

Biogeography-Based Optimization for Hydraulic Prosthetic Knee Control

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ABSTRACT

We discuss open-loop control development and simulation results for a newly-developed cyber-physical system (CPS) used as a semi-active, above-knee prosthesis. The control signal of our CPS consists of two hydraulic valve settings that control a linear cylinder actuator and provide torque to the prosthetic knee. We develop open-loop control using biogeography-based optimization (BBO), which is a recently developed evolutionary algorithm. The research contributes to the field of cyber-physical systems by showing that it is possible to find effective open-loop control signals for our newly proposed semi-active hydraulic knee prosthesis through a dual-system optimization process which includes both human and robot control search parameters.

General Terms

Algorithms, Performance, Design, Reliability, Experimentation, Human Factors, Theory, Verification.

Key Words

Biogeography Based Optimization, Hydraulic Knee Prosthesis, Control Theory.

1. INTRODUCTION

Cyber-physical systems (CPS) include a number of challenges that we address in this research. First, a CPS is an inherently complex system due to the interaction of multiple, distributed subsystems [1]. Therefore, when designing a CPS, subsystems must be designed and optimized in an integrated way. In particular, human behavior and cyber behavior must

be optimized simultaneously. Humans are naturally adaptive, but adaptability needs to be intentionally and specifically integrated into the cyber components of CPS. Second, the hardware/software division needs to be rethought in CPS due to their tight integration [2]. Third, control is a key component of CPS [3]. Fourth, considering the aging population of the US, medical care is one of the most pressing CPS applications [3], [4], [5]. Medical applications comprise a CPS area that has particular challenges due to the combination of embedded systems that coordinate with the dynamics of physical, human bodies [2] and environmental uncertainty [6]. Fifth, CPS is fundamentally multidisciplinary [7]. This research brings together the disciplines of biomedical engineering, computer intelligence, and biomechanics. We recognize that there are many other CPS issues that are critically important, including standardized architectures, reliability, security, dependability, reconfigurability, certifiability, and others. We do not address these issues specifically in this research, although we do partially address some of them to the extent that they overlap with the issues discussed above.

We propose a new CPS design for transfemoral amputees, and also derive open-loop control signals for the prosthesis. The prosthesis harvests energy and provides controlled release of energy during the gait cycle with a spring-loaded high pressure hydraulic chamber, a low pressure hydraulic chamber, and a linear cylinder actuator. The semi-active nature of the CPS allows the device to use less power than its fully active prosthetic counterparts while operating at a quieter noise level. Prostheses have long been known to produce degenerative side effects [1], [9], [10], because of the unnatural and high torques that the user's hip produces when compensating for the prosthesis' inadequacy. Therefore, we place a high priority not only on the appearance of normal gait through tracking reference angles and coordinates, but also on the hip torques that the amputee has to produce to interface with the prosthesis.

Microprocessor controlled knees have been a success in several different prostheses. Most notably, the Otto Bock C-Leg has become the benchmark of prosthetic knees. The performance of the C-Leg depends on the controls embedded in its microcontroller. Otto Bock's leg reacts well to a variety of situations and has proven to decrease detrimental side effects relative to more conventional prostheses [11], [12].

Evaluation tests have shown that microprocessor control has proven to be the best option for high performance prostheses [11], [12]. However, even the most modern and technically sophisticated knee prostheses still do not fully restore normal gait and do not prevent all detrimental side effects [12], [13], [14], [15], [16].

Our open-loop prosthetic control approach focuses on biogeography based optimization (BBO), which is a recently developed evolutionary algorithm (EA). BBO gives better performance than traditional EAs for a wide variety of benchmarks and real-world optimization problems [17], [18]. Solving for an optimal open-loop control by strictly analytical means is intractable for the nonlinear, time-varying prosthetic control problem. We therefore use BBO in this paper to search for an open-loop control by minimizing a cost function through the evaluation of a population of candidate control solutions.

