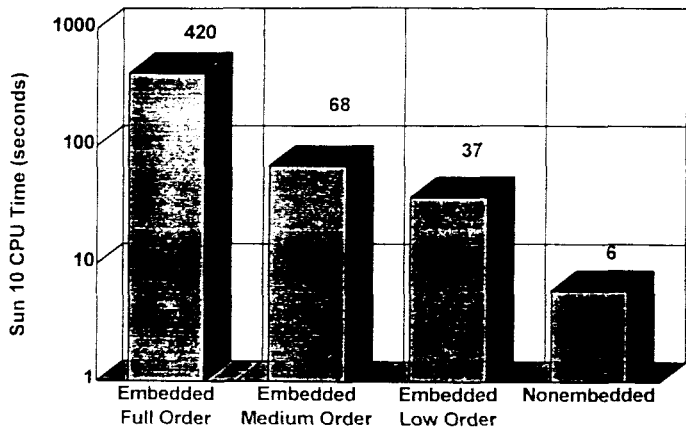
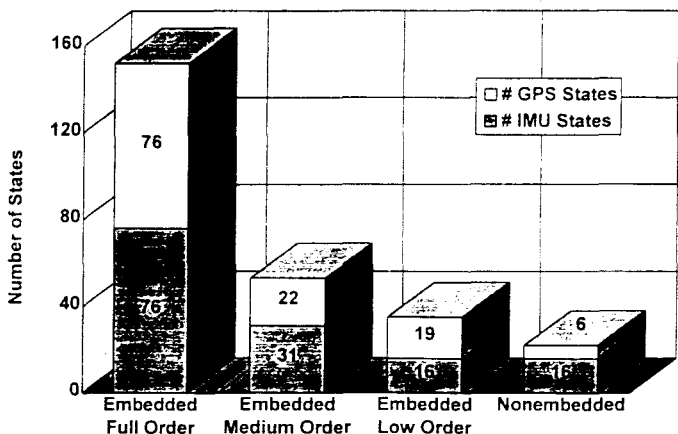


Fig. 2.

## DIGITAL PHASE-LOCKED LOOP DESIGN

A phase-locked loop is used to track the noise-corrupted phase of the carrier component of the GPS signal. The phase information is used to compute frequency, and the frequency information is used to compute the user's velocity. Up to this point we have assumed that the GPS receiver's PLL perfectly maintains lock on the GPS carrier. However, the design of PLLs which can maintain lock on the GPS carrier phase has proven to be a challenging task, particularly if the receiver trajectory is highly dynamic or if the signal is very noisy. For instance, classical PLL tracking techniques (e.g., Costas loops) are able to maintain lock in missile dynamics only if they are aided by the INS. It is preferable if the PLL can track the carrier independently of the INS. If the PLL loses lock on the signal, then the user will not be able to compute the relative doppler frequencies between herself and the GPS satellites, and the error of the GPS-derived velocity

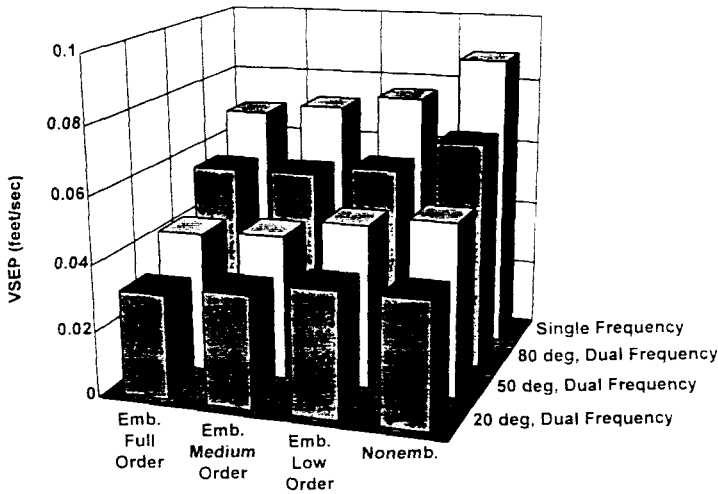


Fig. 4.