Researchers have found various EAs, including genetic algorithms (GAs) and simulated annealing, to be attractive for solving difficult control problems. Control optimization with EAs is done by parameterizing the control signals, and then using the EA as a parameter optimization algorithm to find the parameters that result in the best controls. EAs are often effective tools for parameter optimization, so the conversion of control problems to parameter optimization problems makes them appropriate problems for EAs. For example, GAs are appropriate tools for finding solutions to certain nonlinear, second order, two point boundary value problems [19] because GAs are simple and do not require advanced mathematical tools. EAs can find nonlinear controls for generic trajectory optimization problems [20]. GAs and simulated annealing have found optimal trajectories for trajectory optimization problems [21]. GA-based optimization for missile flight midcourse guidance is another example of their usefulness for control [22]. This method was used to optimize muscle excitation signals for large-scale musculoskeletal systems [23]. The key to all of these studies is the conversion of the control optimization problem to a parameter optimization problem. The GA / Fourier series approach to optimal control was also applied to robotic manipulator control [25].

We convert the prosthetic control problem into a parameter optimization problem by representing the control signals as Fourier series. This idea was first used for the optimization of structural systems [24] with linear dynamics and a quadratic performance index. That reference assumed that the optimal profile of each configuration variable was continuous on the interval $[0, T]$, where T is the fixed time interval of the control problem. In practice, only a finite number of Fourier terms are used to represent the control signals, and this idea converts the control optimization problem to a parameter optimization problem. This approach is a computationally efficient approach for optimal control, and is able to handle boundary conditions and high order problems. We are motivated by the previously referenced research to use the Fourier series approach for the prosthetic control problem. We are further motivated by the recent success of BBO to use it for the optimization of the Fourier series coefficients that represent the control signals.

Section 2 of this paper discusses the prosthetic dynamics, the prosthetic control problem formulation, and the prosthetic system modeling in MATLAB[®]. Section 3 discusses the open-loop control problem formulation, its solution using BBO, and simulation results, including robustness tests. Section 4 contains conclusions and suggestions for future work.

2. PROBLEM FORMULATION

The problem formulation for prosthetic knee control begins with the derivation of the governing dynamic equations. There are two distinct phases of the human gait cycle, swing phase, and stance phase. Stance phase is defined as the period of time when the foot is in contact with the ground. It begins

when the heel first makes contact, and ends when the foot lifts up off the ground. Swing phase follows stance phase, and is defined as the period of time when the foot is not in contact with the ground. Figure 1 shows the stance and swing phase of the human gait during one stride.

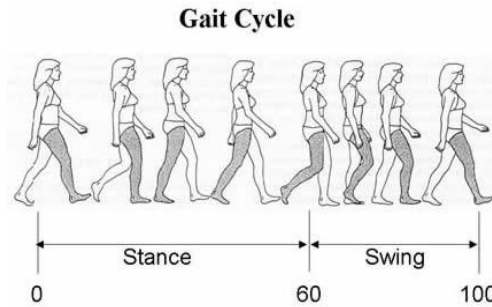


Figure 1: The stance phase of the shaded leg when the heel first makes contact with the ground, and ends when the foot leaves the ground. The swing phase of the shaded leg begins when the foot leaves the ground, and ends when the heel first strikes the ground. Error! Reference source not found..

We derived dynamic equations for limb dynamics (excluding the dynamics of the prosthetic knee actuator) using AutoLev[™] software [26]. The equations are unwieldy and so we do not list them in detail here, but the general form of the dynamic equations is given as follows:

$$\ddot{q} = C(q)Q + d(q, \dot{q}) \quad (1)$$

Note that q is a vector containing the degrees of freedom of the model's motion, given by $q^T = [x_h \ y_h \ \phi_1 \ \phi_k \ \phi_a]$, and Q is a vector of actuations at each of these degrees of freedom, given by $Q^T = [F_{xh} \ F_{yh} \ M_h \ M_k \ M_a]$. Table 1 shows the definitions of the elements of q and Q , and Figure 2 shows the diagram of the limb along with the definition of the angles and forces.

x_h	Horizontal hip position	F_{xh}	Horizontal hip force
y_h	Vertical hip position	F_{yh}	Vertical hip force
ϕ_1	Thigh angle	M_h	Hip moment (torque)
ϕ_k	Knee angle	M_k	Knee moment (torque)
ϕ_a	Ankle angle	M_a	Ankle moment (torque)