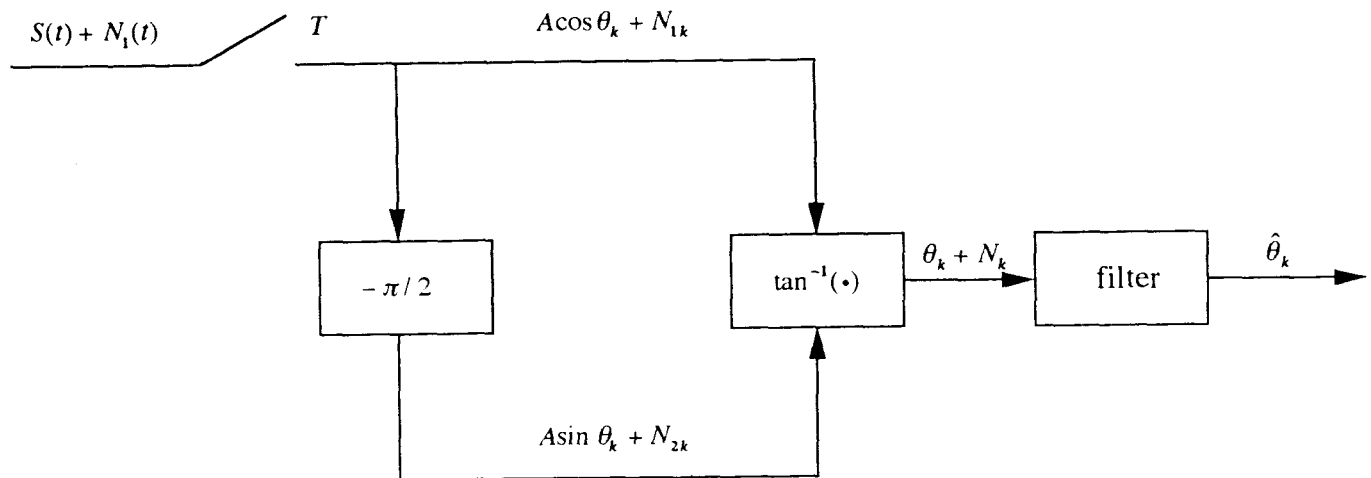


Fig. 5.

dual frequency operation. As expected, single frequency performance is noticeably worse than dual frequency performance.

In summary, the methodologies which we have developed have been applied to a typical ballistic missile mission. We have seen that the critical GPS user segment parameters for the given trajectory are:

- Single versus dual frequency operation
- Filter structure (embedded versus nonembedded)
- Antenna phase response accuracy, but only for dual frequency operation
- Filter size, but only for dual frequency operation.

The methodology which we have presented can also be used to investigate GPS receiver design parameters for other trajectories, missions, or applications.

information will grow without bound. It is therefore desirable to provide robust algorithms for the GPS receiver's PLL.

### Phase Estimation

The PLL architecture considered in this paper is shown in Figure 5. This architecture was introduced by the Jet Propulsion Laboratory specifically for the purpose of tracking in a high dynamics environment [13]. Note from the figure that the output of the arctan phase discriminator is modulo  $2\pi$ . That is, the phase discriminator does not know the difference between  $\theta$  radians and  $\theta + 2\pi$  radians. If the phase estimation error suddenly goes from zero to some multiple of  $2\pi$ , it is said that a cycle slip has occurred. So it is more important for a PLL to prevent cycle slips than it is to maintain a small phase error. If the PLL maintains lock on the phase, the PLL contribution to the GPS receiver's velocity error is small compared to other sources of velocity error [9]. But if a cycle slip occurs, the velocity error increases by an order of

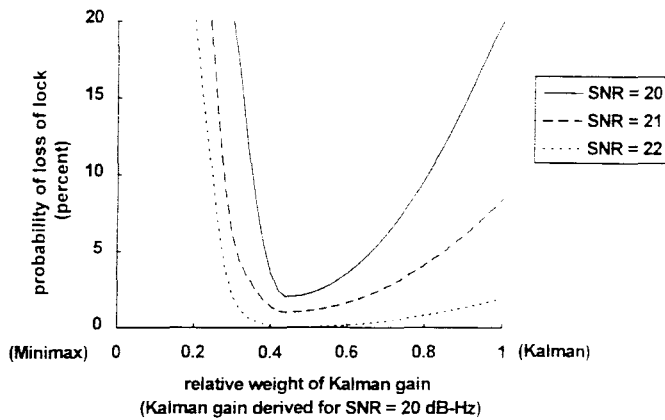


Fig. 6.

magnitude. In some cases, the noise is so high or the phase dynamics are so severe that the estimation error begins growing without bound. In this case it is said that loss of lock has occurred, and the user loses all velocity information from the GPS receiver. So it is primarily loss of lock and secondarily cycle slips which are of greatest concern (rather than phase error).

### Optimal Filtering

Kalman filtering, also known as  $H_2$  filtering, is a well-established technology which dates back to the 1960s and has its roots in the late 1700s [12].  $H_2$  filtering minimizes the variance of the estimation error and assumes that the noisy inputs have known statistical properties. This assumption limits the applicability of Kalman filters. This limitation has given rise to a recent interest in minimax filtering, also known as  $H_\infty$  filtering [14]. The optimality measure which is used in  $H_\infty$  filtering is a certain measure of the worst-case effect of the noise, and no knowledge of the noise statistics is assumed [11].

Recall that in PLL design loss of lock is of greater concern than estimation error. So, since  $H_\infty$  filters are designed to minimize the worst case effect of the noise, PLL filtering is a natural application of  $H_\infty$  theory.

Although  $H_2$  filtering theory assumes too much,  $H_\infty$  filtering theory assumes too little. The engineer typically has less knowledge about the noise than an  $H_2$  filter requires, but more knowledge than an  $H_\infty$  filter can use. This motivates the design of an estimation filter which uses the best characteristics from each type of filter. This type of cross between  $H_2$  and  $H_\infty$  theory can be called a hybrid filter. A hybrid filter uses a weighted combination of the steady-state  $H_2$  and  $H_\infty$  gains in the estimator. That is, the hybrid filter gain is  $K = d K_2 + (1 - d) K_\infty$  where  $d$  (between 0 and 1) is of relative importance given to  $H_2$  performance.

The behavior of the hybrid filter was investigated by simulating its performance for a GPS receiver used on a missile test flight. Recall that our primary interest is in maintaining lock. With this in mind, the probability of loss of lock was obtained for various values of the Kalman weight  $d$ .

Probability of loss of lock was obtained from 100 Monte Carlo samples for each value of  $d$ . The probability is shown in Figure 6 for three values of signal-to-noise ratio (SNR). It is seen that the use of hybrid filtering results in a dramatic improvement over both  $H_2$  and  $H_\infty$  filtering. This improvement becomes more noticeable as noise increases (SNR decreases).

### Fuzzy Filtering

Many engineers have noted that while computers can outperform humans in many tasks, it is also true that humans can outperform computers in tasks which are considered simple (e.g., recognizing faces and voices, or catching a ball). This observation led engineers to the design of computer systems which mimic the imprecise, subjective reasoning processes of humans. The result was fuzzy logic [1].

Fuzzy logic has been successfully applied to many control problems which were very difficult to solve using traditional engineering approaches. Some examples include antilock braking, pattern recognition, and traffic signal control. Fuzzy control is typically applied to systems which are too complex to easily lend themselves to more mathematically rigorous control methods. But some systems are difficult to control even though an analytical system model is known. One source of difficulty in such systems is the conflicting requirements of tracking high frequency inputs while at the same time rejecting noise. A GPS receiver's PLL is such a system. This motivates the application of fuzzy logic to PLL filter design [10].

A fuzzy PLL filter operates according to rules like the following: "The phase measurement is a little larger than expected, and it was also larger than expected one sample period ago, so I'd better increase my phase estimate or I may lose lock." This is called a fuzzy rule. The engineering challenge is designing and quantifying the fuzzy rules in an optimum manner. One way these rules can be determined is through common sense. The fuzzy rules can also be trained to improve the performance of the PLL.