Table 1: Dynamic equation variables

A block diagram of the open-loop controller is shown Figure 4. An effective controller should be able to track the knee and thigh angles, as well as hip position in stance phase. We model the user's forces and torques at the hip with simple proportional-derivative feedback controllers. These controllers produce force and moment responses based on the hip position and thigh angle tracking error in the system. The response from these controllers is added to the reference hip actuations and the sums are applied to the hip in simulation. The actuations applied to the simulated hip are given by:

$$F_{xh} = F_{xhref} + P_{xh}(x_{href} - x_h) + D_{xh} \frac{d}{dt}(x_{href} - x_h) \quad (5)$$

$$F_{yh} = F_{yhref} + P_{yh}(y_{href} - y_h) + D_{xy} \frac{d}{dt}(y_{href} - y_h) \quad (6)$$

$$M_h = M_{href} + P_{\phi_1}(\phi_{1ref} - \phi_1) + D_{\phi_1} \frac{d}{dt}(\phi_{1ref} - \phi_1) \quad (7)$$

Note that we apply different controller gains during stance+ phase than we do in swing phase. In stance phase, the simulated leg is on the ground, and the user's other leg is swinging freely. Therefore, during stance phase, the user is unable to provide large compensative actuations; we model this by applying lower controller gains during stance phase.

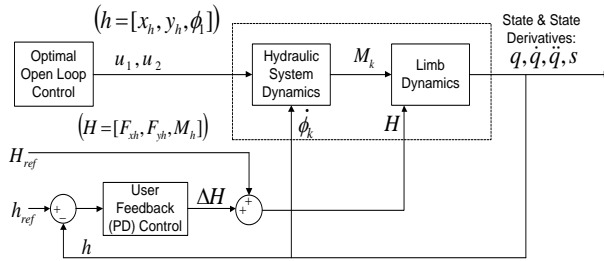


Figure 4: Open-loop control block simulation diagram. The limb dynamics are given in Equations 1–3, and the linear cylinder dynamics are given in Equations 4–7.

3. CPS OPTIMIZATION

As a starting point for prosthetic control, we find the open-loop control that delivers the best tracking performance without any disturbances or unknowns. The prosthesis is controlled in discrete time with a control update frequency of 100 Hz. The open-loop control consists of the sequence of signals, u_1 and u_2 , to the two hydraulic flow valves. The control signals vary between 0 and 1, corresponding to fully closed and fully open, respectively. We want to find the sequence of controls that will give the best overall performance.

Our search techniques rely on BBO combined with brute force. Analytical solutions are intractable since the prosthetic system is nonlinear and time-varying. Since we do not have a power source that provides torque to the knee other than the spring in the high pressure accumulator, we must store and release energy selectively so as to not deplete the stored energy or lose energy expenditure capability at points that might cause the prosthesis to collapse, cause the knee angle to exceed zero (hyper-extension), or cause angle tracking to be poor.

We provide this brief discussion of the complexity of the prosthetic control problem to justify our assertion that analytical control methods, and static control methods, are unsuitable. Evolutionary algorithms often excel at this type of multidimensional, nonlinear optimization problem. Therefore, we choose BBO, a recently developed EA, to optimize the prosthetic controls. Section 3.1 provides a brief overview of the tuning process before BBO was applied. Section 3.2 gives an overview of BBO and how it can be used to find optimal controls. Section 3.3 provides simulation results.

3.1 Manual Tuning Process

Before we apply BBO for optimization, we perform a manual tuning process to improve control performance which will then be feed into a BBO simulation. The 12 parameters we optimize are the knee valve controls (u_1 and u_2), the high pressure accumulator (HPA) initial volume, the hip proportional gains of the controller (3 each for stance and swing phase), an initial y-offset of the vertical hip position, a y-offset of the vertical hip position during swing phase, and a y-offset of the vertical hip position during stance phase. The addition of a y-offset on the vertical hip position was added to the simulation to prevent a toe stub that kept occurring during swing phase with the idea that a human is capable of slight adjustments to hip position. There are an additional 9 state variable initial conditions, but we found through trial and error that these variables have less impact on our simulation results and are not the focus of our work. For the manual tuning process, we run the simulation for one stride and use a brute force approach. The primary means of performance measurement was the cost value, which is discussed further in Section 3.2, but we also perform a visual inspection of the knee angle, thigh angle, and HPA volume plots.