One way of training the fuzzy rules is through gradient descent. Gradient descent is a mathematically precise way of measuring the PLL estimation error, computing its derivative with respect to the parameters of the fuzzy rule base, and then adjusting those parameters in such a way that the estimation error is minimized.

Another way of training the fuzzy rules is through a genetic algorithm (GA). GAs are based on the principle of survival of the fittest in evolutionary theory. A "population" of fuzzy rule bases is evaluated and each member of the population is assigned a "fitness value" based on its performance. Then the least fit are removed from the population, while the most fit "mate" by combining their various features to produce the next generation of fuzzy rule bases. The performance of the fuzzy rule bases gets better with each generation. GAs are more time-consuming than gradient descent, but also more flexible.

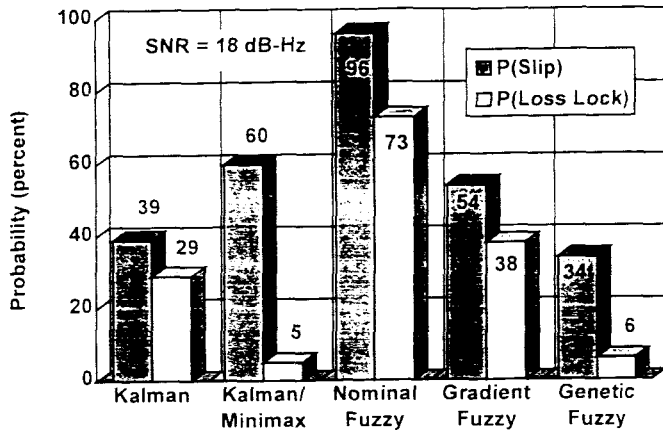


Fig. 7.

The behavior of fuzzy PLL filtering was investigated by simulating its performance for a GPS receiver used on a missile test flight. Recall that our primary interest is in maintaining lock, and our secondary interest is in preventing cycle slips. With this in mind, the probability of loss of lock was obtained for three different fuzzy PLLs: a nominal fuzzy PLL where the fuzzy rules were based on common sense, a fuzzy PLL which was trained using gradient descent, and a fuzzy PLL which was trained with a genetic algorithm. Probability of loss of lock and cycle slippage was obtained from 100 Monte Carlo samples. Figure 7 shows these probabilities for various PLL filter designs. Note that the hybrid  $H_2/H_\infty$  filter and the GA fuzzy PLL are best as far as probability of loss of lock. The Kalman filter and the GA fuzzy PLL are best as far as probability of cycle slip.

The drawback of a fuzzy PLL is that it needs to be trained based on an *a priori* known trajectory. Simulation results have been obtained which show that the fuzzy PLL is robust to 3-sigma departures from the nominal trajectory [10]. Still, some knowledge of the nominal trajectory is required to train the fuzzy PLL. This indicates that a fuzzy PLL may be appropriate for an application where a nominal trajectory is known *a priori* (e.g., missile navigation), but not for an application where there is not a nominal trajectory (e.g., aircraft navigation).

## CONCLUSION

The results presented in this paper compare integrated GPS/INS navigation performance as a function of filter architecture, antenna phase error, single versus dual frequency, and filter size. For the ballistic missile trajectory which we considered, it is seen that the most important decision to be made about a GPS receiver is whether to use a nonembedded or embedded filter structure. The next critical decision is whether to use a single frequency receiver or a dual frequency

receiver. If embedded filtering is chosen, filter order becomes an important design parameter. If a dual frequency receiver is used, antenna accuracy becomes an important issue. The methodology which is demonstrated in this paper can also be used to investigate the effects of GPS receiver parameters for other trajectories, missions, or applications.

Two new phase-locked loop filters were proposed in this paper: a hybrid Kalman/minimax filter and a fuzzy filter. These methods were introduced to preclude the need for INS aiding. Simulation results indicate that these two new filters can offer significant improvement over existing PLL designs.

## ACKNOWLEDGMENT

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