3.2 Biogeography-Based Optimization

BBO is an evolutionary algorithm that has solved optimization problems more effectively than many other evolutionary algorithms [17]. BBO has also solved real-world application problems such as ECG signal classification [18], power system optimization [28], groundwater detection [29], and satellite image classification [30]. BBO is based on the science and study of species migration from one habitat to another. Habitats have different levels of suitability for various species. This is called the habitat suitability index (HSI) of a particular habitat. Habitats with a high HSI tend to have a large number of species, and habitats with a low HSI tend to have a low number of species. Species will immigrate to, and emigrate from, a habitat with a probability that is determined by the HSI. A habitat with a large number of species (high HSI) will tend to have a low immigration rate and a high emigration rate. Conversely, a habitat with a low number of species (low HSI) will tend to have a high immigration rate and low emigration rate. Figure 5 shows the migration curves (actually straight lines) for BBO. Nature will optimize the number of species living in each habitat to achieve equilibrium.

Now picture each habitat as a candidate solution to an optimization problem, and picture each species as a distinguishing feature (independent variable) of that candidate solution. In BBO, each candidate solution shares its features with other candidate solutions, and this sharing process is analogous to migration in biogeography. As migration occurs for many cycles (that is, many generations), the habitats become more suitable for their species, which corresponds to candidate solutions providing increasingly better solutions to an optimization problem. We also implemented common EA concepts in BBO such as elitism and mutation, which we discuss in more detail later in this section.

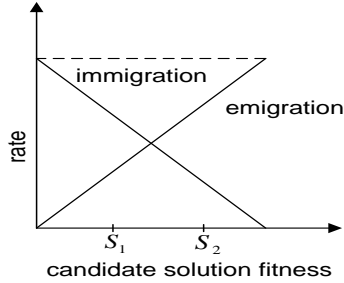


Figure 5: BBO migration curves. This shows two candidate solutions to the same problem. S_1 is a relatively poor solution, and S_2 is a relatively good solution.

In order to use BBO to solve the prosthetic knee control problem, we need to decide two things. First, what to use as features of a candidate control solutions. Second, we need to decide what cost function to use. Our prosthesis candidate control solutions consist of the two valve control signals for the entire period of the gait cycle. Assuming a gait period of $T = 1.26$ seconds, as obtained in our lab from able-bodied test subjects, and assuming a 100 Hz control signal, this requires 126 values for each control signal. In order to reduce the size of the search space and to bias the controls to smooth functions, we represent each control signal as a Fourier series. The Fourier series can point-wise approximate any continuous, periodic, integrable function to any degree of accuracy [31]. The formula for one of the control signals, with a similar formula for the second control signal, is

$$u_1(t) = \frac{a_0}{2} + \sum_{n=1}^{12} [r_n \cos(2\pi n t / T + \theta_n)] \quad (8)$$

The control signals saturate at 0 (fully closed) and 1 (fully open). We compared control signals generated by a Fourier series to those generated by other functions: piecewise linear functions, piecewise constant functions, and cubic splines. Our studies (not shown here) indicate that the Fourier series representation perform best, based on visual comparisons between prosthesis angles and reference angles. As seen in Equation 6, we use 25 coefficients in the Fourier series of each control. Our experiments show that this number of coefficients provides enough resolution to thoroughly search the space of control signals, while not unduly increasing the size of the search space. We chose Fourier coefficients from a polar search space to ensure that the phase for the resulting waveforms is picked from a uniform distribution.

The ranges used are the following: $0 \leq a_0 \leq 2$, and $0 \leq r_n \leq 1$, and $-\pi \leq \theta_n \leq \pi$ for $n > 0$. We know that the control signal must be between 0 and 1 and we want to limit the search space so that a good control can be found with a reasonable amount of computational effort from our BBO algorithm. We found these ranges of coefficient values to provide an appropriate balance between performance and computational effort. Every 0.01 seconds we evaluate the Fourier series for each control and use those values as a constant control for the next 0.01 seconds. This simulates the operation of a zero-order hold microcontroller, which updates the control signals at 100 Hz.

We assign a cost value to each candidate solution. In EAs, the terms “cost” and “fitness” are often used. Generally we want to minimize cost and maximize fitness, two different but functionally equivalent optimization approaches. In this paper we use the convention that we want to minimize cost. That is, as a candidate solution improves, its cost decreases. Our cost function includes the HPA volume difference between the beginning and end of the gait cycle, the thigh angle tracking errors, the knee angle tracking errors, and the amount by which the knee angle exceeds zero. We include the HPA volume in the cost function because we want the HPA volume to be periodic for effective operation over multiple gait cycles. We include the amount by which the knee angle exceeds zero to prevent the prosthetic leg from bending backwards. The cost function is therefore given as

$$J = \int_{t=0}^T [w_1(\phi_1(t) - \phi_{1ref}(t))^2 + w_2(\phi_k(t) - \phi_{kref}(t))^2 + w_3(x_h(t) - x_{href}(t))^2 + w_4(y_h(t) - y_{href}(t))^2 + w_5(\dot{x}_h(t) - \dot{x}_{href}(t))^2 + w_6(\dot{y}_h(t) - \dot{y}_{href}(t))^2 + w_7 U(\phi_k(t)) \phi_k(t)] dt \quad (9)$$

Mutation is a process that probabilistically mutates features of a candidate solution to increase diversity in the population [17]. At each generation, each candidate solution feature has a 5% probability of mutation. If a solution feature is selected for mutation, then it is replaced with a random number uniformly distributed between the minimum and maximum of its search domain.

BBO runs with two elites in our simulations. Elitism involves saving some of the best solutions of the current generation to insert into the population of the next generation. This ensures that BBO will never lose the best solutions from one generation to the next, and the lowest cost value reported at each generation will never increase from one generation to the next. We chose our population size and number of generations based on computational effort and the effect of diminishing returns. Experience shows that for the prosthetic control optimization problem, a BBO run of 100 generations with 100 individuals can find a good solution while not wasting valuable computation time on unneeded generations, or on an unnecessarily large population. The vast majority of the computational effort of the BBO algorithm, as in most

real-world EAs, consists of cost function evaluations (that is, prosthesis control simulations).

3.3 Open-Loop Control Results

Figure 6 shows the best cost at every generation of the BBO algorithm. We reinitialize the population at certain intervals to widen the search space, and to avoid becoming trapped in a local minimum. We keep some of the best results from the previous generation's population to avoid losing good candidate solutions.

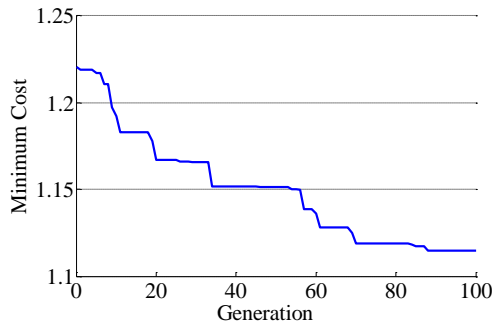


Figure 6: This shows the lowest value of our cost function for the entire population in each BBO generation.

Figure 7 shows the thigh angle tracking that BBO achieved after 100 generations and the subsequent knee angle tracking is shown in Figure 8. The RMS error of the thigh angle is 10.68 degrees, and the RMS error of the knee angle tracking is 25.29 degrees. We see the thigh angle tracks well through stance phase and that most of the RMS error occurs near the end of swing phase before the leg hits the ground. Note that our starting point for a second stride is close to the initial hip position which is what we would expect given the periodic nature of the human gait.

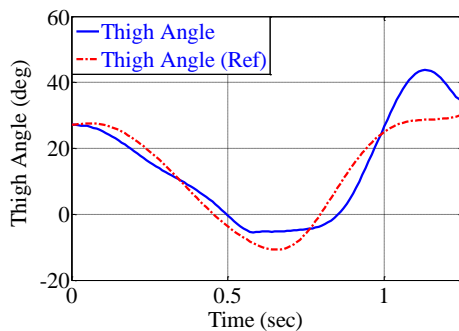


Figure 7 shows the thigh angle tracking for both our BBO simulation results and the able bodies reference data. We little error through the completion of stance phase, and despite the larger error seen at the end of swing phase, our final hip position is in good position to begin a second stride.

Although the knee angle tracking in Figure 8 does not appear to be close, we show in Figure 9 that a walking motion is achieved. We see good tracking at the beginning of stance phase, but the knee does not reach the knee bend we see on the reference data during stance. As the leg begins to enter swing phase, we do see a fuller knee extension that nearly matches the able bodied reference data. The lack of negative

knee angle during swing was a contributing factor to the previously mentioned toe stubs, and as with the thigh position, we see the final knee angle to closely match the initial position of the knee putting the leg in near ideal conditions for a second stride.

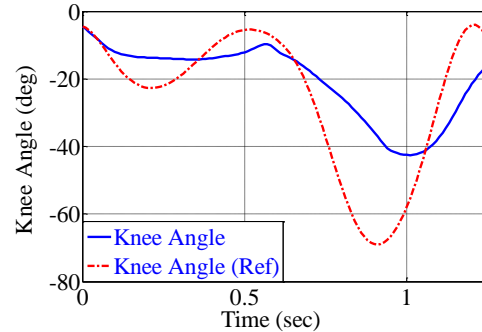


Figure 8 displays knee angle tracking of our BBO simulation along with the able bodied reference data. Knee angle tracking proves to be much harder to achieve, yet we see our final conditions close to the initial conditions which suggests we see a periodic movement.

While the tracking results from Figure 7 and 8 suggest that further optimization is possible, we present the simulation results in the form of a 'walking stick figure' in Figure 9. The top plot in Figure 9 is of the able bodied reference data, and the lower plot is our simulation results that correspond to the tracking data in Figures 7 and 8. We see the reference foot to be higher off the ground than our simulation results, and this is indicative of our inability to achieve the high negative angle that is seen from the knee angle reference data in Figure 8.

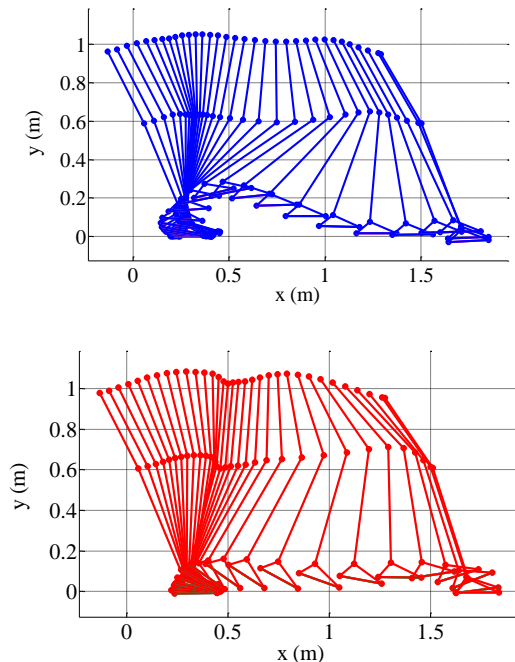


Figure 9: the top plot shows the reference data with the bottom plot showing the simulation stride produced after 100 BBO generations.

As humans walk in many different styles with many different variances in gait, we must keep in mind that perfect knee and thigh angle tracking may not be possible for even two able bodies individuals. It is important that we achieve a walking motion that limits the stress a transfemoral amputee may see on their good leg. Figure 9 shows that despite the RMS error in thigh and knee angle tracking, we are capable of finding control parameters that will produce a walking motion.

4. Conclusions and Future Work

We have proposed a new hydraulic knee design, and have shown that BBO is able to generate near-optimal solutions for our cyber-physical system. The control solution provides reasonable knee and thigh angle tracking while requiring continuous interaction of the human and machine aspects in our CPS.

While computer simulations offer an invaluable tool in the optimization of our cyber-physical system controls, it is necessary that our research also include physical testing of the CPS which includes both the verification and validation of the actual knee prototype. Due to logistical and safety issues that arise with human amputee testing, we avoid this dilemma through the construction of a hip robot capable of simulating various human gaits. Our test plan is to apply the optimal controls found through simulation to the hip robot. This too offers limitations, however, as continued maintenance and replacement of key components are required to extend the life of the robot beyond a few months. We solve this problem by adding a model of the hip robot to our simulation. We are then able to accurately test the knee performance without actually applying stress to the robot. Current work includes applying BBO to find optimal open-loop robot controls as well as the implementation of the embedded systems controller that gives us a smart cyber-physical system. Future work includes the use of our open-loop controls in conjunction with feedback control to provide a more robust control solution.

Closed-loop control is required to obtain a robust knee prosthesis controller. Several intelligent control methods show promise in this area, including artificial neural networks and fuzzy logic. These options are attractive because of universal approximation theorems [33] and because they mimic the way that humans control natural knees. Neural networks and fuzzy logic can both be tuned with either gradient descent, or with an evolutionary algorithm such as BBO [32].

Other issues that need to be addressed by a prosthetic implementation include sensor selection for closed-loop control [34] and gait phase recognition [35], [36], [37], [38]. Also, although we have developed controls only for a normal walking gait, a commercial prosthesis needs to function correctly in various operating modes. A commercial prosthesis also needs to implement user intent recognition [39], [40], and stumble detection and recovery [40], and it needs to have a reliable and long-lasting power source [41].

Acknowledgments

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